Unsteady MHD flow in porous media past over exponentially accelerated inclined plate with variable wall temperature and mass transfer along with Hall current

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Abstract

In this paper we are studying unsteady MHD flow in porous media past over exponentially accelerated inclined plate having variable wall temperature as well as mass transfer along with Hall current. We have used Laplace-transform technique to find the solution of the equations in the flow model. The results obtained are discussed with the help of graphs. The drag force at the boundary has been tabulated.

Keywords: MHD, unsteady flow, inclined plate, Hall current, skin friction.

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1. Introduction

The MHD flow plays important roles in different areas of science and technology, like chemical engineering, mechanical engineering, petroleum engineering, biomechanics, irrigation engineering and aerospace technology. A number of researchers have worked on MHD flow considering the different flow models. In most of the models heat and mass are the prime components. Some of the models are worth mentioning. Hall effect on MHD flow through a speed up plate was investigated in Deka (2008). Seth et al. (2010) have analyzed convective flow of the unsteady MHD type in a rotating channel surrounded by parallel plate having thermal source/sink in a porous medium in slip boundary states. Rajput and Sahu (2011) have investigated effects of rotation and magnetic field on the flow past an exponentially accelerated vertical plate with constant temperature.

Chauhan and Rastogi (2012) have analyzed Hall effect on MHD slip-flow as well as heat transfer by a porous mode above a speed up plate within a revolving system. Mukherjee and Prasad (2014) have analyzed influence of radiation as well as porosity factor on MHD flow as a result of exponentially-stretching sheet for a porous channel. Srinivas and Kishan (2015) have considered Hall effects on unsteady MHD free convection flow over a stretching sheet with variable viscosity and viscous dissipation. We are considering the unsteady MHD flow over an exponentially-accelerated inclined plate having heat as well as mass transfer through porous medium with Hall current’s occurrence. The results are discussed with the help of graphs and table.

2. Mathematical analysis

Geometric model of the problem is shown in Figure 1.
Figure 1. Geometric model of the problem

The x axis is along the vertical which is normal to y-z plane. The z axis lies in the horizontal plane. The plate is inclined at angle \( \alpha \) from vertical. A uniform magnetic field \( B = (0, 0, B_0) \) is employed perpendicular to the flow. During the motion, the direction of the magnetic field changes along with the plate in such a way that it always remains perpendicular to it. This means, the direction of magnetic field is tied with the plate. Let \( (u, v, w) \) be the components of the velocity vector \( V \). Then using the equation of continuity \( \nabla \cdot V = 0 \), we get \( w = 0 \).

The equation of generalized ohm’s law is given as
\[
\frac{j}{B_0} + \frac{m \tau_e}{\sigma} (j \times B) = \sigma (E + V \times B).
\]

However the external electric field due to polarization of charges is as negligible, therefore \( E = 0 \).

Thus the equation becomes
\[
\frac{j}{B_0} + \frac{m \tau_e}{\sigma} (j \times B) = \sigma (V \times B).
\]

Here \( m = \omega_e \tau_e \) is the Hall current parameter. \( \omega_e \) and \( \tau_e \) are the cyclotron frequency and the collision time of electrons, respectively. Other symbols are as usual. Let \( (j_1, j_2, j_3) \) be the components of current density \( j \). Here \( J_1 \) is the current density along the flow, and \( J_2 \) is perpendicular to it. Since \( J_3 \) component is along the plate which is non-conducting, therefore \( J_3 = 0 \).

Using above assumption, generalized ohm’s law gives \( j_1 + m j_2 = \sigma v B_0 \) and \( j_2 - m j_1 = -\sigma u B_0 \).

Solving these equations, we get
\[
\begin{align*}
\dot{j}_1 &= \frac{\sigma B_0(v + mu)}{1 + m^2} \\
\dot{j}_2 &= \frac{\sigma B_0(mv - u)}{1 + m^2}.
\end{align*}
\]

Further, flow through porous media is governed by the following form of Darcy’s Law \( q = -\frac{K}{\mu} \nabla p \), where \( q \) is the flux (discharge per unit area with unit length per unit time), \( \nabla p \) is pressure gradient while \( K \) is the medium’s intrinsic permeability. The relation between Darcy’s flux and porosity is given by \( V = \frac{q}{\phi} \), where \( V \) is fluid velocity and \( \phi \) is the porosity of the medium.

These equations are combined with Navier-Stokes equation to obtain the flow model under study.

Equations of motion
\[
\begin{align*}
\frac{\partial u}{\partial t} &= \nu \frac{\partial^2 u}{\partial z^2} + g \beta C \alpha (T - T_\infty) + g \beta \cdot C \alpha (C - C_\infty) - \frac{\sigma B_0^2 (u + mv)}{\rho (1 + m^2)} - \frac{uv}{K}, \\
\frac{\partial v}{\partial t} &= \nu \frac{\partial^2 v}{\partial z^2} + \frac{\sigma B_0^2 (mu - v)}{\rho (1 + m^2)} - \frac{uv}{K}.
\end{align*}
\]

Diffusion equation
\[
\frac{\partial C}{\partial t} = \frac{D \partial^2 C}{\partial z^2},
\]

Equation of energy
\[ \rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2}. \] (4)

The conditions at the boundary of the flow could be stated as:

\[ \begin{align*}
  t \leq 0 & : u = 0, \; v = 0, \; T = T_w, \; C = C_w, \; \forall z, \\
  t > 0 & : u = u_0 e^{\beta t}, \; v = 0, \; T = T_w + (T_w - T_\infty) \frac{u_0^2 t}{v}, \; C = C_w + (C_w - C_\infty) \frac{u_0^2 t}{v}, \; \text{at } z = 0, \\
  u \to 0, \; v \to 0, \; T \to T_w, \; C \to C_w \; \text{as } z \to \infty.
\end{align*} \] (5)

Here, \( u \) is the primary velocity, \( v \)- secondary velocity, \( g \)- gravitational acceleration, \( \beta \)-volumetric coeff. of thermal expansion, \( b \)-acceleration parameter, \( t \)- time, \( m \)- the Hall current, \( K \)-the permeability parameter, \( T \)-temp of the fluid, \( \beta \)-volumetric coeff. of concentration expansion, \( C \)-concentration of the fluid, \( \nu \)-the kinematic viscosity, \( \rho \)- density, \( C_p \)- specific heat, \( k \)-thermal conductivity of the fluid, \( D \)- mass diffusion coeff., \( T_w \)-temperature near the plate, \( C_w \)-species concentration near the plate, \( B_0 \)-magnetic parameter.

Now to transform equations (1), (2), (3) and (4) into dimensionless form, the following dimensionless quantities are introduced

\[ \bar{z} = \frac{zu_0}{\nu}, \; \bar{u} = \frac{u}{u_0}, \; \bar{v} = \frac{v}{u_0}, \; \Theta = \frac{(T - T_w)}{(T_w - T_\infty)}, \; S_c = \frac{\nu}{D}, \; \mu = \frac{\mu C_p}{k}, \; \rho = \frac{\mu C_p}{k}, \; G_r = \frac{g \beta \nu (T_w - T_\infty)}{u_0^3}, \; M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, \; G_m = \frac{g \beta \nu (C_w - C_\infty)}{u_0^3}, \; \bar{b} = \frac{b \nu}{u_0^3}, \; \bar{C} = \frac{(C - C_w)}{(C_w - C_\infty)}, \; \bar{t} = \frac{tu_0^2}{\nu}, \; \bar{K} = \frac{u_0^2}{\nu^2} K. \] (6)

Here \( \bar{u} \) is dimensionless primary velocity, \( \bar{v} \) - secondary velocity, \( \bar{b} \)-dimensionless acceleration parameter, \( \bar{t} \)- dimensionless time, \( \Theta \) - dimensionless temperature, \( \bar{C} \)- dimensionless concentration, \( \bar{K} \)- the dimensionless permeability parameter, \( G_r \)- the thermal Grashof number, \( G_m \)- the mass Grashof number, \( \mu \)- coeff. of viscosity, \( P_r \)- Prandtl number, \( S_c \)- Schmidt number, \( M \)-magnetic parameter.

Thus the model becomes

\[ \frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + G_r \cos \Theta + G_m \cos \Theta \bar{C} - \frac{M (\bar{u} + m \bar{v})}{(1 + m^2)} - \frac{1}{\bar{K}} \bar{u}, \] (7)

\[ \frac{\partial \bar{v}}{\partial \bar{t}} = \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} \frac{M (m \bar{u} - \bar{v})}{(1 + m^2)} - \frac{1}{\bar{K}} \bar{v}, \] (8)

\[ \frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{S_c} \frac{\partial^2 \bar{C}}{\partial \bar{z}^2}, \] (9)

\[ \frac{\partial \Theta}{\partial \bar{t}} = \frac{1}{P_r} \frac{\partial^2 \Theta}{\partial \bar{z}^2}. \] (10)

The boundary conditions (5) become:

\[ \begin{align*}
  \bar{t} \leq 0 : \bar{u} = 0, \bar{v} = 0, \Theta = 0, \bar{C} = 0, \; \forall \bar{z}, \\
  \bar{t} > 0 : \bar{u} = e^{\bar{t} \bar{r}}, \bar{v} = 0, \Theta = \bar{r}, \bar{C} = \bar{r}, \; \text{at } \bar{z} = 0, \\
  \bar{u} \to 0, \bar{v} \to 0, \Theta \to 0, \bar{C} \to 0, \; \text{as } \bar{z} \to \infty.
\end{align*} \] (11)

For convenience, we remove the bars in the above equations to get
\[
\begin{align*}
\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial z^2} + G, \cos \alpha \theta + G_m \cos \alpha C - \frac{M(u + mv)}{(1 + m^2)} - \frac{1}{K} u, \\
\frac{\partial v}{\partial t} &= \frac{\partial^2 v}{\partial z^2} + \frac{1}{(1 + m^2)} - \frac{1}{K} v, \\
\frac{\partial C}{\partial t} &= \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2}, \\
\frac{\partial \theta}{\partial t} &= \frac{1}{P_r} \frac{\partial^2 \theta}{\partial z^2}.
\end{align*}
\] (12)
(13)
(14)
(15)

The corresponding boundary conditions become:
\[
\begin{align*}
t \leq 0: & \quad u = 0, v = 0, \theta = 0, C = 0, \quad \forall z, \\
t > 0: & \quad u = e^{bt}, v = 0, \theta = t, C = t \text{ at } z=0, \\
u \to 0, \quad v \to 0, \quad \theta \to 0, \quad C \to 0, \quad \text{as } z \to \infty.
\end{align*}
\] (16)

Combining equations (12) and (13), the model becomes
\[
\begin{align*}
\frac{\partial q}{\partial t} &= \frac{\partial^2 q}{\partial z^2} + G, \cos \alpha \theta + G_m \cos \alpha C - qa, \\
\frac{\partial C}{\partial t} &= \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2}, \\
\frac{\partial \theta}{\partial t} &= \frac{1}{P_r} \frac{\partial^2 \theta}{\partial z^2}.
\end{align*}
\] (17)
(18)
(19)

Finally, the boundary conditions become:
\[
\begin{align*}
t \leq 0: & \quad q = 0, \theta = 0, C = 0, \quad \forall z, \\
t > 0: & \quad q = e^{bt}, \theta = t, C = t, \quad \text{at } z=0, \\
q \to 0, \quad \theta \to 0, \quad C \to 0, \quad \text{as } z \to \infty.
\end{align*}
\] (20)

Here \( q = u + i v, \quad a = \frac{M(1 - im)}{1 + m^2} + \frac{1}{K} \).

The equations (17) to (19), are solved by Laplace - transform method. The solution obtained is as under:
\[
q = \frac{1}{2} e^{-\sqrt{\alpha \theta} t} A_5 + \frac{\cos \alpha}{4a^2 \sqrt{\pi}} \left[ \sqrt{\pi} G_m \left\{-A_z z + \sqrt{\alpha \theta} A_z z + \frac{1}{2A_3 A_3} + 2e^{-\sqrt{\alpha \theta} A_z z} \right\} + \frac{1}{2A_3 A_3} \right] - G_m P_r \left\{-aA_0 z + A_1 A_1 + 2A_3 A_3 P_r \right\} + \frac{1}{\sqrt{P_r}} \left[A_3 \sqrt{\pi} A_4 - a \sqrt{\pi} A_1 + \frac{1}{\sqrt{P_r}} \right] - \sqrt{\pi} G_m \left\{-aA_0 z + \sqrt{\alpha \theta} A_z z - 2e^{-\sqrt{\alpha \theta} A_z z} \right\} + 2A_3 A_3 P_r \left\{ A_3 A_3 \right\}
\]
\[
+ \sqrt{S_c} G_m \left\{-aA_0 z + \frac{1}{\sqrt{S_c} A_4 A_4} \right\} + \frac{1}{\sqrt{S_c}} \left[ A_3 \sqrt{\pi} A_4 + \frac{2a \sqrt{\pi} A_1}{\sqrt{P_r}} \right] - \sqrt{\pi} G_m \left\{-aA_0 z + \sqrt{\alpha \theta} A_z z - 2e^{-\sqrt{\alpha \theta} A_z z} \right\} + 2 A_3 A_3 P_r \left\{ A_3 A_3 \right\}.
\]
The expressions for the symbols involved in the solution are mentioned in the appendix.

3. Skin Friction

The non-dimensional skin-friction is given as
\[
\left. \frac{dq}{dz} \right|_{z=0} = \tau_x + i \tau_y.
\]

4. Analysis of the results

The velocity profiles for various parameters like thermal and mass Grashof numbers, magnetic parameter \( M \), Hall current \( m \), acceleration parameter \( b \), permeability parameter \( K \), Prandtl number \( Pr \) and time \( t \) are shown in Figures 1.1 to 2.10. It is deduced from Figures 1.1 and 2.1 that primary and secondary velocities of fluid decrease when the angle of inclination \( \alpha \) is increased. It is observed from Figure 1.2 and 2.2, when the mass Grashof number \( G_m \) is increased then the velocities are increased. From Figures 1.3 and 2.3 it is deduced that velocities increase with \( Gr \). Further we observe that when permeability parameter \( K \) is increased then the velocities are increased (Figures 1.4 and 2.4), which is obvious due to the fact that increase in porosity helps in free movement of the fluid particles. Hence it is in agreement with the natural flow of the fluid. Also, if Hall current parameter \( m \) is increased then \( u \) increases, while \( v \) gets decreased (Figures 1.5 and 2.5). It is observed from Figures 1.6 and 2.6 that when \( M \) increases, \( u \) decreases, but the trend reverses with \( v \). Further, we see that when acceleration parameter \( b \) is increased then the velocities are increased (Figures 1.7 and 2.7). This is again in agreement with the natural flow of the fluid. From Figures 1.8, 2.8, 1.9 and 2.9, it can be seen that the velocities \( u \) and \( v \) decrease when \( Pr \) and \( Sc \) are increased. Further, from Figures 1.10 and 2.10, it is observed that velocities increase with time. This is due to the fact that plate is accelerated continuously. Finally we state that the theoretical results are in agreement with the natural flow phenomenon.

Skin friction is given in Table 1. The value of \( \tau_x \) rises with the rise in \( Gm, Gr, m, K \) and \( t \). However, it decreases with \( \alpha, M, b, Pr \) and \( Sc \). Similar effects are observed with \( \tau_y \), except \( M, b \) and \( m \). Further, when \( M \) and \( b \) are increased then \( \tau_y \) increases. Again, \( \tau_y \) decreases when \( m \) increases. Comparative plots of \( \tau_x \) and \( \tau_y \) for Hall current and magnetic field parameter are shown in Figures 3.1 and 3.2, respectively.
Table 1. Skin friction for different parameters

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<th>$\alpha$ (in degree)</th>
<th>$M$</th>
<th>$m$</th>
<th>$Pr$</th>
<th>$Sc$</th>
<th>$Gm$</th>
<th>$Gr$</th>
<th>$b$</th>
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Table 1. (cont’d): Skin friction for different parameters

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<td>100</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.7779030</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>1</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.4</td>
<td>2.5355800</td>
</tr>
</tbody>
</table>

5. Conclusion

The flow model considered in the research has been solved by Laplace transform technique. The model consists of equations of motion, diffusion equation and equation of energy. To study the solutions obtained, standard sets of the values of the parameters have been considered. The numerical data obtained is discussed with the help of graphs and table. We found that the data obtained is in concurrence with the actual flow phenomenon.

Appendix

\[ A_1 = -1 - A_{16} - e^{2 \sqrt{z r}} (1 - A_{17}) \]
\[ A_2 = -1 + A_{16} - e^{2 \sqrt{z r}} (1 - A_{17}) \]
\[ A_3 = -A_4 \]
\[ A_4 = 1 + A_{23} + A_{13} (1 - A_{24}) \]
\[ A_5 = 1 - A_{25} - A_{13} (1 - A_{26}) \]
\[ A_6 = -A_{27} - A_{13} (1 + A_{23}) \]
\[ A_7 = -A_6 \]
\[ A_8 = 2 e^{\sqrt{z r}} A_1 (1 - at) \]
\[ A_{10} = (2 e^{4 t} \sqrt{r} + 2 \sqrt{r} z A_{11}) \sqrt{P_r} \]
\[ A_{11} = -1 + erf \left[ \frac{z \sqrt{P_r}}{2 \sqrt{t}} \right] \]
\[ A_{12} = -1 + erf \left[ \frac{z \sqrt{S_c}}{2 \sqrt{t}} \right] \]
\[ A_{13} = e^{-at \sqrt{\frac{aS_c}{1 + S_c}}} \]
\[ A_{14} = e^{-at \sqrt{\frac{aS_c}{1 + S_c}}} \]
\[ A_{15} = 1 + A_{12} \]
\[ A_{16} = e^{2 \sqrt{a + bt} z} \]
\[ A_{17} = erf \left[ \frac{2 \sqrt{at - z}}{2 \sqrt{t}} \right] \]
\[ A_{18} = e^{-2z \sqrt{\frac{aS_c}{1 + S_c}}} \]
\[ A_{19} = e^{-2z \sqrt{\frac{aS_c}{1 + S_c}}} \]
\[ A_{20} = erf \left[ \frac{z - 2t \sqrt{aS_c}}{2t} \right] \]
\[ A_{21} = erf \left[ \frac{z + 2t \sqrt{aS_c}}{2t} \right] \]
\[ A_{22} = erf \left[ \frac{z - 2t \sqrt{aS_c}}{2t} \right] \]
\[ A_{23} = erf \left[ \frac{z + 2t \sqrt{aS_c}}{2t} \right] \]
\[ A_{24} = erf \left[ \frac{2 \sqrt{a + bt} - z}{2 \sqrt{t}} \right] \]
\[ A_{25} = erf \left[ \frac{2 \sqrt{a + bt} + z}{2 \sqrt{t}} \right] \]

References


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