Adomian Approximation Approach to Thermal Radiation with Heat Transfer Effect on Compressible Boundary Layer Flow on a Wedge*

F. I. Alao and S. B. Folarin

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Abstract

This paper investigates the effect of radiation with heat transfer on the compressible boundary layer flow on a wedge. Fluid viscosity is assumed to be negligible. The compressible boundary layer equations were transformed using Stewartson transformation. The resulting partial differential equations were further transformed using dimensionless and similarity variables. A third order and a second order coupled with non-linear ordinary differential equation systems corresponding to the momentum and the energy equations were obtained. An approximate analytical method (Adomian decomposition) was used to solve the coupled non linear equations. The result shows that, effect of radiation contributes to the increase in velocity field and decrease in temperature field of the boundary layer flow. The significant finding of this study is that, due to variable thermal radiation, flow separation is controlled. The temperature decreases with increasing value of radiation parameter with fixed Prandtl number and Falkner Skan parameter.

1 Introduction

A body which is introduced into a fluid at different temperature forms a source of equilibrium disturbance due to the thermal interaction between the body and the fluid. Thermal radiation is the movement of heat energy by electromagnetic waves which are composed of radio and light waves. Within the field of aerodynamics, the analysis of boundary layer problem for two dimensional steady and compressible laminar flows passing a wedge is a common area of interest. Also, the study of hydrodynamics flow and heat transfer along a wedge has gained considerable attention due to its vast applications in industry and its important bearing on several technological and natural processes. Though, literature has shown that successful studies had been carried out, but with a limitation of assuming constant thermal radiation on the effect of heat transfer on forced convective boundary layer flow past a wedge in a viscous compressible fluid, several authors have studied the effect of heat transfer on compressible boundary layer flow of various kinds of dynamic systems.

Viskanta (1925) investigated the interaction between conduction, convection and radiation in a fully developed laminar flow of an absorbing, emitting, gray gas between two diffuse, non black, isothermal, parallel surfaces. The two-dimensional non linear integro-differential equation, namely, the energy equation was solved by an approximate solution based on the Taylor series expansion.

The wedge flow was investigated for the first time by Falkner and Skan (1930). They considered two dimensional incompressible wedge flow. Hartee (1937) later investigated the same problem and found the similarity solution for different values of the wedge angle. Koh and Hannet (1945) investigated the incompressible laminar heat transfer of wedge flow and used a differential transformation method where as the transient heat transfer boundary layer flow on a wedge with sudden change of thermal boundary conditions of uniform wall temperature and heat flux. Hsu *et al.*, (1990) studied the temperature and flow field of the flow past a wedge by the series expansion method. Hossain *et al.*, (1996) made large number of investigations on free, forced and mixed convective flow over a wedge. Hossain (1998) investigated the effect of heat transfer on compressible boundary layer flow over a circular cylinder. Hossain (2000) considered the flow of viscous incompressible fluid with temperature dependent viscosity and thermal conductivity past a permeable wedge with uniform surface heat flux. Seddeek and Salem (2002) discussed the effect of variable viscosity and thermal diffusivity on mixed convection flow along vertical stretching sheet. Mukhopadhyay (2009) investigated the effect of radiation and variable fluid viscosity on flow and heat transfer along a symmetric wedge.

This paper proffers solution to the radiant heat transfer on compressible boundary layer flow on a wedge using Adomian decomposition technique.

2 Mathematical formulation

The steady two-dimensional compressible laminar boundary layer flow over a wedge is considered. The

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fluid on the wedge is flowing through the entire surface or locally from various locations on the surface of the wedge.

Where $velocity = U_e(x) = U_\infty \left(\frac{x}{l}\right)^m$ for $m \neq 1$

where L is the characteristic length and m is the Falkner Skan exponent related to the included angle

$\varnothing B$ by $m = \dfrac{b}{2-b}$ for $m < 0$.

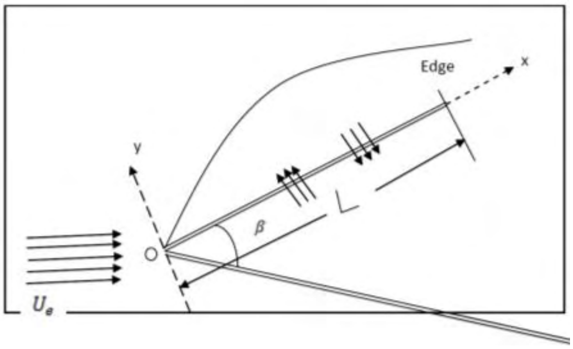

Fig. 1 Flow Configuration and Coordinate System for the Wedge.

Under the above assumption, the equations governing this type of flow are in the form.

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0 \qquad 2.1$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \qquad 2.2$$

$$\ell Cp\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)-u\frac{\partial p}{\partial x}=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho Cp}\left(\frac{\partial q_r}{\partial y}\right) \qquad 2.3$$

Together with $P=\rho RT, \mu=\mu_0\left(T/T_0\right)$ 2.4

and Boundary Conditions are $u=v=0$, $T=T_w$ at $y=0$ 2.5

$$u=u_1, \qquad T=T_1 \qquad \text{at } y=\infty \qquad 2.6$$

$$u_1(x)=u_\infty Sin\left(x/a\right) \qquad 2.7$$

where, T_w is the constant wall temperature, (x,y) are the Cartesian coordinates with x and y axes along and normal to the surface of the cylinder respectively, (u,v) are the velocity components along x and y axes, p is the pressure, ρ is the density, k is the thermal conductivity, C_p is the specific heat at constant pressure, R is the gas constant and the suffix o, refers to some standard state, say, $x=0$.

Stewartson transformation variables are given below as;

$$\text{(a)}\quad Y=\frac{a_1}{a_0\sqrt{v_0}}\int_0^y\frac{\rho}{\rho_0} \qquad \text{(b)}\ \rho u=\rho_0\sqrt{v_0}\frac{\partial\psi}{\partial y} \qquad 2.8$$

where ψ is the stream function, a_1 and a_0 are velocities of sounding the main stream.

Then,

$$u=\left(\frac{a_1}{a_0}\right)\frac{\partial\psi}{\partial Y}\ \text{(i)} \qquad v=-\left(\frac{\rho_0}{\rho}\right)\sqrt{v_0}\left(\frac{\partial\psi}{\partial x}+\frac{\partial\psi}{\partial Y}\cdot\frac{\partial Y}{\partial x}\right)\ \text{(ii)} \qquad 2.9$$

$$\mu\frac{\partial u}{\partial y}=\frac{\mu_0 P}{P_0}\frac{a_1^{\,2}}{a_0^{\,2}\sqrt{\mu_0}}\frac{\partial^2\psi}{\partial Y^2}\ \text{(i)} \qquad \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)=\frac{\ell}{\ell_0}\frac{a_1^{\,3}}{a_0^{\,3}}\frac{\partial^3\psi}{\partial Y^3}\ \text{(ii)} \qquad 2.10$$

$$k\frac{\partial T}{\partial y}=\mu_0 Cp\frac{P}{P_0}\frac{a_1}{a_0\sqrt{v_0}}\frac{\partial T}{\partial Y} \qquad 2.11$$

with, $$\frac{\rho}{\rho_0}=\left(\frac{a_1}{a_0}\right)^{\frac{2\gamma}{\gamma-1}} \qquad 2.12$$

Applying these Stewartson transformation variables of Equations (2.8) to (2.12) on Equations (2.1), (2.2) and (2.3), we obtain,

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=\left(\frac{a_1}{a_0}\right)^{\frac{3\gamma-1}{\gamma-1}}\left[\frac{\partial^2\psi}{\partial x\partial Y}\rho_0-\frac{\partial^2\psi}{\partial x\partial Y}\rho_0\right] \qquad 2.13$$

$$\left(\frac{a_1}{a_0}\right)^2\left[\frac{\partial^2\psi}{\partial x\partial Y}\frac{\partial\psi}{\partial Y}-\frac{a_1}{a_0}^{\frac{2\gamma-1}{\gamma-1}}\frac{\partial^2\psi}{\partial Y^2}\frac{\partial\psi}{\partial x}\right]=-\frac{1}{\rho}v_1\frac{\partial v_1}{\partial x}\frac{T}{T_1}+\frac{1}{\rho}\left(\frac{a_1}{a_0}\right)^{\frac{5\gamma+3}{\gamma-1}}\frac{\partial^3\psi}{\partial Y^3} \qquad 2.14$$

$$\rho C_p\left(\frac{\partial\psi}{\partial Y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial Y}\frac{\partial T}{\partial Y}\right)=C_p\left(\frac{a_1}{a_0}\right)^{\frac{3\gamma-1}{\gamma-1}}\frac{\partial^2 T}{\partial Y^2}-\frac{1}{\rho C_p}\left(\frac{a_1}{a_0}\right)^{-1}\frac{\partial q_r}{\partial y} \qquad 2.15$$

Using the Roseland approximation for radiation (Brewester (1972), Mukhopadhyay (2009)) the radiative heat flux can be written as,

$$q_r=\frac{-4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \qquad 2.16$$

where, k^* is the absorption coefficient and σ is the Stefan-Boltzman, assuming the temperature within the flow is such that T^4 may be expanded in Taylor series about T_∞ and neglecting the higher orders terms we have,

$$T^4=4T_\infty^{\ 3}T-3T_\infty^{\ 4} \qquad 2.17$$

where, T_∞ is the free stream temperature.

Also, considering the functions relating to the absolute temperature T, with Mach number relation, we have,

$$\left(1+\frac{\gamma-1}{2}M_1^{\ 2}\right)S=\frac{T}{T_1}-\frac{\gamma-1}{2}M_1^{\ 2}\left(1-\frac{u^2}{U_1^{\ 2}}\right)-1 \qquad 2.18$$

where, $M_1 \ll 1$ (subsonic flow) $M_1=\frac{v}{a}$

We sufficiently consider a flow in which the Mach number <<1, replacing the factor a_0/a_1 by unity and substituting Equations (2.16) - (2.18) into Equations (2.13) to (2.15), the equations describing the flow and heat transfer reduce to,

$$\frac{\partial^2\psi}{\partial x\partial Y}\frac{\partial\psi}{\partial Y}-\frac{\partial^2\psi}{\partial Y^2}\frac{\partial\psi}{\partial x}=\frac{\partial^3\psi}{\partial Y^3}+\frac{T}{T_1}v_1\frac{\partial v_1}{\partial x} \qquad 2.19$$

$$\frac{\partial\psi}{\partial Y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial Y}=\frac{\partial^2 T}{\partial Y^2}+\frac{1}{(C_p)^2}\frac{1}{v_0}\frac{16\sigma T_\infty^{\ 3}}{3k^*}\frac{\partial^2 T}{\partial Y^2} \qquad 2.20$$

Using dimensionless variable in Equations (2.20),

$$\theta=\frac{T-T_\infty}{T_w-T_\infty} \qquad 2.21$$

Equations (2.19) and (2.20) become,

$$\frac{\partial^2\psi}{\partial x\partial Y}\frac{\partial\psi}{\partial Y}-\frac{\partial^2\psi}{\partial Y^2}\frac{\partial\psi}{\partial Y}=\frac{\partial^3\psi}{\partial Y^3}+\frac{T}{T_1}v_1\frac{\partial v_1}{\partial x} \qquad 2.22$$

$$\frac{\partial\psi}{\partial Y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial Y}=\frac{\partial^2\theta}{\partial Y^2}+R_a\frac{\partial^2\theta}{\partial Y^2} \qquad 2.23$$

With boundary conditions,

$$\psi=\frac{\partial\psi}{\partial Y}=0, \theta=0 \qquad \text{at } Y=0 \qquad 2.24$$

$$\frac{\partial\psi}{\partial Y}\to Ue=x^m, \theta=0 \qquad \text{at } Y=\infty \qquad 2.25$$

where, $R_a=\frac{1}{(C_p)^2}\frac{1}{v_0}\frac{16\sigma T_\infty{}^3}{3k^*}$ (radiative parameter)

Considering the similarity variables,

$$\psi(x,Y)=x^\alpha F(\eta) \qquad 2.26$$

$$S=\theta(x,Y)=G(\eta) \qquad 2.27$$

where, $\eta=\frac{Y}{x^\gamma}$,

$$\text{Let, } \frac{(T_w-T_\infty)}{T_1}=\alpha \text{ and } \frac{T_\infty}{T_1}=\alpha_2. \qquad 2.28$$

$$\text{Then, } 1+S=G\alpha_1+\alpha_2 \qquad 2.29$$

Substituting equations (2.26) to (2.29) into equations (2.22) to (2.25), yields,

$$F'''=-\left(\frac{1+m}{2}\right)FF''+mF'^2-m\alpha_1 G+m\alpha_2 \qquad 2.30$$

$$G''=-\frac{3}{2}\left(\frac{1+m}{3\Pr+4Q}\right)\Pr FG' \qquad 2.31$$

with boundary conditions,

$$F(0)=0, F'(0)=0, G(0)=1 \text{ at } \eta=0 \qquad 2.32$$

$$F'(\infty)=1, G(\infty)=0 \text{ at } \eta=\infty \qquad 2.33$$

4 Method of Solution: Adomian Decomposition Method

According to Makinde *et al* (2007), the Adomian decomposition method is useful for obtaining the closed form and numerical approximations of linear or nonlinear differential equations. This method has been applied to obtain formal solutions to a wide class of stochastic and deterministic problems in science and engineering involving algebraic, differential, integro-differential, differential delay, integral and partial differential equations.

The method of Adomian decomposition introduced by the American mathematician, G Adomian, (1923–1996) is based on the search for a solution in the form of a series and on decomposing the nonlinear operator into a series in which the terms are calculated recursively using Adomian polynomials. (Adomian 1994; Adomian 1988; Adomian and Rach 1991). This technique has many advantages over classical techniques. It avoids perturbation in order to find solutions of given nonlinear equation.

This method provides an accurate approximation of the solution. As a main advantage of this method over traditional numerical methods, the decomposition procedure of Adomian does not require discretization of the solution. Unlike other numerical methods, this method does not result in any large system of linear or nonlinear equations, therefore, Adomian decomposition method provides a closed form of the solution.

The Adomian decomposition method is applied to find the solution of Equations (2.30) and (2.31) satisfying the boundary conditions in Equations (2.32) and (2.33).

$$F = A + B\eta + \frac{C\eta^2}{2} - L^{-1}\left(\frac{1+m}{2}\right)FF'' - L^{-1}mF'^2 + m\alpha_1 L^{-1}G + mL^{-1}\alpha_2 \qquad 4.1$$

$$G = D + E\eta - L^{-1}\frac{3}{2}\left(\frac{1+m}{3\Pr + 4Q}\right)\Pr FG' \qquad 4.2$$

where, $L^{-1} = \iiint (.)\,d\eta d\eta d\eta$ 4.3

By decomposition, we have,

$$F = \sum_{n=0}^{\infty} Fn = F_0 + F_1$$

$$G = \sum_{n=0}^{\infty} Gn = G_0 + G_1 \qquad 4.4$$

$$F_0 = A + B\eta + \frac{c\eta^2}{2} + L^{-1}m\alpha_2 \qquad 4.5$$

$$F_1 = -L^{-1}\left(\frac{1+m}{2}\right)F_0F_0'' - L^{-1}mF'^2{}_0 + m\alpha_1 L^{-1}G_0 \qquad 4.7$$

Also, $G_0 = D + E\eta$ 4.8

$$G_1 = -L^{-1}\frac{3}{2}\left(\frac{1+m}{3\Pr + 4Q}\right)\Pr F_0G_0' \qquad 4.9$$

Therefore, the general term for the Adomian solution to Equations (2.30) and (2.31), is given as,

$$F_{n-1} = -L^{-1}\left(\frac{1+m}{2}\right)\sum_{n=0}^{\infty} An - L^{-1}m\sum_{n=0}^{\infty} Bn + L^{-1}\alpha_1\sum_{n=0}^{\infty} G_0 \qquad 4.10$$

$$G_{n+1} = -L^{-1}\frac{3\Pr}{2}\left(\frac{1+m}{3\Pr + 4Q}\right)\sum_{n=0}^{\infty} An \qquad 4.11$$

For n ≥ 0, we obtained values of F

applying boundary conditions on equations on equations (4.6) and (4.8) we have

A=0, B=0 and $F''(0) = C = \gamma$ also D=1 and $G'(0) = E = \sigma$

Where An, Bn are gotten through Adomian polynomials.

Using the computer programming tool (MATLAB), we iterate equations (4.10) and (4.11), and the result is presented graphically in Figs 2 to 5.

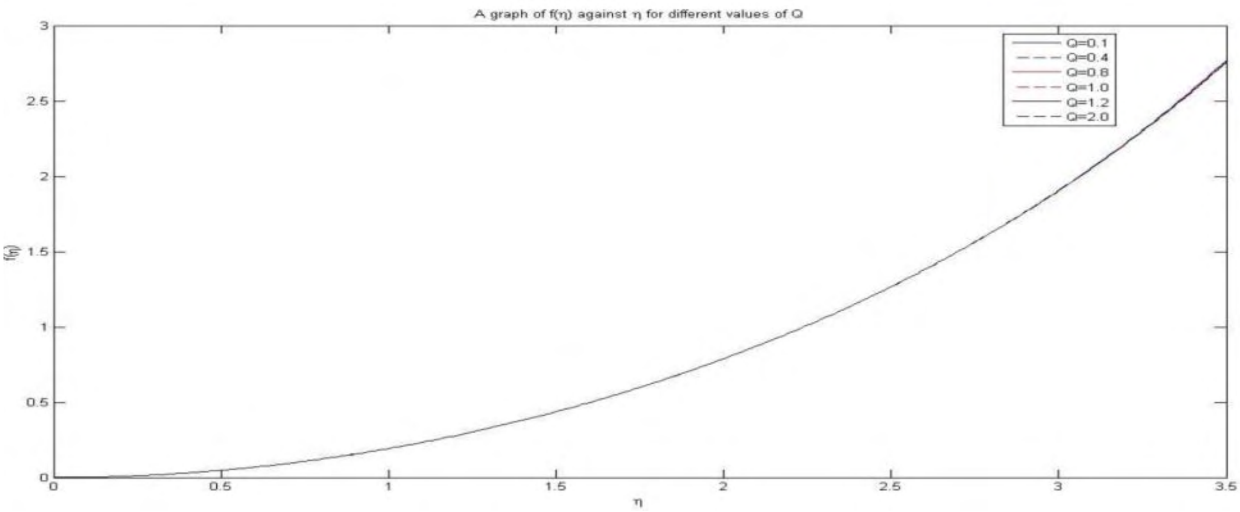

Fig. 2 Variation of Velocity Profile f(ɖ) with ɖ for several values of Qôs with oth

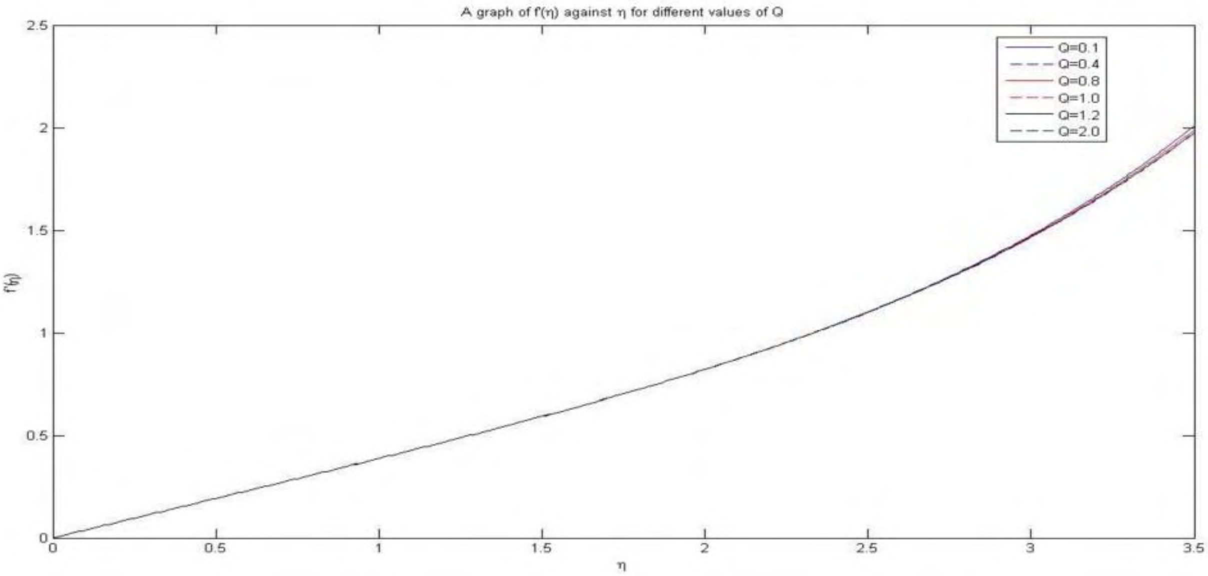

Fig. 3 Variation of Velocity Profile F^I(ɖ)with ɖ f o r s e v e r a l v a l u e s o f Q ô s w i t h o

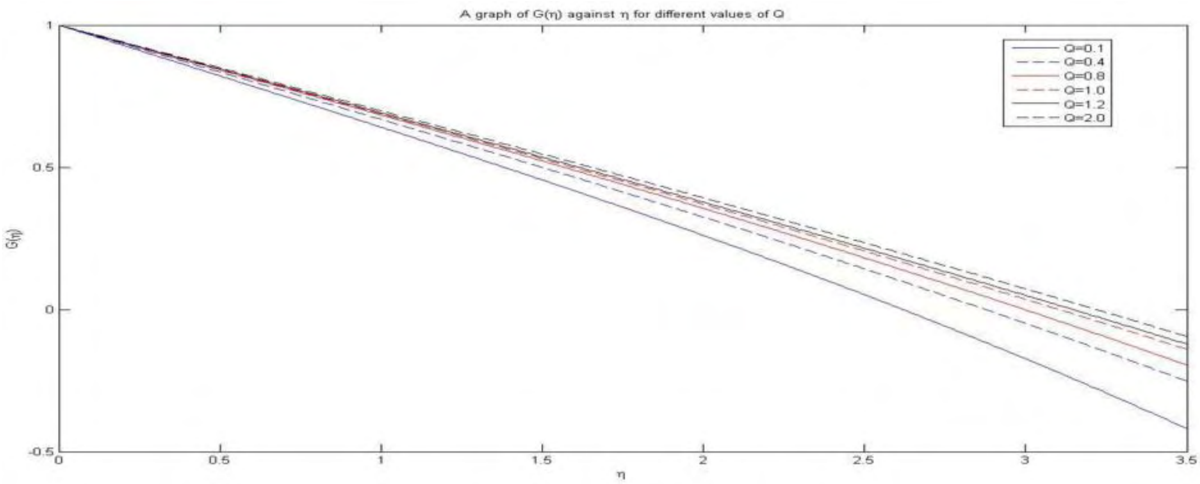

Fig. 4 Variation of Temp Profile G(ɖ)with ɖ f o r s e v e r a l v a l u e s o f Q ô s w i t h o

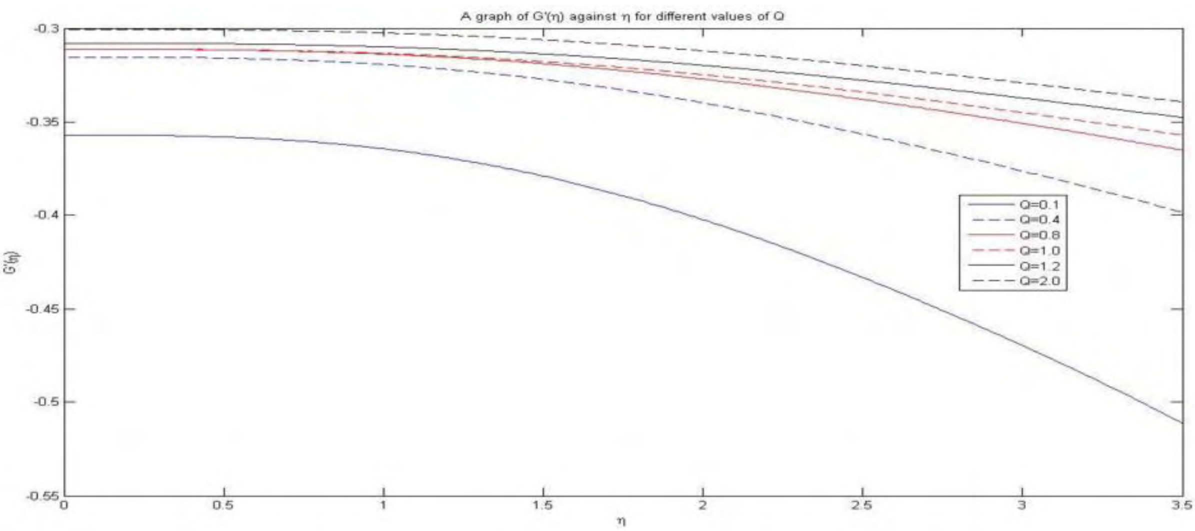

Fig. 5 Variation of Temp Profile G^I(ɖ)with ɖ f o r s e v e r a l v a l u e s o f Q ô s w i t h o

5 Results and Discussion

In order to analyze the result, numerical computation has been carried out for various values of parameters, α_1, α_2, m, Q and prandtl number (that is, m = 0.01, $\alpha_1 = 0.2$, $\alpha_2 = 0.1$, Pr = 0.5 and Q = 0.1), while some others parameters were fixed.

For clearer interpretation of the results, the numerical values were plotted as shown in Figs. 2 to 5. The accuracy of the results of this method was tested by comparing the results with those available in the literatures. Also, the results (in the absence of thermal radiation) also agreed with those in related literatures. (Mukhopadhyay, 2009).

Fig. 2 shows the results of the variation of the thermal radiative term with fixed Falkner Skan exponent m and the prandtl number Pr in the region of the boundary layer of the wedge. It is discovered that increase in radiation causes increase in fluid velocity along the wedge and there is uniform increase in velocity along the boundary layer flow of the wedge, despite the varying thermal radiation in the wedge.

Figure 3 gives similar interpretation between the velocity field and the radiation in the boundary layer of the wedge, but slight change in the in radiation as the heat transfer is moving from lower region to the upper region of the boundary layer of the wedge causes the increase in fluid velocity.

Also, Fig. 4 demonstrates the effects of radiation parameters Q on temperature field in the presence of some fixed parameter. The temperature $G(\eta)$ decreases as thermal radiation Q decreases. This is in agreement with the physical fact that the thermal boundary layer decreases with decreasing Q. The effect of radiation in the thermal boundary layer is effective in the boundary layer flow.

In Fig. 5, the rate of decrease in temperature $G(\eta)$ is very wide as thermal radiation decreases in the boundary layer of the both region of the wedge.

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Authors

Dr. Alao, Felix Ilesanmi is an Applied Mathematician. His area of research is Fluid Mechanics and Combustion theory. His PhD work, Self-Similar Solution for Viscous Reacting Flows, was carried out at Department of Mathematical Sciences, Federal University of Technology, Akure, Nigeria. He started his teaching career as a Graduate/ Research Assistant in 1992, at Department of Mathematical Sciences, Federal University of Technology, Akure, Nigeria. He is presently a Reader (Assoc. Professor) in the same institution. He has also worked as a visiting Senior Lecturer at University of Mines and Technology (UMaT), Tarkwa, Ghana (2009 – 2010). He has attended many local and international conferences and summer schools. He has many research publications in both local and international journals.

Folarin Samson. B is an applied Mathematician in the area of Fluid dynamics, and currently a research assistant at Department of Mathematical Sciences, King Fahd University of Petroleum and Minerals Saudi Arabia and will soon proceed to United State of America to continue his PhD research program. He holds a B.Tech in Industrial Mathematics and M.Tech in Applied Mathematics, both from, The Federal University of Technology Akure, Nigeria. His present activity and research include Fluid Transport through porous media.