Double-diffusive convection of compressible rotating Walters’ (B’) fluid with Hall currents saturating a porous medium

Parul Aggarwal¹, Urvashi Gupta²*

¹Energy Research Centre, Panjab University, Chandigarh-160014, INDIA
²University Institute of Chemical Engineering and Technology, Panjab University, Chandigarh-160014, INDIA
* Corresponding Author: e-mail: dr_urvashi_gupta@yahoo.com. Tel +91-8427777887, Fax.+91-172-2779173

Abstract

Keeping in view the conflicting tendencies of rotation and Hall currents (magnetic field) while acting together; combined effects of Hall currents and rotation are considered on the hydromagnetic stability of a compressible Walters’ (Model B’) elastico-viscous fluid heated and soluted from below saturating a porous medium. Boussinesq approximation is used to simplify the complex hydromagnetic equations and the perturbation equations are analyzed in terms of normal modes. A dispersion relation governing the effects of visco-elasticity, salinity gradient, rotation, Hall currents and medium permeability is derived. It has been found that for stationary convection, Walters’ (Model B’) fluid behaves like an ordinary Newtonian fluid due to the vanishing of the visco-elastic parameter. Compressibility, solute gradient, rotation and magnetic field postpone the onset of instability as such their effect is to stabilize the system. Hall currents and medium permeability are found to hasten the onset of instability for permissible range of values of various parameters. The dispersion relation is analyzed numerically and the effects of various parameters for permissible range of values are depicted graphically. The visco-elasticity, solute gradient and Hall currents (hence magnetic field) introduce oscillatory modes in the system which were non-existent in their absence. Also the case of overstability is discussed and sufficient conditions for the non-existence of overstability are derived.

Keywords: Walters’ (Model B’) fluid, rotation, Hall currents, thermosolutal instability, compressibility, porous medium.

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1. Introduction

Chandrasekhar (1981) in his celebrated monograph considered a detailed account of the theoretical and experimental results for the onset of thermal instability (Bénard convection) for Newtonian viscous/inviscid fluids under varying assumptions of hydrodynamics and hydromagnetics. In the standard Bénard problem instability is driven by density difference caused by a temperature difference between upper and the lower planes bounding the fluid. If the fluid additionally has salt dissolved in it then there are potentially two destabilizing sources for the density difference, the temperature field and the salt field. The heat and solute being two diffusing components, double-diffusive convection/thermosolutal convection is a general term dealing with such phenomenon. This double-diffusive phenomenon has been extensively studied recently due to its direct relevance in the field of chemical engineering, astrophysics and oceanography.

The investigation of flow of fluids through porous medium has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. A great number of applications of such a flow in geophysics are found in a book by Philips (1991). The effect of the earth’s magnetic field on the stability of such a flow is of interest in geophysics particularly in the study of earth’s core where the earth’s mantle, which consists of conducting fluids, behaves like a porous medium. When fluid flow is considered in a porous medium, some additional complexities arise which are principally due to the interactions between the fluid and the porous material. We will consider those fluids for which Darcy’s law is applicable, which states that the gross effect, as the fluid slowly percolates through the pores of rock, is that usual viscous term in the equation of elastico-viscous fluid motion will be
replaced by the resistance term \[
\left(-\frac{1}{k_1}\left(\mu - \mu' \frac{\partial}{\partial t}\right)\right) q,
\]
where \(\mu\) and \(\mu'\) are the coefficients of viscosity and viscoelasticity, \(k_1\) is the medium permeability and \(q\) is the Darcian (filter) velocity of the fluid. The stability of flow of a single component fluid through porous medium taking into account the Darcy’s resistance has been studied by Lapwood (1948) and Wooding (1960). When the fluids are compressible, the equations governing the system become quite complicated. Spiegel and Veronis (1960) simplified the set of equations governing the flow of compressible fluids assuming that the depth of the fluid layer is much smaller than the scale height, as defined by the author’s, and the motions of infinitesimal amplitude are considered. Sharma and Gupta (1993) investigated the effect of porosity on the thermal instability of compressible fluid with Hall currents and suspended particles. Thermal instability of compressible, finite Larmor radius Hall plasma has been studied by Sharma and Sunil (1996) in a porous medium. The Hall current is important in flows of laboratory plasmas as well as in many geophysical and astrophysical situations. Sherman and Sutton (1962) have considered the effect of Hall currents on the efficiency of a magneto-hydro dynamic (MHD) generator. Numerous problems on effect of Hall currents under varying assumptions of hydrodynamics and hydromagnetics for Newtonian fluids have been attempted by many researchers in the past, e.g. Gupta (1967), Sharma and Gupta (1990), Sharma and Sunil (1995), Chauhan and Agrawal (2011), Guchhait et al. (2011) and Prasad and Kumar (2012) to name a few among several others. In all the above mentioned studies, fluids have been considered to be Newtonian.

In the last two decades with the advancement of studies for polymeric solutions and other viscoelastic fluids, many scientists and researchers focused their attention to study non-Newtonian fluid flow problems. The pioneering and fundamental works are of Bhatia and Steiner (1972), Oldroyd’s (1958), Rivlin and Ericksen (1955) and Walters (1960) on various viscoelastic fluids. Walters (1962) reported that the mixture of polymethyl methacrylate and pyridine at 25°C containing 30.5 g of polymer per liter behaves very nearly as the Walters’ (Model B’). In the last decade, quite a number of authors investigated fluid flow problems on these viscoelastic fluids. Some of these are Sunil et al. (2000a), Sharma et al. (2006), Gupta and Sharma (2007, 2008), Kumar et al. (2004), Gupta and Kumar (2010), Gupta and Aggarwal (2011). But none of the authors have studied the combined effect of rotation and Hall currents on thermosolutal instability problem for Walters’ (Model B’) fluid. This combined effect in the study of Walters’ (Model B’) fluid is very important and interesting due to the interacting and conflicting effects of rotation and Hall currents when applied together. It is worth while to mention here that magnetic field has stabilizing effect where as Hall currents and permeability have destabilizing effects in the absence of rotation. But in the presence of rotation the effects of magnetic field, Hall currents and permeability are stabilizing / destabilizing depending upon the conditions as derived later in the results and discussion section. Therefore keeping in view the conflicting tendencies of Hall currents, and rotation applied together and various applications of viscoelastic fluids in chemical technology and paper industry, the present problem of double-diffusive convection for compressible Walters’ fluid has been investigated. Some earlier known results have been recovered from the present formulation.

2. Mathematical Formulation of the Problem and Perturbation Equations

We have considered an infinite, horizontal, compressible electrically conducting Walters’ (Model B’) fluid layer of thickness \(d\) in a homogenous medium of porosity \(\varepsilon\) and medium permeability \(k_1\) which is heated and soluted from below (\(z = 0\)) so that temperature and concentration at bottom is \(T_0\) and \(C_0\) and at the upper layer (\(z = d\)) is \(T_d\) and \(C_d\) respectively, as shown in Figure 1. A uniform temperature gradient \(\beta (= |dT/dz|)\) and concentration gradient \(\beta’ (= |dC/dz|)\) are maintained. The fluid is acted upon by the gravity force \(g = (0, 0, -g)\), uniform vertical magnetic field \(H = (0, 0, H)\) and uniform vertical rotation \(\Omega = (0, 0, \Omega)\).

Let \(T, p, \rho, C, \alpha, \alpha’, g, \eta, \mu_c, N, e, \nu, \nu’, \kappa, \kappa’\) and \(q = (u, v, w)\) denote, respectively, temperature, pressure, density, concentration, thermal coefficient of expansion, solvent coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability, electron number density, charge on an electron, kinematic viscosity, kinematic viscoelasticity, thermal diffusivity, solute diffusivity and fluid velocity.
The equations expressing conservation of momentum, mass, temperature, solute concentration and equation of state after using Boussinesq approximation (see Chandrasekhar, 1981; Walters, 1960 and Joseph, 1976) are

\[
\frac{1}{\varepsilon} \left\{ \frac{\partial q}{\partial t} + \frac{1}{\varepsilon} (q \nabla) q \right\} = - \left( \frac{1}{\rho_m} \right) \nabla p - \frac{1}{k_f} \left( \mathbf{v} - \mathbf{v'} \frac{\partial}{\partial t} \right) q + g \left( 1 + \frac{\partial \rho}{\partial \rho_m} \right) + \frac{\mu_e}{4\pi \rho_m} (\nabla \times \mathbf{H}) \times \mathbf{H} + \frac{2}{\varepsilon} (q \times \Omega),
\]

(1)

\[
\nabla \cdot q = 0,
\]

(2)

\[
E \frac{\partial T}{\partial t} + (q \nabla) T = \kappa \nabla^2 T,
\]

(3)

\[
E' \frac{\partial C}{\partial t} + (q \nabla) C = \kappa' (\nabla^2 C),
\]

(4)

\[
\rho = \rho_m \left[ 1 - \alpha(T - T_0) + \alpha'(C - C_0) \right],
\]

(5)

where \( E = \varepsilon + (1 - \varepsilon) \left( \frac{\rho_s c_s}{\rho_0 c_f} \right) \) is a constant and \( E' \) is a constant analogous to \( E \) but corresponding to solute rather than heat and \( \rho_s, c_s, \rho_0 \) and \( c_f \) denote the density and heat capacity of solid (porous) matrix and fluid matrix, respectively.

In the present model, we have ignored the non-Newtonian effects of second-order fluids on heat transportation in comparison to other terms in heat equation and assume that viscoelastic effects influence the heat transport only through velocity. From Maxwell’s equations for a porous medium, we have

\[
\varepsilon \frac{dH}{dt} = (H \nabla) q + \varepsilon \eta \nabla^2 H - \frac{\varepsilon}{4\pi Ne} \nabla \times [(\nabla \times H) \times H],
\]

(6)

\[
\nabla \cdot H = 0,
\]

(7)
\[ f(x, y, z, t) = f_m + f_0(z) + f'(x, y, z, t), \]  
\hspace{1cm} (8) \]

\( f_m \) stands for constant space distribution of \( f \), \( f_0 \) is the variation in the absence of motion and \( f'(x, y, z, t) \) is the fluctuation resulting from motion. For initial state, we have

\[ p = p(z), \rho = \rho(z), T = T(z), C = C(z), \mathbf{q} = (0, 0, 0) \text{ and } \mathbf{H} = (0, 0, H), \]

where \[ p(z) = p_m - g \frac{z}{\rho_0} (\rho_m + \rho_0)dz, \]

\[ \rho(z) = \rho_m \left[ 1 - \alpha_m (T - T_0) + \alpha'_m (C - C_0) + K_m (p - p_m) \right]. \]

\[ T(z) = -\beta z + T_0, C(z) = -\beta' z + C_0, \]

\[ \alpha_m = -\left( \frac{1}{\rho} \frac{\partial \rho}{\partial T} \right)_m (= \alpha, \text{say}), \quad \alpha'_m = -\left( \frac{1}{\rho} \frac{\partial \rho}{\partial C} \right)_m (= \alpha', \text{say}), \quad K_m = \left( \frac{1}{\rho} \frac{\partial \rho}{\partial p} \right)_m. \]  
\hspace{1cm} (9) \]

Here \( p_m \) and \( \rho_m \) stand for a constant space distribution of \( p \) and \( \rho \). Linearized stability theory and normal mode analysis method is used to study infinitesimal perturbation and depth of fluid layer is assumed to be much less than the scale height as defined by Spiegel and Veronis (1960). Using these assumptions and results for compressible fluids, the flow equations are found to be the same as those for incompressible fluids except that the static temperature gradient \( \beta \) is replaced by its excess over the adiabatic \( \left( \beta - \frac{g}{C_p} \right) \).

In our analysis we have considered a small perturbation on steady state solution and let

\[ \delta p, \delta \rho, \theta, \gamma, \mathbf{h} = (h_x, h_y, h_z) \text{ and } \mathbf{q} = (u, v, w) \]

denote the perturbations in pressure, density, temperature, solute concentration, magnetic field and velocity respectively. The change in density \( \delta \rho \) is given by

\[ \delta \rho = -\rho_m (\alpha \theta - \alpha' \gamma). \]  
\hspace{1cm} (10) \]

Then the linearized hydromagnetic perturbation equations are

\[ \frac{1}{\varepsilon} \left( \frac{\partial \mathbf{q}}{\partial t} \right) = -\frac{1}{\rho_m} (\nabla \delta p) - \frac{1}{k_1} (v - v') \frac{\partial}{\partial t} \mathbf{q} - g (\alpha \theta - \alpha' \gamma) + \frac{\mu_e}{4\pi \rho_m} (\nabla \times \mathbf{h}) \times \mathbf{H} + \frac{2}{\varepsilon} (\mathbf{q} \times \Omega), \]  
\hspace{1cm} (11) \]

\[ \nabla \cdot \mathbf{q} = 0, \]  
\hspace{1cm} (12) \]

\[ \frac{\partial \theta}{\partial t} = \left( \beta - \frac{g}{C_p} \right) w + \kappa \nabla^2 \theta, \]  
\hspace{1cm} (13) \]

\[ \frac{\partial \gamma}{\partial t} = \beta' w + \kappa' (\nabla^2 \gamma), \]  
\hspace{1cm} (14) \]

\[ \nabla \cdot \mathbf{h} = 0, \]  
\hspace{1cm} (15) \]

\[ \frac{\partial \mathbf{h}}{\partial t} = (\mathbf{H} \cdot \nabla) \mathbf{q} + \varepsilon \eta \nabla^2 \mathbf{h} - \frac{\varepsilon}{4\pi N_e} \nabla \times [(\nabla \times \mathbf{h}) \times \mathbf{H}]. \]  
\hspace{1cm} (16) \]
3. Normal Mode Analysis Method and Dispersion Relation

In the present study, we have used normal mode analysis method and assumed that perturbation quantities are of the form

\[ x, y, z, \Theta, \Gamma, Z(z) \times \exp(ik_x x + ik_y y + nt), \]

where \( k_x \) and \( k_y \) are the wave numbers along \( x \) and \( y \) directions and resultant wave number is given by \( k = \left( k_x^2 + k_y^2 \right)^{1/2} \) and \( n \) is the growth rate. Also, \( \zeta = \partial \psi / \partial x - \partial \phi / \partial y \) is the z-component of vorticity and \( \xi = \partial h_z / \partial x - \partial h_x / \partial y \) is the z-component of current density.

Using expression (17), equations (11) – (16) can be rewritten as

\[
\begin{align*}
\left[ \frac{\sigma}{\epsilon} + \frac{1}{P_1} (1 - \sigma F) \right] \left( D^2 - a^2 \right) W + \frac{g a^2 d^2}{v} (a \Theta - a T) - \frac{\mu_e H d}{4 \pi \sigma_m v} (D^2 - a^2) D K + \frac{2 \Omega d^3}{\epsilon v} D Z &= 0, \\
\left[ \frac{\sigma}{\epsilon} + \frac{1}{P_1} (1 - \sigma F) \right] Z - \frac{\mu_e H d}{4 \pi \sigma_m v} D X - \frac{2 \Omega d^3}{\epsilon v} D W &= 0,
\end{align*}
\]

(18)

\[
\begin{align*}
[D^2 - a^2 - E p_i \sigma] \Theta + \frac{\beta d^2}{\kappa} \left( \frac{G - 1}{G} \right) W &= 0, \\
[D^2 - a^2 - E' q \sigma] \Gamma + \left( \frac{\beta' d^2}{\kappa'} \right) W &= 0,
\end{align*}
\]

(20)

\[
\begin{align*}
[D^2 - a^2 - p_2 \sigma] K + \left( \frac{H d}{\epsilon \eta} \right) D W - \frac{H d}{4 \pi \sigma_m \eta} D X &= 0, \\
[D^2 - a^2 - p_2 \sigma] X + \left( \frac{H d}{\epsilon \eta} \right) D Z + \frac{H}{4 \pi \sigma_m \eta} (D^2 - a^2) K &= 0,
\end{align*}
\]

(22) (23)

where various non-dimensional parameters used are as follows

\[ a = k d, \quad \sigma = \frac{nd^2}{v}, \quad p_1 = \frac{v}{\kappa}, \quad p_2 = \frac{v}{\eta}, \quad q = \frac{v}{\kappa'}, \quad F = F', \quad P_1 = \frac{k_1}{d^2}, \quad G = \left( \frac{C p}{g} \right), \beta, x^* = \frac{x}{d}, \gamma^* = \frac{y}{d}, \zeta^* = \frac{z}{d}, \quad D = \frac{d}{dz^*}. \]

Consider the case of two free boundaries which are perfect conductors of both heat and solute concentration. For the case of free boundaries the boundary conditions are (see Chandrasekhar, 1981)

\[ W = D^2 W = 0, \quad D Z = 0, \quad \Theta = 0, \quad \Gamma = 0 \text{ at } z = 0 \text{ and } 1, \quad K = 0, \]

on perfectly conducting boundaries

and \( h_x, h_y, h_z \) are continuous. Since the components of magnetic field are continuous and the tangential components are zero outside the fluid, we have

\[ D K = 0, \] on the boundaries.

(24) (25)

Using the boundary conditions (24) and (25), it can be shown that all the even order derivatives of \( W \) must vanish for \( z = 0 \) and 1. Therefore the proper solution of \( W \) characterizing the lowest mode is

\[ W = W_0 \sin \pi z, \]

(26)

where \( W_0 \) is a constant. After eliminating \( \theta, X, Z, \Gamma \) and \( K \) between equations (18) - (23), we obtain
where

\[
R_1 = \left( \frac{G}{G-1} \right) \left[ \frac{1}{P} \left( \frac{1}{x} \right)^2 + S_1 + \frac{1}{x} \right] \left[ Q_1 \left( \frac{1}{x} \right)^2 + P Q_1 + 2P \sqrt{T_1 M} + P T_1 \left( 1 + x + M \right) \right],
\]

Equation (27) is the dispersion relation including the effects of rotation, Hall currents, compressibility and solute gradient on the thermosolutal instability of Walters’ (Model B’′) fluid in porous medium.

4. Results and Discussion

(i) Case of Stationary Convection

Consider the case when instability sets in the form of stationary convection. For stationary convection, \( \sigma_1 = 0 \) and the dispersion relation (27) reduces to

\[
R_1 = \left( \frac{G}{G-1} \right) \left[ \frac{1}{P} \left( \frac{1}{x} \right)^2 + S_1 + \frac{1}{x} \right] \left[ Q_1 \left( \frac{1}{x} \right)^2 + P Q_1 + 2P \sqrt{T_1 M} + P T_1 \left( 1 + x + M \right) \right].
\]

The above equation expresses the modified Rayleigh number \( R_1 \) as a function of dimensionless wave number, \( x \), and the parameters \( S_1, Q_1, G, M \) and \( T_1 \). For stationary convection, the viscoelastic parameter \( F \) vanishes with \( \sigma_1 \) and the Walters’ (Model B’′) fluid behaves like an ordinary Newtonian fluid. Keeping the non-dimensional number \( G \) (accounting for compressibility) as fixed, we get

\[
R_c = \left( \frac{G}{G-1} \right) R_c,
\]

where \( R_c \) and \( \overline{R_c} \) denote, respectively, the critical Rayleigh numbers in the absence and presence of compressibility. Thus, the effect of compressibility is to postpone the onset of thermosolutal instability. The cases \( G < 1 \) and \( G = 1 \) correspond to negative and infinite values of Rayleigh numbers due to compressibility which are not relevant in the present study. To investigate the effect of combined presence of Hall currents and rotation for thermosolutal convection in porous medium, we examine the natures of \( dR_1/dS_1, dR_1/dQ_1, dR_1/dM, dR_1/dT_1 \) and \( dR_1/dP \) analytically and numerically. Equation (28) yields

\[
\frac{dR_1}{dS_1} = \left( \frac{G}{G-1} \right),
\]

which shows that solute gradient has a stabilizing effect on thermosolutal convection. Numerically, \( R_1 \) as given in equation (28), is plotted against \( x \) for \( G = 10, Q_1 = 100, M = 10, T_1 = 10^3 \) and for different values of \( S_1 = 100, 200, 300, 400, 500 \) in Figure 2. Here we would like to mention that the values of \( R_1 \) are calculated in MS-Excel for different values of other parameters involved. It is clear from the figure that the Rayleigh number increases with an increase in \( S_1 \) and establishes the stabilizing effect of solute gradient. To analyze the effect of magnetic field, expression (28) yields
\[
\frac{dR_1}{dQ_1} = \left( \frac{G}{G-1} \right) \left( \frac{1+x}{x} \right) \left\{ \left[ (1+x+M) + (1+x) + P(2Q_1 + 2\overline{T_1}M - PT_1) \right] + P^2 Q_1^2 \right\}^{-1},
\]

which implies that magnetic field has stabilizing/distabilizing effect depending upon whether

\[
\left[ (1+x) + P(2Q_1 + 2\overline{T_1}M) \right] > P^2 T_1 \quad \text{or} \quad \left[ (1+x) + P(2Q_1 + 2\overline{T_1}M) \right] < P^2 T_1.
\]

This destabilizing effect of magnetic field occurs in the presence of rotation. For \( T_1 = 0 \),

\[
\frac{dR_1}{dQ_1} = \left( \frac{G}{G-1} \right) \left( \frac{1+x}{x} \right) \left\{ \left[ (1+x+M) + (1+x) + 2PQ_1 \right] + P^2 Q_1^2 \right\}^{-1},
\]

which shows the stabilizing effect of magnetic field. We have plotted \( R_1 \) against the scaled wavenumber, \( x \), for \( G = 10, P = 0.001, S_1 = 100, M = 10, T_1 = 10^3 \) and for various values of \( Q_1 = 100, 150, 200, 250 \) and 300 in Figure 3. It is clear from the figure that \( R_1 \) increases with an increase in \( Q_1 \), confirming the stabilizing effect of magnetic field. Here, it is worthwhile to mention that the former condition mentioned above, i.e.,

\[
\left[ (1+x) + P(2Q_1 + 2\overline{T_1}M) \right] > P^2 T_1
\]

holds true for the values of various parameters under consideration. The result is in agreement with that of Gupta and Sharma (2008) for Rivlin-Ericksen fluids. In Figure 4, \( R_1 \) is plotted against the scaled wavenumber, \( x \), for \( T_1 = 0 \). The figure clearly exhibits the stabilizing effect of magnetic field. The earlier work of Sunil et al. (2004) is a particular case of the present work in the absence of rotation for a non-porous medium.

In the absence of Hall currents, the above expression for the derivative reduces to

\[
\frac{dR_1}{dQ_1} = \left( \frac{G}{G-1} \right) \left( \frac{1+x}{x} \right) \left[ \left( (1+x) + PQ_1 \right)^2 - P^2 T_1(1+x) \right] \left[ (1+x) + PQ_1 \right]^2,
\]

which is in agreement with the counterpart presented by Sharma and Bhardwaj (1993), reflecting the stabilizing/distabilizing effect of magnetic field in the presence of rotation. Thus, in the absence of Hall currents and presence of rotation, magnetic field has stabilizing or destabilizing effect depending on whether

\[
\left[ (1+x) + PQ_1 \right]^2 > P^2 T_1(1+x) \quad \text{or} \quad \left[ (1+x) + PQ_1 \right]^2 < P^2 T_1(1+x).
\]
But for permissible values of various parameters involved, the already mentioned effect is stabilizing as
\[ [(1 + x) + PQ_1]^2 > P^2 T_1 (1 + x), \]
is the only condition which is satisfied. This can also be seen graphically as shown in Figure 5, where \( R_1 \) is plotted against \( x \) for different values of \( Q_1 \) in the absence of Hall currents and presence of rotation.

For analyzing the effect of Hall currents, we obtain the expression
\[
\frac{dR_1}{dM} = - \left( \frac{G}{G-1} \right) \left( \frac{1+x}{x} \right) Q_1 \left\{ \left[ \left( 1 + x \right) + PQ_1 + P \sqrt{T_1 / M} \right] \left[ 1 - P \sqrt{T_1 / M} \right] \right\}^{-1} \\
\left\{ \left[ \left( 1 + x \right) + PQ_1 + M \right] \right\}^{-1},
\]
which states that Hall currents have stabilizing/destabilizing effect depending on whether \( P \sqrt{T_1 / M} > 1 \) or \( P \sqrt{T_1 / M} < 1 \).

But for the permissible range of values of various parameters under consideration, this effect is destabilizing since \( P \sqrt{T_1 / M} < 1 \) is the only condition which is satisfied. In the absence of rotation, the above expression reduces to
\[
\frac{dR_1}{dM} = - \left( \frac{G}{G-1} \right) \left( \frac{1+x}{x} \right) Q_1 \left\{ \left[ 1 + x + PQ_1 \right] \right\}^{-1},
\]
establishing the usual destabilizing effect of Hall currents. These results are in agreement with the numerical/graphical results of Figures 6 and 7 where \( R_1 \) is plotted against the scaled wavenumber, \( x \), for \( G = 10, Q_1 = 100, S_1 = 100 \) and for \( T_1 = 10^3 \) and \( T_1 = 0 \), respectively, for various values of \( M (10, 30, 50) \). Expression for observing effect of rotation is obtained as
\[
\frac{dR_1}{dT_1} = \left( \frac{G}{G-1} \right) \left( \frac{1+x}{x} \right) P \left\{ \left[ 1 + x + M + \frac{Q_1 \sqrt{M / T_1}}{T_1} \right] \right\}^{-1} \\
\left\{ \left[ 1 + x + PQ_1 + M \right] \right\}^{-1},
\]
which reflects the stabilizing influence of rotation. This is in agreement with the corresponding result for Rivlin-Erickson fluids as derived by Gupta and Sharma (2007). In Figure 8, \( R_1 \) increases with the increase in \( T_1 \) which confirms the aforementioned result.
For analyzing the effect of medium permeability, we obtain

\[
\frac{dR_1}{dP} = \left(\frac{G}{G-1}\right) \left(\frac{1+x}{x}\right) \frac{1}{P^2} \left\{ P^2 \left[ Q_1^2 M + 2Q_1\sqrt{T_1 M (1+x+M)} + T_1 M^2 + 2T_1 M (1+x) \right] + \\
(1+x) \left[ P^2 T_1 (1+x) - \left\{ (1+x) + P Q_1 + M \right\}^2 \right]\right\} \left\{ (1+x) + P Q_1 + M \right\}^{-2}.
\]

(37)

Pondering this equation meticulously, it is seen that permeability has a stabilizing/destabilizing effect depending upon whether

\[ P^2 T_1 (1+x) > \left\{ (1+x) + P Q_1 + M \right\}^2 \quad \text{or} \quad P^2 T_1 (1+x) < \left\{ (1+x) + P Q_1 + M \right\}^2. \]

In the absence of rotation, the expression reduces to

\[
\frac{dR_1}{dP} = \left(\frac{G}{G-1}\right) \left(\frac{1+x}{x}\right) \frac{1}{P^2} \left\{ P^2 M Q_1^2 - (1+x) \left\{ (1+x) + P Q_1 + M \right\}^2 \right\} \left\{ (1+x) + P Q_1 + M \right\}^{-2},
\]

(38)

which shows that the permeability has destabilizing/stabilizing effect depending on whether

\[ P^2 M Q_1^2 < (1+x) \left\{ (1+x) + P Q_1 + M \right\}^2 \quad \text{or} \quad P^2 M Q_1^2 > (1+x) \left\{ (1+x) + P Q_1 + M \right\}^2. \]
Figure 10 confirms the destabilizing influence of permeability in the absence of rotation as the later of the two inequalities does not hold true for the permissible values of various parameters. This result is identical to that of Gupta and Sharma (2008). Further, in the absence of Hall currents, the expression reduces to

\[
\frac{dR_1}{dP} = \left(\frac{G}{G-1}\right) \left(\frac{(1+x)^2}{x}\right) \frac{1}{P^2} \left\{ \frac{P^2 T_i (1+x) - [(1+x) + PQ_i]^2}{(1+x) + PQ_i} \right\},
\]

which shows that the permeability has destabilizing/stabilizing effect depending on whether

\[ P^2 T_i (1+x) < \left[(1+x) + PQ_i \right]^2 \quad \text{or} \quad P^2 T_i (1+x) > \left[(1+x) + PQ_i \right]^2. \]

Again Figure 11 confirms the destabilizing influence of medium permeability in the absence of Hall currents as the later of the two inequalities does not hold true for the considered values of various parameters. In the absence of both rotation and Hall currents, the expression becomes

\[
\frac{dR_1}{dP} = \left(\frac{G}{G-1}\right) \left(\frac{(1+x)^2}{x}\right) \frac{1}{P^2},
\]

which accounts for the usual destabilizing influence of medium permeability in the absence of rotation and Hall currents. This is in agreement with the result of Figure 12, where \( R_1 \) is plotted against \( x \) for various values of \( P = 0.001, 0.002, 0.003, 0.004, 0.005. \)
(ii) Stability of the System and Oscillatory Modes

To determine the possibility of oscillatory modes we multiply equation (18) by $W^*$, the complex conjugate of $W$ and using equations (19) - (23) together with the boundary conditions (24) and (25), we obtain

$$
\left[ \frac{\sigma + 1}{\epsilon P_1} (1 - \sigma F) \right] I_1 + \left[ \frac{G}{G - 1} \right] \frac{g \alpha \kappa a^2}{\nu \beta} \left[ I_2 + \frac{\mathcal{E}q_\sigma}{\nu \beta} I_3 \right] + \frac{g \alpha \kappa a^2}{\nu \beta'} \left[ I_4 + \frac{\mathcal{E}q_\sigma}{\nu \beta} I_5 \right] + \frac{\mu \eta e}{4 \pi \rho_m v} \left[ I_6 + p_2 \sigma I_7 \right] +
$$

$$+ \frac{d^2}{\epsilon P_1} \left[ \frac{\sigma^* + 1}{\epsilon P_1} (1 - \sigma F) \right] I_8 + \frac{\mu \eta d^2 e}{4 \pi \rho_m v} \left[ I_9 + p_2 \sigma I_{10} \right] = 0,
$$

(41)

where

$$I_1 = \int_0^1 \left[ |D\mathbf{W}^2 + a^2 |\mathbf{W}|^2 \right] dz, \quad I_2 = \int_0^1 \left[ |\mathbf{D}\phi|^2 + a^2 |\phi|^2 \right] dz,$$

$$I_3 = \int_0^1 \left[ |\phi|^2 \right] dz, \quad I_4 = \int_0^1 \left[ |D\mathbf{I}|^2 + a^2 |\mathbf{I}|^2 \right] dz, \quad I_5 = \int_0^1 \left[ |\mathbf{I}|^2 \right] dz,$$

$$I_6 = \int_0^1 \left[ |D^2 \mathbf{K}^2 + 2a^2 |D\mathbf{K}|^2 + a^4 |\mathbf{K}|^2 \right] dz,$$

$$I_7 = \int_0^1 \left[ |D\mathbf{K}|^2 + a^2 |\mathbf{K}|^2 \right] dz, \quad I_8 = \int_0^1 \left[ |\mathbf{K}|^2 \right] dz,$$

$$I_9 = \int_0^1 \left[ |D\mathbf{X}|^2 + a^2 |\mathbf{X}|^2 \right] dz, \quad I_{10} = \int_0^1 \left[ |\mathbf{X}|^2 \right] dz,$$

where integrals $I_1, I_2, \ldots, I_{10}$ are all positive and definite. Putting $\sigma = \sigma_r + i \sigma_i$ and equating real and imaginary parts of equation (41), we get

$$\left\{ \frac{1 - F}{\epsilon P_1} \right\} I_1 - \left( \frac{G}{G - 1} \right) \frac{g \alpha \kappa a^2}{\nu \beta} E_p I_3 + \frac{g \alpha \kappa a^2}{\nu \beta'} I_2 + \frac{\mathcal{E}q_\sigma}{\nu \beta} I_5 + \frac{\mu \eta e}{4 \pi \rho_m v} p_2 I_7 + d^2 \left[ \frac{1 - F}{\epsilon P_1} \right] I_8 + \frac{\mu \eta d^2 e}{4 \pi \rho_m v} p_2 I_{10} \right\} \sigma_r =
$$

$$- \left\{ \frac{1}{\epsilon P_1} \right\} I_1 - \left( \frac{G}{G - 1} \right) \frac{g \alpha \kappa a^2}{\nu \beta} I_2 - \frac{g \alpha \kappa a^2}{\nu \beta'} I_4 + \frac{\mathcal{E}q_\sigma}{\nu \beta} I_5 + \frac{\mu \eta e}{4 \pi \rho_m v} I_6 + d^2 \left[ \frac{1 - F}{\epsilon P_1} \right] I_8 + \frac{\mu \eta d^2 e}{4 \pi \rho_m v} I_{10} \right\} \sigma_i = 0.
$$

(42)

(43)

It is inferred from equation (42) that $\sigma_r$ may be positive or negative, which means that the system may be stable or unstable. Also, from equation (43), $\sigma_i$ may be zero or non-zero, signifying that the modes may be non-oscillatory or oscillatory. The oscillatory modes appear due to the presence of viscoelasticity, solute gradient and magnetic field (hence Hall currents) which, would not exist in the absence of such effects. This result is in agreement with the result from the study of Sunil et al. (2000b) where the effect of Hall currents has been investigated on thermal instability of Walters’ (Model B’) fluid.

(iii) Case of Overstability

Let us now discuss the possibility whether instability may occur as an overstability. Since for overstability, we wish to find Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which equation (27)
will admit solutions with real values of $\sigma_1$. Equating real and imaginary parts of equation (27) and eliminating $R_1$ between them, we obtain

$$A_4c_1^4 + A_3c_1^3 + A_2c_1^2 + A_1c_1 + A_0 = 0,$$

(44)

where we have set $c_1 = \sigma_1^2$, $b = 1 + x$, and

$$A_4 = p_4^2E^2q^2b^2\left[\frac{Ep_1}{P} + b\left(\frac{1}{e} - \frac{\pi^2F}{P}\right)\right]\left(\frac{1}{e} - \frac{\pi^2F}{P}\right)^2,$$

(45)

$$A_3 = E^2q^2p_2^2b^2\left[\frac{1}{e} - \frac{\pi^2F}{P}\right]\left[Q_1\left(\frac{1}{e} - \frac{\pi^2F}{P}\right) (Ep_1 - p_2) + \frac{p_2^2}{P^2} + 2bP^2\left(\frac{1}{e} - \frac{\pi^2F}{P}\right)^2\right] + p_2b\left(\frac{1}{e} - \frac{\pi^2F}{P}\right)^2 \times$$

$$\times \left[\frac{b^2Ep_1}{P} + \frac{p_2}{P} + (2 + b^2)\left(\frac{1}{e} - \frac{\pi^2F}{P}\right)\right] + \frac{p_2^2}{P^2}E^2q\left(\frac{1}{e} - \frac{\pi^2F}{P}\right) + 2p_2b^2E^2q^2\frac{Ep_1}{P}\left(1 - \frac{\pi^2F}{P}\right) \times$$

$$\times \left[2b\left(\frac{1}{e} - \frac{\pi^2F}{P}\right) - (b + M)\right] + p_2^3b\left[S_1(b - 1)p_2(EP_1 - E'q)\left(\frac{1}{e} - \frac{\pi^2F}{P}\right)^2 + p_2E^2q^2\frac{Ep_1}{P^3} - T_1\left(1 - \frac{\pi^2F}{P}\right)\right]\right] +$$

$$+ \left[4E^2q^2p_2^3\left(\frac{1}{e} - \frac{\pi^2F}{P}\right)^2 \left\{\frac{1}{P} + \frac{1}{2}Q_1Ep_1b + b\left(\frac{1}{e} - \frac{\pi^2F}{P}\right)\right\}\right] + T_1Ep_1E^2q^2\frac{p_2^4}{P}.$$  

(46)

Since $\sigma_1$ is real for overstability, the four values of $c_1(= \sigma_1^2)$ should be positive. The sum of roots of equation (44) is $-A_3/A_4$ and this should be positive if each of the roots is positive. Now, it is clear from expression (45) and (46) that $A_3$ and $A_4$ are always positive if

$$\frac{\pi^2F}{P} < \frac{1}{e}, \quad Ep_1 > p_2, \quad Ep_1 > E'q, \quad b > M, \quad b > 1, \quad 2b\left(\frac{1}{e} - \frac{\pi^2F}{P}\right) > (b + M) \quad \text{and} \quad \frac{Ep_1}{P^3} > T_1\left(\frac{1}{e} - \frac{\pi^2F}{P}\right),$$

i.e. if

$$\kappa < \min \left\{\frac{E\eta}{E'}, \kappa', \frac{\nu^2\varepsilon^3 d^2}{4\Omega^2 k_1^2 \pi^2 (k_1 - \varepsilon v')}\right\},$$

(47)

and

$$k > \max \left\{\frac{H}{4N\eta d} \left(\frac{\varepsilon k_1}{2(k_1 - \varepsilon v') - \varepsilon k_1}\right)^{1/2}, \left(\frac{H}{4N\eta d}\right)^{1/2}\right\}. $$

(48)

Thus, for conditions (47) and (48), overstability cannot occur and the principle of exchange of stabilities is valid. Hence, these are the sufficient conditions for the non-existence of over stability, the violation of which does not necessarily imply the occurrence of overstability. The analogous conditions $\kappa < \min \left\{\frac{E\eta}{E'}, \kappa', \frac{\nu^2\varepsilon^3 d^2}{4\Omega^2 k_1^2 \pi^2 (k_1 + \varepsilon v')}\right\}$ and

$$k > \max \left\{\frac{\mu H^2E^2\pi}{2\rho_{\kappa}\kappa^2 d^2}, \left(\frac{cH}{4N\eta d}\right)^{1/4}\right\}$$

are derived by Gupta and Sharma (2008) for the case of Rivlin-Erickson elastico-viscous fluid. Further, in the absence of rotation, magnetic field (and hence Hall currents) and viscoelasticity, the above conditions, as expected, reduce to $\kappa < \min \{\eta, \kappa\}$ (see Chandrasekhar, 1981 and Veronis, 1965).
5. Concluding Remarks

In the present paper, the combined effect of Hall currents and rotation on the stability of a compressible Walters’ (Model B′) elastico-viscous fluid heated and soluted from below in porous medium is considered. The effects of various parameters such as magnetic field, compressibility, Hall currents, rotation and medium permeability have been investigated analytically as well as numerically. The main results from this paper are as follows:

1. The expressions \( dR_1/dS_1, dR_1/dQ_1, dR_1/dM, dR_1/dT_1 \) and \( dR_1/dP \) are examined analytically and it has been found that the solute gradient (\( S_1 \)) and rotation (\( T_1 \)) have stabilizing effect. Very interestingly, magnetic field which has stabilizing effect for \( T_1 = 0 \) has stabilizing/destabilizing influence for \( T_1 \neq 0 \), while Hall currents and permeability have destabilizing effect for \( T_1 = 0 \) and also have stabilizing/destabilizing influence for \( T_1 \neq 0 \). Various conditions for stabilizing/destabilizing influence of magnetic field, Hall currents and permeability are derived.

2. The effects of the above mentioned parameters are also studied numerically for permissible range of values of various parameters through Figures (2) - (12). It is found that magnetic field postpones the onset of instability, while Hall currents and permeability hasten the same for the considered allowed range. The reason for stabilizing effects of magnetic field and rotation are accounted for by Chandrasekhar (1981) and for solute gradient by Veronis (1965). These are found to be valid for second-order fluids as well.

3. The effect of compressibility is to postpone the onset of instability as is clear from equation (29).

4. The oscillatory modes appear due to the presence of viscoelasticity, solute gradient and Hall currents. In the absence of these effects, the principle of exchange of stabilities is found to hold.

5. The conditions \( \kappa < \min \left\{ \frac{E\eta f}{E'}, \frac{e}{4\Omega^2 k^2 \pi^2 (k_1 - ev')} \right\} \) and \( k > \max \left\{ \frac{H}{4\eta d}, \frac{ek_i}{4Nerd(2(k_1 - ev') - ek_i)} \right\}^{1/2}, \left( \frac{H}{4\eta d} \right) \) are sufficient for the non-existence of overstability. In the absence of viscoelasticity, rotation, magnetic field (hence Hall currents) and permeability, the above conditions, as expected, reduce to \( \kappa < \min \left\{ \eta, \kappa' \right\} \).

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_p )</td>
<td>specific heat of the fluid at constant pressure,</td>
</tr>
<tr>
<td>( c_s )</td>
<td>heat capacity of solid matrix,</td>
</tr>
<tr>
<td>( c_f )</td>
<td>heat capacity of the fluid,</td>
</tr>
<tr>
<td>( C )</td>
<td>concentration,</td>
</tr>
<tr>
<td>( C_0 )</td>
<td>concentration at bottom layer,</td>
</tr>
<tr>
<td>( C_d )</td>
<td>concentration at upper layer,</td>
</tr>
<tr>
<td>( d )</td>
<td>depth of fluid layer,</td>
</tr>
<tr>
<td>( D )</td>
<td>derivative with respect to ( z = (d/dz) ),</td>
</tr>
<tr>
<td>( E )</td>
<td>constant due to porous medium for heat,</td>
</tr>
<tr>
<td>( E' )</td>
<td>constant due to porous medium for solute,</td>
</tr>
<tr>
<td>( e )</td>
<td>charge of an electron,</td>
</tr>
<tr>
<td>( F )</td>
<td>factor due to kinematic viscoelasticity,</td>
</tr>
<tr>
<td>( f )</td>
<td>an arbitrary function of ( x, y, z, t ).</td>
</tr>
<tr>
<td>( f_m )</td>
<td>constant space distribution of ( f ),</td>
</tr>
<tr>
<td>( f_0 )</td>
<td>variation of ( f ) in absence of motion,</td>
</tr>
<tr>
<td>( G = \left( \frac{C_p}{g} \right) \beta )</td>
<td>factor due to compressibility,</td>
</tr>
<tr>
<td>( g = (0, 0, -g) )</td>
<td>acceleration due to gravity,</td>
</tr>
<tr>
<td>( h = (h_x, h_y, h_z) )</td>
<td>perturbation in magnetic field ( H = (0, 0, H) ),</td>
</tr>
<tr>
<td>( H = (0, 0, H) )</td>
<td>magnetic field vector having components ( (0, 0, H) ),</td>
</tr>
<tr>
<td>( k = \left( k_x^2 + k_y^2 \right)^{1/2} )</td>
<td>wave number of the disturbance,</td>
</tr>
<tr>
<td>( k_x, k_y )</td>
<td>wavenumbers in ( x ) and ( y ) directions respectively,</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>thermal coefficient of expansion,</td>
</tr>
<tr>
<td>( \alpha' )</td>
<td>solute coefficient of expansion,</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( (= dT / dz) ) temperature gradient,</td>
</tr>
<tr>
<td>( \beta' )</td>
<td>( (= dC / dz) ) concentration gradient,</td>
</tr>
<tr>
<td>( \delta )</td>
<td>( (\partial ) perturbation in the respective physical quantity,</td>
</tr>
<tr>
<td>( \eta )</td>
<td>resistivity,</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>porosity,</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>( (\partial ) perturbation in temperature,</td>
</tr>
<tr>
<td>( \theta )</td>
<td>( (\partial ) perturbation in solute concentration,</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>thermal diffusivity,</td>
</tr>
<tr>
<td>( \kappa' )</td>
<td>solute diffusivity,</td>
</tr>
<tr>
<td>( \xi )</td>
<td>( z )-component of current density,</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>( z )-component of vorticity,</td>
</tr>
<tr>
<td>( \mu )</td>
<td>viscosity of the fluid,</td>
</tr>
<tr>
<td>( \mu' )</td>
<td>viscoelasticity of the fluid,</td>
</tr>
<tr>
<td>( \mu_e )</td>
<td>magnetic permeability,</td>
</tr>
<tr>
<td>( \nu )</td>
<td>kinematic viscosity,</td>
</tr>
<tr>
<td>( \nu' )</td>
<td>kinematic visco-elasticity,</td>
</tr>
<tr>
<td>( \rho )</td>
<td>density of the fluid,</td>
</tr>
</tbody>
</table>

\[ k_1 \quad \text{medium permeability}, \]

\[ M \quad \text{Hall current parameter}, \]

\[ n \quad \text{growth rate of the disturbance}, \]

\[ N \quad \text{electron number density}, \]

\[ P \quad \text{factor due to permeability}, \]

\[ p \quad \text{fluid pressure}, \]

\[ p_m \quad \text{constant space distribution of } p, \]

\[ p_1 \quad \text{thermal Prandtl number}, \]

\[ p_2 \quad \text{magnetic Prandtl number}, \]

\[ q \quad \text{effective thermal conductivity of the pure fluid}, \]

\[ Q_1 \quad \text{Chandrasekhar number}, \]

\[ R_1 \quad \text{Rayleigh number}, \]

\[ S_1 \quad \text{solute Rayleigh number}, \]

\[ T_1 \quad \text{Taylor number}, \]

\[ T \quad \text{temperature}, \]

\[ T_0 \quad \text{temperature at bottom layer}, \]

\[ T_d \quad \text{temperature at upper layer}, \]

\[ q = (u, v, w) \quad \text{fluid velocity vector having components } (u, v, w), \]

\[ W_0 \quad \text{constant}, \]

\[ (x, y, z) \quad \text{x, y, z directions}, \]

\[ x \quad \text{wavenumber}. \]

References


**Biographical notes**

**Dr. Urvashi Gupta** is an Associate Professor in University Institute of Engineering and Technology, Panjab University, Chandigarh, India. She has more than twenty years of teaching and research experience. Her current area of research includes Fluid flow problems for Newtonian, non Newtonian, Micropolar and Nano fluids. She has published more than twenty five papers in referred international journals and presented many papers in national/ international conferences.

**Parul Aggarwal** is an Assistant Professor in the Department of Applied Sciences, Chandigarh College of Engineering and Technology, Chandigarh, India. She has more than 12 years of experience in teaching and research. Her current area of research includes fluid flow problems for viscoelastic/micropolar fluids. She received her M.Sc degree from Panjab University, Chandigarh in year 2000.

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