

# Determination of short circuit stresses in an air core reactor using finite element analysis

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## Abstract

This paper shows the use and effectiveness of finite element method while designing an air core reactor for determining the short circuit forces and stress level due to short circuit. A 500 Amp air core series reactor having nominal voltage rating of 600 Volt was to be designed and to be subjected to a short circuit current of 8 KA for 2 second. A finite element approach was adopted in designing the stress level due to short circuit forces and the results were utilized in developing a reactor that finally withstood the short circuit test.

**Keywords:** Reactor, Short circuit forces, von Mises stress, Finite Element Method, Inductor

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## 1. Introduction

The use of finite element method is evident since long for various applications such as determination of forces in a civil structure, stress analysis in a mechanical structure, effect of vibration and shocks on a structure or components, thermal analysis of an equipment or part therein etc. The FEM is being also effectively utilized in analyzing the performance parameters of electrical equipments such as transformers, generators, inductors etc. Kladas *et al.* (1994) represent one such application of FEM for calculating the leakage flux and force on a power transformer winding under short circuit condition. Reddy and Vijaykumar (2008) have evaluated life of a power transformer by determining the hottest spot in the transformer using FEM. Using quasi-static finite element and circuit based method. Barzegaram *et al.* (2009) carried out the frequency response analysis in power transformer for detecting the winding short circuits due to inter-turn faults or disk to disk faults. Pierce (1995), Hong *et al.* (2005), Bell and Bodger (2007) and Tsili *et al.* (2008) also describe various approaches for designing transformer using FEM.

Kumbhar and Kulkarni (2007) presented short circuit analysis of a split winding transformer using coupled field circuit approach. In their study they estimated short circuit forces on an individual winding using FEM. However, actual deformation due to such short circuit was not estimated. These mechanical forces have been taken care of by Ahn *et al.* (2012) on a 50 KVA dry type transformer with experimental verification.

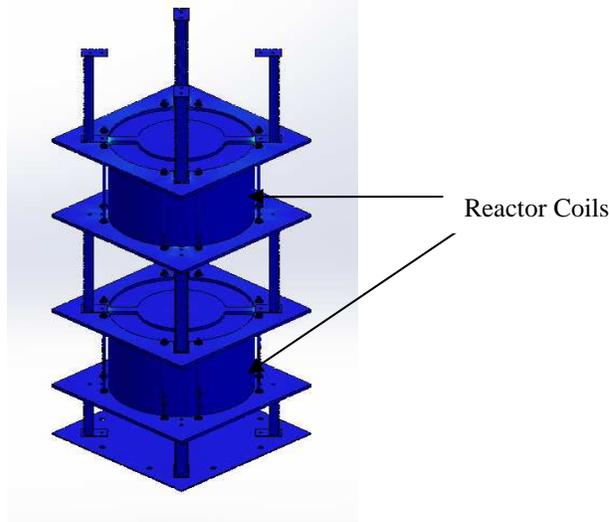
In this paper, FEM has been used to determine the short circuit forces in an air core, single phase, 50 Hz, 600 V, 500 A, (continuous duty) dry type series reactor assembly (stack of two reactors one over other) as shown in fig. 1, when being subjected to thermal short circuit condition at 16 times rated current for 2 sec according to IEC 60076-6, 2007. FEM using Matlab by Kwon *et al.* (2011) and minute details of alternating machine by Say (1983) provided the needed insight. The short circuit forces, radial and axial forces have been estimated using electromagnetic model and then these equivalent mechanical forces were used to determine the stress level using Von Mises stress equations. Using FEM the deformations in the reactor coil in radial and axial direction have been identified along with the stresses on the supports. Based on FEM results, a factor of safety was determined and a reactor coil was developed which then subjected to an actual thermal short circuit testing at CPRI, Bhopal, India.

## 2. Electromagnetic Force Analysis

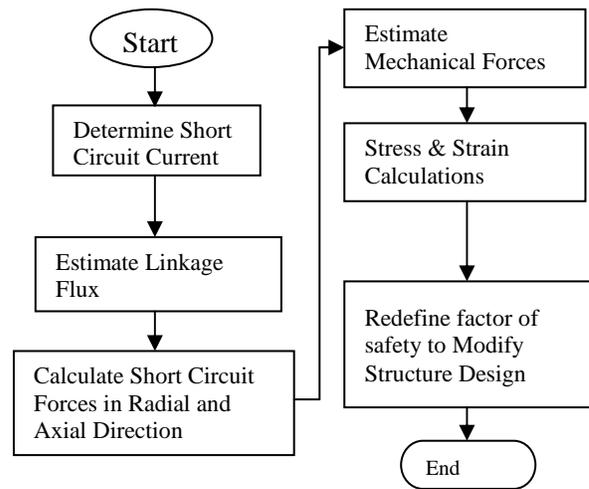
The model of an air core reactor is as shown in Figure 1 and its specifications are as given in Table 1.

**Table 1.** Specification of Air core reactor

Type of Reactor	Air Core Dry Type
Category	Series Reactor
Line Voltage	600 V
Current Rating	500 A
Duty	Continuous
Thermal Short Circuit Ability	16 Times Rated Current For 2 sec
Reference Standard	IEC 60076-6, 2007



**Figure 1.** Reactor Model



**Figure 2.** Flow Chart for Short Circuit Analysis of Reactor

**2.1 Short Circuit Current and Force Calculation:** The Lorentz’s force due to short circuit current and linkage flux causes mechanical forces and deformation in the winding. To estimate the resultant mechanical forces on the winding, a sequential FEM as presented in [1] has been used. The flow chart for the same is shown in Figure 2.

The short circuit current under transient condition can be approximated as

$$I_{sc}(t) = I_o e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2 + X^2}} \cos \omega t \tag{1}$$

Where,  $I_{sc}$  is the short circuit current and  $I_o$  is the initial current in Ampere (A), R, L and X are resistance in Ohm ( $\Omega$ ) Inductance in Henry (H) and reactance in  $\Omega$  respectively.

The value of initial current depends on the instant of switching the circuit. At switching instant  $t = 0$ , if voltage is zero then  $I_o$  will have maximum value and if voltage is at its peak then it will be zero.

The transient magnetic flux depends on the magnetizing characteristics. Since this is an air core reactor, the linking flux will increase in proportion to the short circuit current. The component of the flux using vector potential can be expressed as

$$B_r = -\frac{\partial A_\phi}{\partial z}, B_\phi = 0, B_z = \frac{1}{r} \frac{\partial r A_\phi}{\partial r} \tag{2}$$

According to Lorentz’s force, the expression for electromagnetic force is given as in equation (3). It is unidirectional pulsating type since it is proportional to the square of the current.

$$\vec{F} = F_m \left( \frac{1}{2} + e^{-\frac{2R}{L}t} - 2e^{-\frac{R}{L}t} \cos \omega t + \frac{1}{2} \cos 2\omega t \right) \quad (3)$$

where,  $\vec{F}$  is the electromagnetic force in Newton (N) at any instant t and  $F_m$  represents the maximum value of force in N.

This force  $\vec{F}_t$  comprises of two forces, one in radial direction and other in axial direction. The reactor is consisting of one single coil only and hence the radial force will be in outward direction. The axial force will be having predominant effect in the downward direction for bottom coil and in the upward direction for top coil. This is because the stack of these coils is to be connected with one single phase only. The maximum calculated value of radial force found is 605 N and that of axial force is 363 N. To find out the structural deformation, these forces are used as an input data to the stress strain model using von Mises Criterion.

**2.2. Stress Strain Calculation and Deformation Prediction:** The von Mises yield criterion is used to determine the stress level in the reactor coil during short circuit condition. Mathematically the von Mises yield criterion is expressed as:

$$J_2 = K^2 \quad (4)$$

where,  $K$  is the yield stress of the material in pure shear. At the onset of yielding, the magnitude of the shear yield stress in pure shear is  $\sqrt{3}$  times lower than the tensile yield stress in the case of simple tension. Thus, we have:

$$K = \frac{\sigma_y}{\sqrt{3}} \quad (5)$$

where,  $\sigma_y$  is the yield strength of the material. If we set the von Mises stress equal to the yield strength and combine the above equations, the von Mises yield criterion can be expressed as:

$$\sigma_v = \sigma_y = \sqrt{3J_2} \quad (6)$$

In terms of Cauchy's stress tensor components, this criterion can be expressed as

$$\sigma_v^2 = \frac{1}{2} \left[ (\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{22}^2 + \sigma_{31}^2) \right] \quad (7)$$

This equation (7) defines the yield surface as a circular cylinder whose intersection with the deviatoric plane is a circle. It can be reduced and reorganized for practical use in different loading conditions.

In case of uni-axial stress or simple tension,  $\sigma_1 \neq 0$ , and  $\sigma_2 = \sigma_3 = 0$ , then the von Mises criterion simply reduces to  $\sigma_1 = \sigma_y$  which means that the material starts to yield when normal stress (uni-axial stress) reaches the yield strength of the material.

Using the results obtained from simple uni-axial tensile tests, yielding of material can be predicted under multi-axial loading conditions. In this situation, the equivalent tensile stress or von Mises stress  $\sigma_v$  can be expressed as in equation (8).

$$\sigma_v^2 = \frac{1}{2} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \quad (8)$$

The short circuit forces calculated above are converted into equivalent mechanical load with an adequate factor of safety (SF). The stress/strain values are then computed using von Mises criterion. The stress and strain values have been calculated for the following conditions.

1. Transverse (horizontal/radial) load and Longitudinal (vertical/axial) load with a factor of safety 1.5.
2. Axial and radial load with increased factor of safety (SF 2) by strengthening the mechanical structure and coil supports.

**2.3 Stress and Deformation with Factor of Safety 1.5 for Transverse and Longitudinal Loading:** The determination of factor of safety is done based on the average value of force calculated and stress likely to be exerted on the available surface area of the support structure. It is the ratio of yield stress to the actual stress.

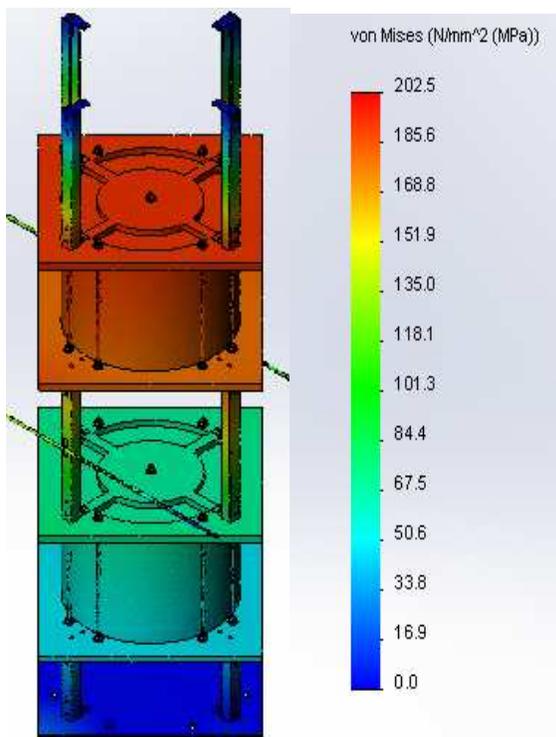
The boundary conditions for these entire analysis are as shown in table II. The Stress, Strain and Deformation values are summarized in table III for a safety factor of 1.5. The values shown are the maximum values obtained during the analysis at a particular node.

**Table 2.** Boundary Conditions

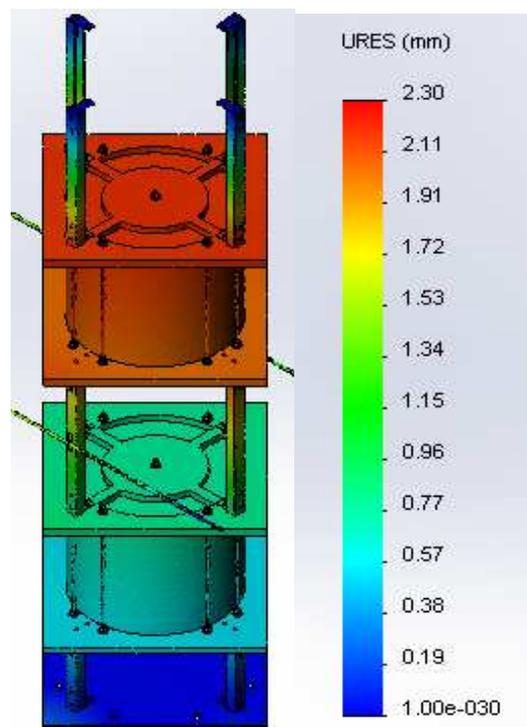
Load Pattern	No. of Nodes	No. of Elements	Boundary Conditions	
			Total Stress in MPa	Total Deformation in mm
Transverse	230029	690087	≤ 270	≤ 2.0
Longitudinal	243154	729462	≤ 70	≤ 1.0

**Table 3.** Stress Strain Values and Deformation for SF 1.5

Load Pattern	Stress in MPa	StrainX10 <sup>-3</sup>	Deformation in mm
Transverse	202.5	4.692	2.300
Longitudinal	74.4	3.560	0.710



**Figure 3.** Stress with Transverse Loading. SF-1.5



**Figure 4.** Deformation with Transverse Loading. SF-1.5

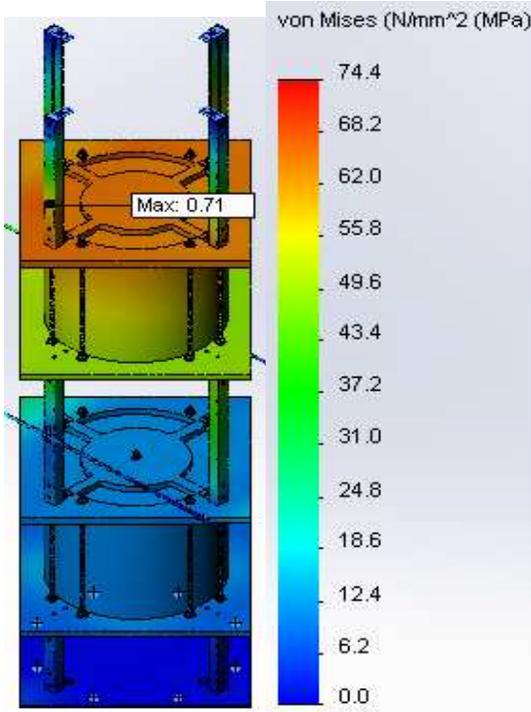


Figure 5. Stress with Longitudinal Loading. SF-1.5

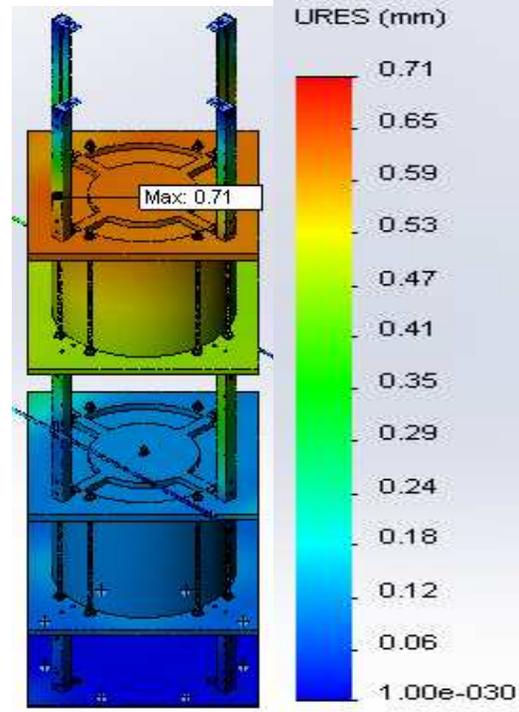


Figure 6. Deformation with Longitudinal Loading. SF-1.5

2.4 Stress and Deformation with Factor of Safety 2.0 for Transverse and Longitudinal Loading: Based on the results obtained for a safety factor of 1.5 it was decided to increase it to 2, since in Longitudinal loading with a safety factor of 1.5, the stress developed has crossed the boundary condition limit of 70 MPa whereas the deformation obtained in Transverse loading has also crossed the boundary condition limit of 2.0 mm.

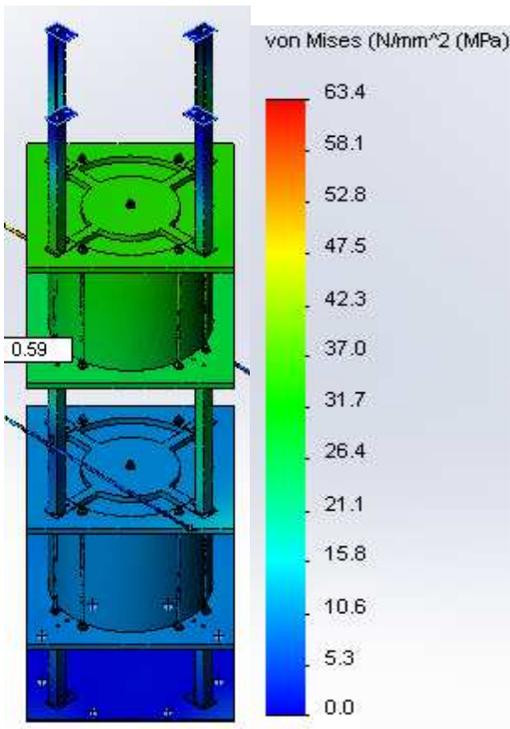


Figure 7. Stress with Transverse Loading. SF-2

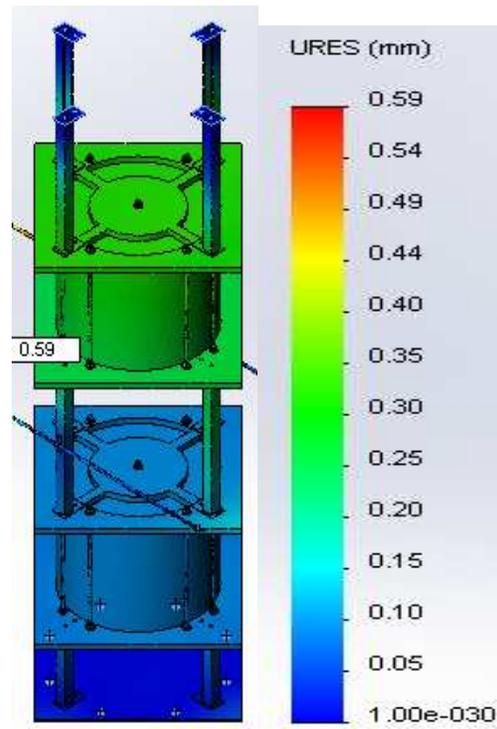


Figure 8. Deformation with Transverse Loading. SF-2

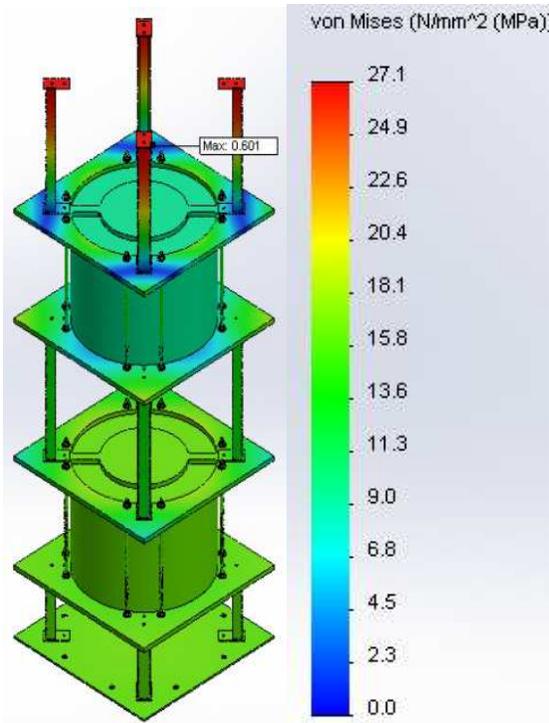


Figure 9. Stress with Longitudinal Loading. SF-2

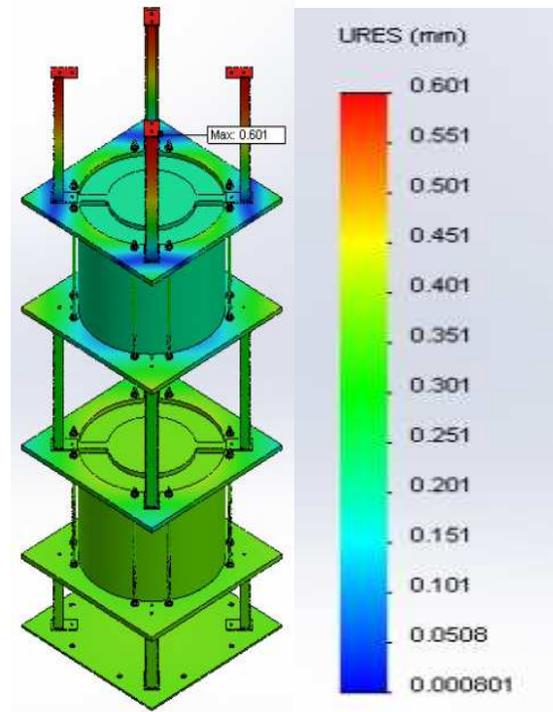


Figure 10. Deformation with Longitudinal Loading. SF-2

Table 4. Stress Strain Values and Deformation for SF 2

Load Pattern	Stress in MPa	StrainX10 <sup>-3</sup>	Deformation in mm
Transverse	63.4	2.136	0.590
Longitudinal	27.1	3.000	0.601

In this analysis, the effect of thermal stresses due to heat generated during short circuit condition has not been taken into account due to the fact that the testing at 16 times rated current will be for 2 seconds only and being a thermal short circuit test, there will not be any consecutive shots for which thermal effects are needed to be accounted for. Also the current density taken is less than 1 Ampere/square mm.

### 3. Results

The summary of the results obtained using the FEM by applying von Mises criterion is given in Table IV for a safety factor of 2. Above results shows that with increased value of safety factor, the stress value in Transverse loading and deformation therein have reduced substantially. This indicates that with strengthening of mechanical structure and coil supports, the geometry has become more rigid to sustain the electromagnetic forces. Based on the above results, the reactor coil was subjected to the actual thermal short circuit test at CPRI, Bhopal, India.

A short circuit current of 8000 A rms was passed for 2 seconds. The reactor coil withstood the current successfully. The passing criteria was – “variation in inductance value of the reactor must be within +/- 2% of the design/declared value”. At the end of test, the variation in inductance value found was 0.081%, which was quite below 2%. Thus it successfully passed the test.

### 4. Conclusion

This paper proves that the short circuit stresses can be evaluated in terms of mechanical deformation using finite element method to an extent whereby considering proper factor of safety, the short circuit proof equipment (reactor in this case) can be built for a specified short circuit level. This approach is accurate, less time consuming and helps in identifying the factor of safety without developing any prototype model. The same approach can be extended to determine the short circuit withstands level of other electrical equipments such as transformer, generators, motors etc. The method can be used to identify the inter layer forces between the layers of a concentric winding machine. Also the deformation in a winding can be predicted under transient condition especially in case of large power and distribution transformers.

## References

- Kladas A. G., Papadopoulos M. P. and Tegopoulos J. A. 1994. Leakage flux and force calculation on power transformer windings under short-circuit: 2-D and 3-D models based on the theory of images and the finite element method compared to measurements. *IEEE Transaction on Magnetics*, Vol. 30, No. 5, pp. 3487-3490.
- Reddy A. S. and Vijaykumar M. 2008. Hottest spot and life evaluation of power transformer design using finite element method. *Journal of Theoretical and Applied Information Technology*, pp. 238-243.
- Kumbhar G. B. and Kulkarni S. V. 2007. Analysis of short-circuit performance of split-winding transformer using coupled field-circuit approach. *IEEE Trans. Power Del.*, Vol. 22, No. 2, pp. 936-943.
- Ahn H. M., Oh Y. H., Kim J. K., Song J. S. and Hahn S. C., 2012. Experimental verification and finite element analysis of short circuit electromagnetic force for dry type transformer. Vol. 48, No. 2, pp. 819-822.
- Hong J., Heyun L., Zihong X. 2005. Three dimensional finite element analysis of electric fields at winding ends of dry type transformer. *ICEMS 2005, Proceedings of the 8th International Conference on Electrical Machines and Systems*, Vol. 3, pp. 2136-2139.
- Tsili M.A., Kladas A.G., Georgilakis P.S., 2008. Computer aided analysis and design of power transformers, *Computers in Industry*, Vol. 59, pp. 338-350.
- Barzegaram M. R., Mirzaie M. and Akmal A. S. 2009. Frequency response analysis in power transformer for detection of winding short circuit using quasi-static finite element and circuit based method, *World Applied Sciences Journal*, Vol. 7, No. 8, pp. 1006-1015.
- McLyman C. W. 2004. *Transformer and Inductor Design Hand book*. New York, USA: Marcel Dekker Inc.
- Pierce, L. W. 1995. Transformer design and application considerations for non sinusoidal load currents. *IEEE IAS*, Vol. 0-7803-256-0/95, pp. 35-47.
- Bell S. C., Bodger, P.S. 2007. Power transformer design using magnetic circuit theory and finite element analysis – A Comparison of Techniques. *AUPEC 2007*, pp. 607-612. Perth Western Australia.
- Say M. G. (1983). *The Performance and Design of Alternating Current Machines*. Delhi, India: CBS Publishers & Distributors.
- Kwon Y. W., Bang H. B. 2011. *The Finite Element Method using Matlab*. New Delhi, India: CRC Press.

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