

One-dimensional spatially dependent solute transport in semi-infinite porous media: an analytical solution

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Abstract

The present study is an attempt to describe analytical solution of spatially dependent solute transport in one-dimensional semi-infinite homogeneous porous domain. In this mathematical model the dispersion coefficient is considered spatially dependent while seepage velocity is considered exponentially decreasing function of space. Dispersion parameter and velocity are directly proportional to each other. Space dependent retardation factor is also taken. The nature of porous media and solute pollutant are considered chemically non-reactive. Initially porous domain is considered solute free and the input source condition is considered uniformly continuous. A new transformation is introduced to solve the advection dispersion equation. The analytical solution is obtained by using Laplace Transformation Techniques (LTT). The effects of spatial dependence on the solute concentration dispersion of various physical parameters are explained with help of different graphs.

Keywords: Diffusion, Seepage velocity, Retardation, Aquifer, Laplace transformation, Pollutant.

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1. Introduction

A solute transport phenomenon is always an important issue for groundwater contamination which depends on the different chemical and physical processes caused by advection-dispersion. Solute transports in the surfaces and sub surfaces have been one of the most important and interesting research topics in hydrological sciences, engineering and mathematics in the last few decades. Development of an analytical solutions for groundwater pollution problems are major interesting area for civil engineers, hydrologist, chemical engineer and mathematicians, because these solutions are much helpful to understand or explain the mechanism of contaminant transport, measure the field parameters related to solute transport, predict the movement of contaminant plumes, and verify the numerical results. Most of the theories of solute transports in the porous media included limited assumptions are available in published literature. Groundwater velocities and hydrodynamic dispersion coefficients are key parameters for description of solute transport in porous media. Solute transport in soil, reservoir, and aquifer is generally governed by advection-dispersion equation. This equation infer from two laws one is Darcy's law and other is conservation of mass (Freeze and Cherry, 1979).

A number of mathematical models describing groundwater flow and solute transport in homogeneous and heterogeneous porous domain have been developed in the past and are available in literature. Huang *et al.* (1996) obtained a general analytical solution for solute transport in one-dimension with asymptotic scale-dependent dispersion in heterogeneous porous domain with assumption that the dispersivity increases linearly with space. Logan (1996) obtained analytical solutions of advection dispersion equation for solute transport in heterogeneous porous media. Leij *et al.* (1993) obtained an analytical solution assuming velocity and dispersion coefficient constant with space and time in three-dimensional porous domain. Fry *et al.* (1993) developed analytical solution of one-dimensional advection-dispersion equation in homogeneous isotropic porous medium. Van Kooten (1995) developed a method for predicting the advective-dispersive transport of a contaminant toward a well in a confined aquifer. Yule and Gardener (1978) obtained analytical solution to describe solute transport for line source neglecting longitudinal dispersion.

Perez Guerrero *et al.* (2009) presents an analytical solution of dispersion equation with constant coefficients in both transient and steady state conditions by using integral transform and change of variable techniques in a finite spatial domain. Hunt (2006) has given solution for instantaneous, continuous and steady pollution sources in uniform groundwater and Zoppou and Knight (1997) obtained an analytical solutions for the spatially dependent dispersion Yates (1990, 1992) obtained analytical solutions of advection dispersion equation in one-dimension considering dispersion coefficient as linear and exponential increasing. Zhang *et al.* (1994) presents an analytical solution of dispersion equation with distance-dependent dispersion coefficient. Pang and Hunt (2001) obtained analytical solutions for advection dispersion equation with scale-dependent dispersion where as Leij *et al.* (1991) presents several analytical solutions of advection-dispersion equation for three-dimensional in semi-infinite homogeneous porous media with unidirectional solute flow assuming uniform initial concentration with first and third type boundary conditions. Aral and Liao (1996) developed analytical solutions for the two-dimensional advection dispersion equation while Hunt (1998) discussed one, two and three-dimensional advection dispersion equation with scale-dependent dispersion coefficients. Su *et al.* (2005) developed closed form solutions of dispersion equation with time and scale-dependent disparity for solute transport in heterogeneous porous media while Singh and Kumari (2014) obtained one-dimensional solute transport for prediction of contaminant along unsteady groundwater flow.

Huang *et al.* (2006) presents an evidence of scale-dependent fractional advection-dispersion in one-dimension. Yadav *et al.* (2010) present analytical solutions of advection dispersion equation in semi-infinite longitudinal porous domain with temporally dependent coefficients. A linear advection-diffusion equation with variable coefficients in a one-dimensional semi-infinite medium is solved analytically using a Laplace transformation technique, for two dispersion problems: temporally dependent dispersion along a uniform flow and spatially dependent dispersion along a non-uniform flow by Jaiswal *et al.* (2009). Longitudinal solute transport from a pulse type source along temporally and spatially dependent flow was discussed by Yadav *et al.* (2012). A power series solution for solute transport in a convergent flow field with scale-dependent advection-dispersion equation with variable coefficients was obtained by Chen *et al.* (2003). Yadav *et al.* (2010) present solution of one dimensional temporally dependent advection dispersion equation in homogeneous porous media. Jaiswal and kumar (2011) obtained analytical solutions of advection-dispersion equation in one-dimension with variable coefficients for two cases one time dependent and other spatially dependent dispersion while analytical solutions of one-dimensional advection-diffusion equation with temporally dependent coefficients was obtained by Jaiswal *et al.* (2011). Sanskrityayn *et al.* (2016) obtained analytical solution of advection dispersion equation with spatially and temporally dependent dispersion using Green's function. Gao *et al.* (2010) was presented a new mobile-immobile model to account for solute transport with scale-dependent dispersion in heterogeneous porous media and the dispersivity was assumed to be a function of travel distance, also the linear adsorption and the first-order degradation of solute were considered. The linear and exponential functions were chosen to describe the relationship between dispersivity and distance. Kumar and Yadav (2015) developed analytical solution for conservative solute transport in one dimensional heterogeneous porous medium and solute dispersion parameter is considered uniform, while the seepage velocity spatially dependent. Singh and Das (2015) present an analytical solution of one-dimensional scale dependent solute dispersion in semi-infinite heterogeneous porous medium. Sharma *et al.* (2016) presented the behavior of solute transport through mobile-immobile soil column based on the laboratory study. Singh *et al.* (2012) present solution of one-dimensional solute transport with space-time-dependent contaminant concentration along uniform flow in a semi-infinite homogeneous porous medium. Singh and Chatterjee (2016) presented analytical solution for non-point source of concentration in semi-infinite aquifer using Laplace transform technique.

In the present study, an analytical solution of solute transport equation is solved using Laplace transform technique in a semi-infinite, homogeneous porous media. The hydrodynamic dispersion, seepage velocity and retardation factor are considered exponentially decreasing function of space dependent. Retardation factor and dispersion parameter are directly proportional to the velocity. The space dependent retardation factor indicates that entire mass of the dissolved substance is not uniform. Continuous input point source is introduced at the beginning of the domain. Initially porous domain is considered solute free; it means there is no concentration present in the medium. Concentration gradient is considered zero at infinity. Solution is demonstrated graphically with a hypothetical input data taken from the previous published works. Ground water velocity ranges from 2 m/year to 2 m/day, intermediate values are taken in present study (Todd, 1980).

2. Mathematical formulation of the Problem

The advection-dispersion equation in one-dimension may be written as follows (Freeze and Cherry, 1979 and Bear, 1972),

$$R(x, t) \frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left(D(x, t) \frac{\partial C}{\partial x} - u(x, t) C \right) \quad (1)$$

In which $C[ML^{-3}]$ is the solute concentration. $D[L^2T^{-1}]$ and $u[LT^{-1}]$ are the dispersion coefficient and seepage velocity respectively. These may be constants or functions of time or space or both. The left hand side of equation (1) is represent change in solute concentration with time and R is retardation factor which is a dimensionless quantity. The right-hand side of the equation (1) represents the influence of the dispersion on the solute concentration distribution by first term and the change of the solute concentration due to advective solute transport by second term. Velocity at the scale of pores causes a solute particle to spread

from initial position. This spreading phenomenon is described at the Darcy scale through dispersion coefficient. In view of established concept hydraulic conductivity causes water velocity at each point in the porous media.

2.2 Uniform Input Point Source Condition

The model simulates concentration along one-dimensional space dependent flow through homogeneous semi-infinite porous domain. Groundwater flow is considered along the x -axis, means the direction of the flow of water is from $x = 0$ to $x \rightarrow \infty$. Initially the porous domain is supposed to be solute free this means that before solute injection in the domain there is no concentration present in the domain. A continuous mass injection of solute is introduced into the aquifer at $x = 0$. To formulate the present problem mathematically the initial and boundary conditions are given by Eqs.(2), (3) and (4).

$$C(x, t) = 0 \quad ; \quad t = 0, x \geq 0 \tag{2}$$

$$C(x, t) = C_0 \quad ; \quad t > 0, x = 0 \tag{3}$$

$$\frac{\partial C(x, t)}{\partial x} = 0 \quad ; \quad t \geq 0, x \rightarrow \infty \tag{4}$$

Eq.(2) represent initially there is no concentration in the domain. The boundary condition (3) represents the continuous injection of pollutant while boundary condition (4) represents solute concentration gradient is zero at the end of the domain.

2.3 Analytical Solution

Let us write

$$\left. \begin{aligned} u(x, t) &= u_0 e^{-mx} \\ D(x, t) &\propto u \Rightarrow D(x, t) = D_0 e^{-mx} \\ R(x, t) &\propto u \Rightarrow R(x, t) = R_0 e^{-mx} \end{aligned} \right\} \tag{5}$$

where u_0, D_0 and R_0 are initial constants of velocity, dispersion and retardation factor respectively. $m[L^{-1}]$ is the flow resistance coefficient whose dimension is inverse of space. Thus parameter mx is the dimensionless variable. $m = 0$ represents the spatially independent parameters.

Substituting these values in Eq. (1) we have

$$R_0 e^{-mx} \frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left(D_0 e^{-mx} \frac{\partial C}{\partial x} - u_0 e^{-mx} C \right)$$

This reduces into following equation

$$R_0 \frac{\partial C}{\partial t} = D_0 \frac{\partial^2 C}{\partial x^2} - U_0 \frac{\partial C}{\partial x} + \gamma_0 C \tag{6}$$

where $U_0 = (u_0 + mD_0)$ and $\gamma_0 = mu_0$

Now Laplace transformation technique is used to get the analytical solution. For which to eliminate the convection term from Eq.(6), we introduce a new dependent variable $K(x, t)$ by following transformation.

$$C(x, t) = K(x, t) \exp \left\{ \frac{U_0}{2D_0} x - \frac{1}{R_0} \left(\frac{U_0^2}{4D_0} - \gamma_0 \right) t \right\} \tag{7}$$

With help of above transformation Eq.(7), the boundary value problem Eqs. (6, 2, 3) and (4) reduces into following three equations in terms of $K(x, t)$ as follows

$$R_0 \frac{\partial K}{\partial t} = D_0 \frac{\partial^2 K}{\partial x^2} \tag{8}$$

$$K(x, t) = 0 \quad ; \quad t = 0, x \geq 0 \tag{9}$$

$$K(x, t) = C_0 \exp(\delta^2 t) \quad ; \quad t > 0, x = 0 \tag{10}$$

$$\frac{\partial K(x, t)}{\partial x} + \frac{U_0}{2D_0} K = 0 \quad ; \quad t \geq 0, x \rightarrow \infty \tag{11}$$

where $\delta^2 = \frac{1}{R_0} \left(\frac{U_0^2}{4D_0} - \gamma_0 \right)$

Applying Laplace transformation on it which comprise of following three equations

$$\frac{d^2\bar{K}}{dx^2} - \frac{pR_0}{D_0}\bar{K} = 0 \tag{12}$$

$$\bar{K}(x,p) = \frac{C_0}{p - \delta^2} \quad ; \quad x = 0 \tag{13}$$

$$\frac{d\bar{K}}{dx} + \frac{U_0}{2D_0}\bar{K} = 0 \quad ; \quad x \rightarrow \infty \tag{14}$$

where $\bar{K} = \int_0^\infty K(x,t)e^{-pt} dt$ and p is Laplace parameter.

The general solution of Eq.(12) is written as

$$\bar{K}(x,p) = c_1 \exp\left(-x\sqrt{\frac{pR_0}{D_0}}\right) + c_2 \exp\left(x\sqrt{\frac{pR_0}{D_0}}\right) \tag{15}$$

Using boundary conditions Eqs.(13) and (14) we have the particular solution as

$$\bar{K}(x,p) = \frac{C_0}{p - \delta^2} \exp\left(-x\sqrt{\frac{pR_0}{D_0}}\right) \tag{16}$$

Applying inverse Laplace transformation on it, using appropriate tables (Van Genuchten and Alves, 1982) and the necessary transformation defined as above backwards, we get

$$C(x,t) = \frac{C_0}{2} \exp\left(\frac{xU_0 - x\sqrt{U_0^2 - 4\gamma_0 D_0}}{2D_0}\right) \times \operatorname{erfc}\left(\frac{R_0 x - t\sqrt{U_0^2 - 4\gamma_0 D_0}}{2\sqrt{D_0 R_0 t}}\right) + \frac{C_0}{2} \exp\left(\frac{xU_0 + x\sqrt{U_0^2 - 4\gamma_0 D_0}}{2D_0}\right) \times \operatorname{erfc}\left(\frac{R_0 x + t\sqrt{U_0^2 - 4\gamma_0 D_0}}{2\sqrt{D_0 R_0 t}}\right) \tag{17}$$

where $U_0 = (u_0 + mD_0)$ and $\gamma_0 = mu_0$.

This is the desired solution of one-dimensional advection-dispersion equation in semi-infinite homogeneous porous medium with spatially dependent coefficients.

3. Result and Discussions

The analytical solution obtained as in Eq. (17) is demonstrated with set of input data to understand the solute concentration distribution behavior in the domain. The chosen set of data is taken from the experimental and theoretical published literatures (Todd, 1980, Jaiswal et.al 2009). The domain is considered semi-infinite but the solute concentration decreases with position and time in a finite domain at different values of time. The solute concentration distribution values C/C_0 are calculating by taking reference concentration as $C_0 = 1.0$, in a finite longitudinal domain $0 \leq x \text{ (km)} \leq 5.0$ at various parameters. The various parameters are taken to illustrate the concentration profile and are given in concern figures. Figure 1 is drawn for the parameters $D_0 = 1.05 \text{ km}^2/\text{yr}$, $R_0 = 1.15$, $m = 0.001 \text{ (km)}^{-1}$ at different time. It demonstrates that the trends of contaminant concentration profiles are similar and decreases with position. At particular position the concentration level is lower for lower time and higher for larger time. It shows that tendency of solute concentration is reducing in nature with time and distance travelled in presence of source contaminant. It may help to rehabilitate the solute concentration. Figure 2 is drawn for various retardation factor for the parameters $D_0 = 1.05 \text{ km}^2/\text{yr}$, $t = 1.0 \text{ yr}$, $m = 0.001 \text{ (km)}^{-1}$. It depict that the solute concentration profile at particular position is lower for larger retardation value while higher for smaller retardation value. Figure 3 is drawn for various flow resistance for the parameters $D_0 = 1.05 \text{ km}^2/\text{yr}$, $t = 1.0 \text{ yr}$, $R_0 = 1.15$. It demonstrates that the contaminant concentration profile at particular position is faster for the higher flow resistance and slower for lower flow resistance. The tendency of solute concentration is reducing in nature with distance travelled in presence of source contaminant. Figure 4 is drawn for various dispersion parameter for the parameters $t = 1.0 \text{ yr}$, $R_0 = 1.15$, $m = 0.001 \text{ (km)}^{-1}$. It represents that the contaminant concentration profile at particular position is lower for the lower dispersion parameter and higher for higher dispersion parameter with distance travelled in presence of source contaminant. From graphs we conclude that the contaminant concentration reduces with increasing retardation factor and

increasing with increasing dispersion parameter. The tendency of contaminant concentration with distance travelled in presence of source contaminant and time is reducing in nature. It may help to predict the harmless concentration level in the domain.

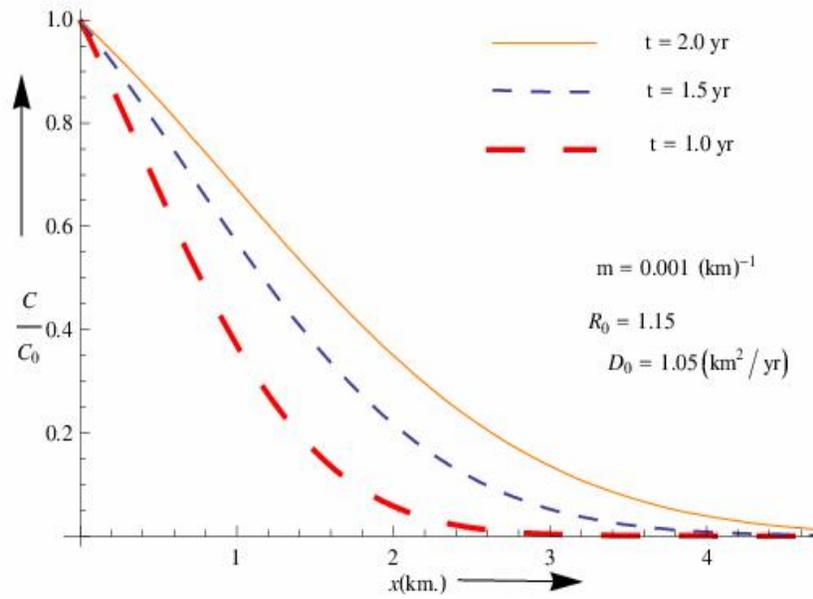


Figure 1. Dimensionless concentration profiles at time $t(\text{yr}) = 1.0, 1.5, 2.0$.

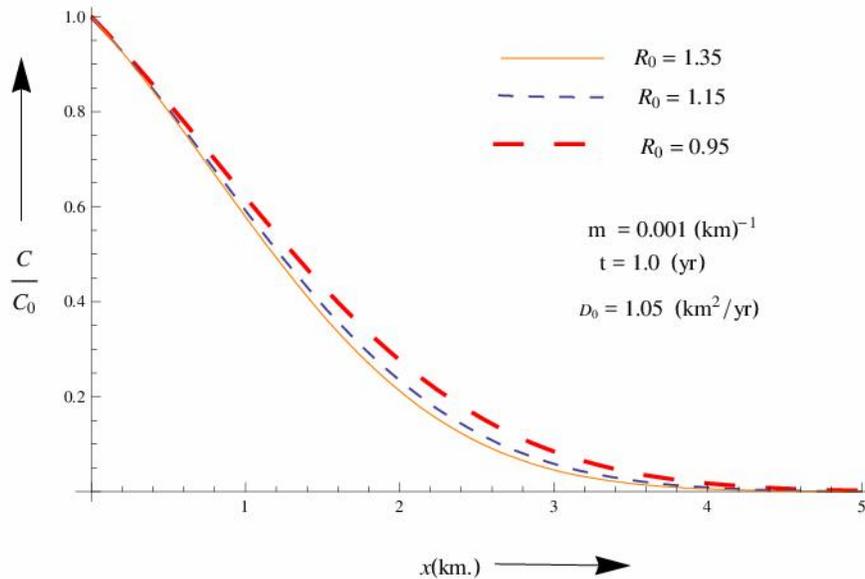


Figure 2. Dimensionless concentration profiles at $R_0 = 0.95, 1.15, 1.35$ for time $t(\text{yr}) = 1.0$.

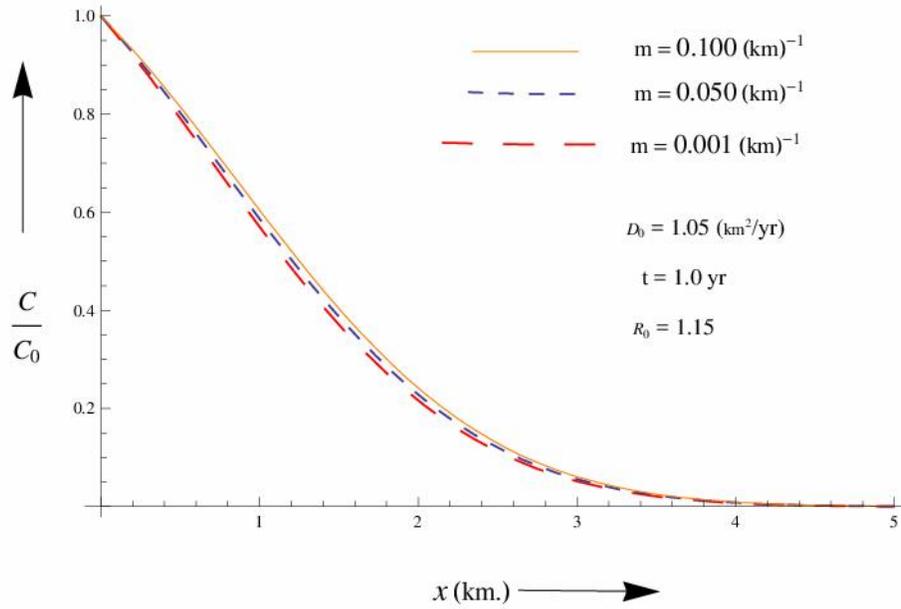


Figure 3. Dimensionless concentration profiles at $m(\text{km})^{-1} = 0.001, 0.050, 0.100$ for time $t(\text{yr}) = 1.0$

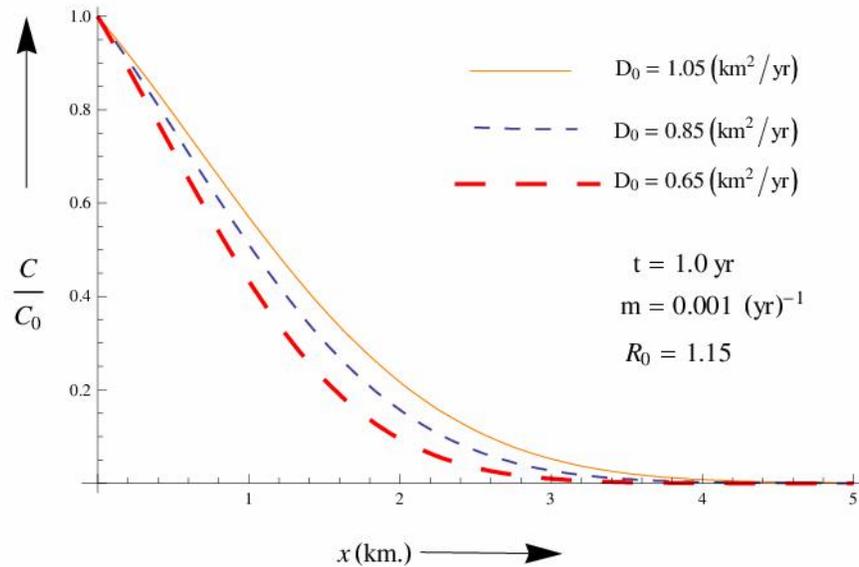


Figure 4. Dimensionless concentration profiles at $D_0(\text{km}^2/\text{yr}) = 0.65, 0.85, 1.05$ for time $t(\text{yr}) = 1.0$

4. Conclusions

A mathematical model is presented for accounting concentration distribution in semi-infinite homogenous porous media using Laplace transform technique. In this work, advective dispersive term, transient velocity and retardation factor are considered spatially dependent function. Dispersion parameter and retardation factor are considered directly proportional to the seepage velocity while seepage velocity is assumed exponentially decreasing function of space. The solution in all possible combinations of spatially dependent coefficients is compared with help of graphs. The trends of solute concentration with distance travelled in presence of source contaminant and time are reducing in nature which may help to understand rehabilitation tendency of the contaminated aquifer in the domain which may help as the primary predictive tools in groundwater management system. From the

obtained solution of dispersion equation and graphs, we also conclude that the contaminant concentration reduces with increasing retardation factor where as increases with increasing time, dispersion parameter and flow resistance coefficients and vice-versa. The tendency of concentration profile is reducing in nature with distance travelled in presence of source contaminant and time. It may help to predict the harmless concentration level in the domain. The obtained solution of advection dispersion equation can be imposed to predict the field problems where hydrological properties of the medium, initial and boundary conditions are same as or can be approximated by ones considered in this study. Results obtained in the proposed problem demonstrate the several features of scale dependent dispersion and retardation in homogeneous porous media

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