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# Bending analysis of laminated composite plates using finite element method

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#### Abstract

Laminated composite plate structures find numerous applications in aerospace, military and automotive industries. The role of transverse shear is very important in composites, as the material is weak in shear due to its low shear modulus compared to extensional rigidity. Hence, an accurate understanding of their structural behaviour is required, such as deflections and stresses. In this paper, a number of finite element analyses have been carried out for various side-to-thickness ratios, aspect ratios and modulus ratios to study the effect of transverse shear deformation on deflection and stresses of laminated composite plates subjected to uniformly distributed load. The numerical results showed, on the deflections and stresses, that the effect of coupling is to decrease the deflections with the increase in the aspect ratio and modulus ratio and increase the stresses with the increase in the side-to-thickness ratio and modulus ratio.

Keywords: Laminated composite plate, bending analysis, uniformly distributed load, finite element method.

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# 1. Introduction

Laminated composite materials are increasingly being used in a large variety of structures including aerospace, marine and civil infrastructure owing to the many advantages they offer: high strength/stiffness for lower weight, superior fatigue response characteristics, facility to vary fiber orientation, material and stacking pattern, resistance to electrochemical corrosion, and other superior material properties of composites. At the same time, the fabricated material poses new problems, such as failure due to delamination and pronounced transverse shear effects due to the high ratio of in-plane modulus to transverse shear modulus ((Kant *et al*, 1998; Zhang and Yang, 2009). An accurate understanding of their structural behaviour is required, such as the deflections, the through thickness distributions of stresses and strains, the large deflection behaviour and, of extreme importance for obtaining strong, reliable multi-layered structures. The finite element method is especially versatile and efficient for the analysis of complex structural behaviour of the composite laminated structures.

In the past, the structural behavior of plates and shells using the finite element method has been studied by a variety of approaches. Choudhary and Tungikar (2011) analyzed the geometrically nonlinear behavior of laminated composite plates using the finite element analysis. They studied the effect of number of layers, effect of degree of orthotropy (both symmetric and antisymmetric) and different fibre orientations on central deflections. Ganapathi *et al.* (1996) presented an eight-node  $C^0$  membrane-plate quadrilateral finite element-based on the Reissner–Mindlin plate theory to analyse moderately large deflection, static and dynamic problems of moderately thick laminates including buckling analysis and membrane-plate coupling effects. Han *et al.* (1994) used the hierarchical finite element method to carry out the geometrically nonlinear analysis of laminated composite rectangular plates. Based on the first-order shear deformation theory and Timoshenko's laminated composite beam functions, the current authors developed a unified formulation of a simple displacement based 3-node, 18degree-of-freedom flat triangular plate/shell element (Zhang and Kim, 2005) and two simple, accurate, shear-flexible displacement based 4-node quadrilateral elements (Zhang and Kim, 2006) and for linear and geometrically nonlinear analysis of thin to moderately thick laminated composite plates. The deflection and rotation functions of the element boundary were obtained from Timoshenko's laminated form Timoshenko's laminated composite plates.

composite beam functions. Reddy *et al.* (2012) applied the artificial neural networks (ANN) in predicting the natural frequency of laminated composite plates under clamped boundary condition. They used the D-optimal design in the design of experiments to carry out the finite element analysis. Wen *et al.* (2010a,b,c) used the finite element method to predict the damage level of the materials. They studied the prediction of the elastic-plastic damage and creep damage using Gurson model and creep damage model, which is based on the Kachanov-Rabothov continuum creep damage law. They also studied the creep damage properties of thin film/substrate systems by bending creep tests and carried the Simulation of the interface characterization of thin film/substrate systems.

Khoa and Thinh (2007) developed a rectangular non-conforming element based on Reddy's higher order shear deformation plate theory to analyze the laminated composite plates. They concluded that, the size of the mesh and the convergence of the method is involved by thickness ratio (h/a). A procedure for the reliability analysis of laminated composite plate structures with large rotations but moderate deformation under random static loads was presented via a corotational total Lagrangian finite element formulation which was based on the Von Karman assumption and first-order shear deformation theory (Kam *et al*, 1993). Based on a higher-order shear deformation theory involving four dependent unknowns and satisfying the vanishing of transverse shear stresses at the top and bottom surfaces of the plate, geometrically nonlinear flexural response characteristics of shear deformable asymmetrically laminated rectangular plates were investigated using a four-node rectangular C<sup>1</sup> continuous finite element having 14 degrees of freedom per node (Singh *et al*, 1994). Polit and Touratier (2002) used a high-order plate model which exactly ensured both the continuity conditions for displacements and transverse shear stresses at the interfaces between layers of a laminated structure, and the boundary conditions at the upper and lower surfaces of the plates to study the geometrically nonlinear finite element was developed, with the Argyris interpolation used for transverse displacement, the Ganev interpolation used for membrane displacements and transverse shear strain distributions represented by cosine functions.

Zinno and Barbero (1995) developed a three-dimensional element with two-dimensional kinematic constraints for the geometric nonlinear analysis of laminated composite plates using a total Lagrangian description and the principle of virtual displacements. Sridhar and Rao (1995) studied the large deformation analysis of circular composite laminated plates using a 48 degrees of freedom (DOF) four-node quadrilateral laminated composite shell finite element. Civalek *et al.* (2011) presented the nonlinear static analysis of a rectangular laminated composite thick plate resting on nonlinear two-parameter elastic foundation with cubic nonlinearity. They used the first-order shear deformation theory (FSDT) for plate formulation and investigated the effects of foundation and geometric parameters of plates on nonlinear deflections. Dharma Raju and Suresh Kumar (2011) developed the analytical procedure to investigate the bending characteristics of anti-symmetric and cross ply laminated composite plates based on a higher order shear displacement model with zig-zag function. They concluded that the effect of bending-stretching-coupling is significant for all modulus ratios except for those close to unity on anti-symmetric angle-ply laminated composite plates of same thickness of any number of layers. Zhang and Zhang (2011) investigated the global bifurcations and multi pulse chaotic dynamics of a simply supported laminated composite piezoelectric rectangular thin plate under combined parametric and transverse excitations. To analyze the complex nonlinear dynamic behaviour of the laminated composite piezoelectric rectangular thin plate they used phase portraits and Lyapunov exponents. Transverse bending of shear deformable laminated composite plates in Green–Lagrange sense accounting for the transverse shear and large rotations are presented by Singh and Dash (2010).

Salehi and Falahatgar (2010) studied the geometrically non-linear behavior of unsymmetrical, fiber-reinforced, laminated, annular sector composite plates. They applied the first order shear deformation theory to the von Karman type non-linear behavior of unsymmetrically, laminated, annular sector composite plates. Naghipour et al. (2008) performed the numerical simulations of laminated composite plates to decrease the weight of Military Mobile Bridges (MMB) using first order shear deformation theory and classical laminate plate theory. They studied the effects of fiber orientation, number of layers and stiffness ratio on the displacement and stress response of symmetric and anti-symmetric laminated composite plates subjected to uniform pressure loads. Tahani and Naserian Nik (2009) developed an analytical method for bending analysis of laminated composite plates with arbitrary lamination and boundary conditions within the displacement field of a first-order shear deformation theory (FSDT), they also employed the Levy-type solution in order to demonstrate the accuracy of the developed method.

Our previous study (Reddy *et al*, 2011) employed a distance-based optimal design in the design of experimental techniques and artificial neural networks to optimize the stacking sequence for a sixteen ply simply supported square laminated composite plate under uniformly distributed load (UDL) for minimizing the deflections and stresses using finite element method. The Present work is concerned with the bending analysis of a simply supported composite laminated plate under uniformly distributed load for various aspect ratios (a/b), modulus ratios ( $E_1/E_2$ ) and side-to-thickness ratios (a/h) using finite element method.

### 2. Geometry of the shell element

In ANSYS software, there are many element types available to model layered composite materials. In our FE analysis, the linear layered structural shell element is used. It is designed to model thin to moderately thick plate and shell structures with a side-to-thickness ratio of roughly 10 or greater. The linear layered structural shell element allows a total of 250 uniform-thickness layers. Alternatively, the element allows 125 layers with thicknesses that may vary bilinearly over the area of the layer. An accurate

representation of irregular domains (i.e. domains with curved boundaries) can be accomplished by the use of refined meshes and/or irregularly shaped elements. For example, a non-rectangular region cannot be represented using only rectangular elements; however, it can be represented by triangular and quadrilateral elements. Since, it is easy to derive the interpolation functions for a rectangular element, and it is much easier to evaluate the integrals over rectangular geometries than over irregular geometries, it is practical to use quadrilateral elements with straight or curved side assuming you have a means to generate interpolation functions and evaluate their integrals over the quadrilateral elements (Reddy, 1997). The linear layered structural shell element is shown in Figure 1. Nodes are represented by I, J, K, L, M, N, O, and P.



Figure 1. Geometry of 8-node element with six degrees of freedom

2.1. 2-D 8-Node quadrilateral shell displacement function The displacement equations are given as

$$\begin{cases}
 u \\
 v \\
 w
\end{cases} = \sum_{i=l}^{8} S_i \begin{cases}
 u_i \\
 v_i \\
 w_i
\end{cases} + \sum_{i=l}^{8} N_i \frac{rt_i}{2} \begin{vmatrix}
 a_{l,i} & b_{l,i} \\
 a_{2,i} & b_{2,i} \\
 a_{3,i} & b_{3,i}
\end{vmatrix} \begin{cases}
 \theta_{x,i} \\
 \theta_{y,i}
\end{cases}$$
(1)

where:  $S_i$  = shape functions,  $u_i$ ,  $v_i$ ,  $w_i$  = motion of node i; r = thickness coordinate  $t_i$  = thickness at node i; {a} = unit vector in s direction; {b} = unit vector in plane of element and normal to {a};  $\theta_{x,i}$  = rotation of node i about vector {a};  $\theta_{y,i}$  = rotation of node i about vector {b}. Note that the nodal translations are in global Cartesian space, and the nodal rotations are based on the element (s-t) space.

2.2. Stress-strain relationship

According to Hooke's law, The stress is related to the strains by

$$\{\sigma\} = [D]\{\varepsilon\} \text{ or } \varepsilon = [D]^{-1}\{\sigma\}$$
(2)

Where **Error! Bookmark not defined.** Total strain vector =  $[\mathcal{E}_x \ \mathcal{E}_y \ \mathcal{E}_z \ \mathcal{E}_{xy} \ \mathcal{E}_{yz} \ \mathcal{E}_{xz}]^T$ 

 $[D]^{-1}$  is the flexibility or compliance matrix and is given by

$$[D]^{-1} = \begin{bmatrix} 1/E_x & -v_{xy}/E_x & -v_{xz}/E_x & 0 & 0 & 0\\ -v_{yx}/E_y & 1/E_y & -v_{yz}/E_y & 0 & 0 & 0\\ -v_{zx}/E_z & -v_{zy}/E_z & 1/E_z & 0 & 0 & 0\\ 0 & 0 & 0 & 1/2G_{xy} & 0 & 0\\ 0 & 0 & 0 & 0 & 1/2G_{yz} & 0\\ 0 & 0 & 0 & 0 & 0 & 1/2G_{yz} \end{bmatrix}$$
(3)

Where  $E_x =$  Young's modulus in the x-direction;  $E_y =$  Young's modulus in the y-direction

E<sub>z</sub>= Young's modulus in the z-direction;  $v_{xy}$  =major Poisson's ratio;  $v_{yx}$  =minor Poisson's ratio;

 $v_{xz}$ =Major poi; on's ratio X-Z plane;=shear modulus in the xy plane

Also, the [D]<sup>-1</sup> matrix is presumed to be symmetric, so that:

$$\frac{\nu_{y\chi}}{E_y} = \frac{\nu_{\chi y}}{E_\chi} \quad \frac{\nu_{Z\chi}}{E_z} = \frac{\nu_{\chi Z}}{E_\chi} \quad \frac{\nu_{Zy}}{E_z} = \frac{\nu_{yZ}}{E_y}$$

Expanding Eq. (2) with Eq.(3), the strain equations are obtained as

$$\varepsilon_{x} = \frac{\sigma_{x}}{E_{x}} - \frac{\upsilon_{xy}\sigma_{y}}{E_{x}} - \frac{\upsilon_{xz}\sigma_{z}}{E_{x}}; \varepsilon_{y} = \frac{\sigma_{y}}{E_{y}} - \frac{\upsilon_{xy}\sigma_{x}}{E_{x}} - \frac{\upsilon_{yz}\sigma_{z}}{E_{y}}; \varepsilon_{z} = \frac{\sigma_{z}}{E_{z}} - \frac{\upsilon_{xz}\sigma_{x}}{E_{x}} - \frac{\upsilon_{yz}\sigma_{y}}{E_{y}}$$
$$\varepsilon_{xy} = \frac{\sigma_{xy}}{G_{xy}}; \varepsilon_{yz} = \frac{\sigma_{yz}}{G_{yz}}; \varepsilon_{xz} = \frac{\sigma_{xz}}{G_{xz}}$$

Where  $\mathcal{E}_x$  =direct strain in the x-direction;  $\mathcal{E}_y$  =direon strain in the x-direction;  $\mathcal{E}_{xy}$  =shear strain in the x-y plane

 $\sigma_x$  =direct stress in the x-direction;  $\sigma_y$  =direct stress in the x-direction;  $\sigma_{xy}$  =shear stress on the x-y plane and

 $\{\sigma\}$ =stress vector= $[\sigma_x \sigma_y \sigma_{xy} \sigma_{yz} \sigma_{xz}]^T$ 

[D] = elasticity or elastic stiffness matrix or stress-strain matrix and is defined in Eq. (4) through Eq.(7) or inverse is defined in Eq.(3) /

$$\sigma_{x} = \frac{E_{x}}{h} \left( 1 - V_{yz}^{2} \frac{E_{z}}{E_{y}} \right) \varepsilon_{x} + \frac{E_{y}}{h} \left( V_{xy} - V_{xz} V_{yz} \frac{E_{z}}{E_{y}} \right) \varepsilon_{y} + \frac{E_{z}}{h} \left( V_{yz} V_{xy} + V_{xz} \right) \varepsilon_{z}$$

$$\tag{4}$$

$$\sigma_{y} = \frac{E_{x}}{h} \left( v_{xy} - v_{xz} v_{yz} \frac{E_{z}}{E_{y}} \right) \varepsilon_{x} + \frac{E_{y}}{h} \left( 1 - v_{xz}^{2} \frac{E_{z}}{E_{x}} \right) \varepsilon_{y} + \frac{E_{z}}{h} \left( v_{yz} + v_{xy} v_{xz} \frac{E_{y}}{E_{x}} \right) \varepsilon_{z}$$

$$\tag{5}$$

$$\sigma_{z} = \frac{E_{x}}{h} \left( v_{yz} v_{xy} + v_{xz} \right) \varepsilon_{x} + \frac{E_{y}}{h} \left( v_{yz} + v_{xy} v_{xz} \frac{E_{y}}{E_{x}} \right) \varepsilon_{y} + \frac{E_{z}}{h} \left( 1 - v_{xy}^{2} \frac{E_{y}}{E_{x}} \right) \varepsilon_{z}$$
(6)

$$\sigma_{xy} = 2G_{xy}; \ \sigma_{yz} = 2G_{yz}; \ \sigma_{xz} = 2G_{xz}$$
Where
$$(7)$$

wnere

$$h = 1 - v_{xy}^{2} \frac{E_{y}}{E_{x}} - v_{yz}^{2} \frac{E_{z}}{E_{x}} - v_{xz}^{2} \frac{E_{z}}{E_{x}} - 2v_{yz}v_{xz}v_{xy} \frac{E_{z}}{E_{x}} \text{ and shear moduli } G_{xy}, G_{yz} \text{ and } G_{xz} \text{ are computed as}$$
$$G_{xy} = \frac{\sqrt{E_{x}E_{y}}}{2\left(1 + \sqrt{v_{xy}v_{yx}}\right)}, G_{yz} = \frac{\sqrt{E_{y}E_{z}}}{2(1 + \sqrt{v_{yz}v_{zy}})} \text{ and } G_{xz} = \frac{\sqrt{E_{x}E_{z}}}{2(1 + \sqrt{v_{xz}v_{zx}})}$$

By integrating the stresses through the plate thickness, we obtain the generalized force -strain relation and moment-curvature relationships for a linear variation of strain through the thickness of the plate and may be defined as

$$\left\{ \frac{N}{M} \right\} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \left\{ \frac{\varepsilon}{K} \right\}$$
(8)

Where N=In-plane force resultants per unit length= $[N_x, N_y, N_{xy}]$ ; M=Bending moments per unit length= $[M_y, M_y, M_{xy}]$ A=Extensional stiffness matrix, relates in-plane forces to the in-plane strains

B=Coupling stiffness matrix, which couples the forces and moments to the mid-plane strain-curvature.

D=Bending moment stiffness matrix which relates bending moments to the plate curvature.

 $\mathcal{E}$  =Membrane strains; K = Curvature strains

The in-plane force and moment resultants per unit length are computed as

$$\begin{cases}
N_x \\
N_y \\
N_{xy}
\end{cases} = \int_{-h/2}^{h/2} \begin{cases}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{cases} dz$$
(9)

$$\begin{cases}
 M_x \\
 M_y \\
 M_{xy}
\end{cases} = \int_{-h/2}^{h/2} \begin{cases}
 \sigma_x \\
 \sigma_y \\
 \sigma_{xy}
\end{cases} zdz$$
(10)

The values of A, B and D matrix are evaluated with the material properties ( $E_1$ ,  $E_2$ ,  $E_3$ ,  $v_{12}$ ,  $v_{23}$ ,  $v_{13}$ ,  $G_{12}$ ,  $G_{23}$ , and  $G_{13}$ ). For details see (Reddy, 1997).

2.3. Validation of linear layered structural shell element- case study

In order to validate the usage of the linear layered structural shell element, a numerical example is solved in static analysis. The boundary condition is simply supported and the geometry and material properties are as follows:

 $E_1/E_2=40$ ,  $G_{12}=G_{13}=0.6E_2$ ,  $G_{23}=0.5E_2$ ,  $v_{12}=0.25$ , a/h=10, a=10. The center deflection and stresses are presented here in non-dimensional form using the following:

$$\overline{w} = w \times \frac{E_2 h^3}{aa^4} \times 10^3$$
,  $\overline{\sigma}_x = \sigma_x \times \frac{h}{aq}$ ,  $\overline{\sigma}_y = \sigma_y \times \frac{h}{aq}$  and  $\overline{\tau}_{xy} = \tau_{xy} \times \frac{h}{aq}$ 

h

Table 1 and Table 2 represent the mesh convergence study and comparison of results of non-dimensional displacement obtained from Reddy (1997) and the ANSYS computer program. The results using a free mesh show an excellent correlation to the results given by Reddy (1997).

Table 1. Nondimensional displacement of composite plates (cross- ply)

Mesh	0/90	0/90/0	0/90/90/0	0/90/0/90
$2 \times 2$	14.222	6.8178	6.5423	6.7662
$4 \times 4$	14.478	-	6.7402	6.9897
10×10	14.488	6.9904	-	6.9965
20×20	14.488	6.9905	6.7459	6.9966
$40 \times 40$	14.475	6.9857	6.7405	6.9904
FSDT (Reddy)	14.069	6.919	6.682	6.9260
Difference (%)	2.8857	0.9640	0.8754	0.929

**Table 2.** Nondimensional displacement of composite plates  $(\theta / - \theta / \theta / - \theta)$ 

Mesh	5	15				
$2 \times 2$	6.7716	6.3811				
$4 \times 4$	-	6.6625				
10×10	6.9652	-				
20×20	-	6.6668				
$40 \times 40$	6.9623	6.6631				
FSDT (Reddy)	6.741	6.086				
Difference (%)	3.2828	9.4824				

### 3. Finite Element Analsysis

The physical structure that was used in this work is a fibre reinforced composite plate, shown in Figure 2. The length (a) and width (b) of the plate is 250mm and thickness (h) of the plate is 25mm. A number of analyses are performed in this design study, using a finite element model of the plate. The model was developed using 1600 linear layered structural shell elements in ANSYS 10.0. The global x-coordinate is taken along the length of the plate; the global y-coordinate is taken along the width of the plate while the global z-direction is taken out the plate surface. There are 40 elements in the axial direction and 40 along the width one. In this Finite element analysis, all the sides are constrained in the Z direction only. The pressure applied on the plate is  $1N/mm^2$ . In this study, sixteen ply ( $[45/-45/45/-45/-45/90/45/0]_s$ ) symmetric laminated composite plate (Reddy *et al.*, 2011) is considered in the analysis. The plate is analyzed for deflections and stresses under a simply supported boundary condition when the plate is subjected to a uniformly distributed load working along the Z - direction for various side-to-thickness ratios (a/h), aspect ratios (a/b) and modulus ratios ( $E_1/E_2$ ). The centre deflection and stresses are presented here in non-dimensional form using the following.



Figure 2. Uniformly loaded simply supported composite plate

The material properties used throughout this study are shown in Table 3 (Baker et al., 2004).

Table 3. Material properties (Boron/epoxy)

$E_1(GPa)$	$E_2(GPa)$	$E_3(GPa)$	$G_{12}(GPa)$	$G_{23}(GPa)$	$G_{13}(GPa)$	$v_{12}$	$v_{23}$	$v_{13}$
210	19	19	4.8	4.8	4.8	0.25	0.25	0.25

# 4. Results and discussion

The classical plate/shell theory which is adequate only for thin shells. However, the linear layered structureal shell element allows to model thin to moderately thick plate and shell structures with a side-to-thickness ratio of roughly 10 or greater. The plate studied here is 25 mm thick and especially for a/h=10 to 40, the thin shell model as well as element should not be used. Since in thick plates, the bending and through-thickness transverse shear stresses are dominant than membrane stress, which are not captured by the thin plate model where the thickness and out of plane stresses are assumed negligible.

The linear layered structural shell element is used to study the effect of transverse shear deformation, material orthotropy and aspect ratio on nondimensional maximum transverse deflections and stresses of a sixteen ply simply supported symmetric laminated composite plate under uniformly distributed load. The results obtained for deflections and stresses from the finite element analysis are plotted in nondimensional quantities as a function of aspect ratios (a/b), modulus ratios (E<sub>1</sub>/E<sub>2</sub>) and side-to-thickness ratios (a/h). In the analysis, the length of the plate and young modulus in the x-direction is changed. Figures 3 to 6 show the effect of bending-stretching coupling and plate aspect ratio on the transverse deflections ( $\overline{w}$ ), normal stresses and shear stresses ( $\overline{\sigma}_x, \overline{\sigma}_y$  and  $\overline{\tau}_{xy}$ ). It is observed that the non-dimensional deflection is maximum for E<sub>1</sub>/E<sub>2</sub> =1 (and aspect ratio=1), and

the minimum for  $E_1/E_2=11$  (and aspect ratio=5). This is due to the fact that, as the Young's modulus of the material increases, the effect of coupling is to decrease the deflections and stresses. The coupling coefficients increase in magnitude (hence the effect of coupling increases) with the increase of the modulus ratio for deflections ( $\overline{w}$ ) and decrease of the modulus ratio for stresses and also, the effect of coupling on deflections is quite significant for aspect ratio less than 3 and is negligible for all values of a/b greater than 3. The effect of coupling is to decrease the stresses with the increase in aspect ratios. The stresses ( $\overline{\sigma}_x, \overline{\sigma}_y$  and  $\overline{\tau}_x$ )

are (maximum at  $E_1/E_2 = 11$  (and aspect ratio=1) and minimum at  $E_1/E_2 = 1$ ( and aspect ratio=5). This is because the plate area increases as the aspect ratio increases and hence, the applied load per unit area decreases. Figures 7 to 10 show the effect of the transverse shear deformation and bending extensional coupling and material orthotropy on transverse deflections of a simply supported sixteen ply symmetric laminated composite plate under uniformly distributed load. The degree of orthotropy has less influence on the deflections for larger modulus ratios and has considerable influence on the stresses. The effect of transverse shear deformation is to decrease the deflections and increase stresses with the increase in the modulus ratio and side-to-thickness ratio. It is observed that, the non-dimensional deflection is maximum for side to thickness ratio is 10 (and  $E_1/E_2=1$ ), and minimum of side to thickness ratio is 40 (and  $E_1/E_2=11$ ). This is due to the fact that the plate area increases as the side to thickness ratio increases and hence, the applied load per unit area decreases. As the Young's modulus of the material increases, the stresses ( $\overline{\sigma}_x$ ,  $\overline{\sigma}_y$  and

to thickness ratio is equal to 40 and higher at side to thickness ratio is equal to 10. For the selected stacking sequence, for modulus

 $<sup>\</sup>bar{\tau}_{xy}$ ) are maximum at side to thickness ratio is 40 and minimum at 10. This is due to the fact that the modulus ratio is lower at side

ratio 2 and side to thickness ratio 10, the variation of deflection (mm) and stresses (Sx, Sy and Sxy, N/mm<sup>2</sup>) plots obtained in ANSYS are shown in Fig 11-14.



**Figure 3.** Nondimensional displacement( $\overline{w}$ ) versus plate aspect ratio (a/b) for different modulus ratios (E<sub>1</sub>/E<sub>2</sub>)



Figure 4. Nondimensional Normal stress ( $\bar{\sigma}_x$ ) versus plate aspect ratio (a/b) for different modulus ratios (E<sub>1</sub>/E<sub>2</sub>)



Figure 5. Nondimensional Normal stress ( $\bar{\sigma}_y$ ) versus plate aspect ratio (a/b) for different modulus ratios (E<sub>1</sub>/E<sub>2</sub>)



**Figure 6.** Nondimensional Shear stress ( $\bar{\tau}_{xy}$ ) versus plate aspect ratio (a/b) for different modulus ratios (E<sub>1</sub>/E<sub>2</sub>)



Figure 7. Effect of shear deformation and material orthotropy on transverse deflections of simply supported laminated plate under uniformly distributed load



Figure 8. Effect of shear deformation and material orthotropy on Normal stress ( $\overline{\sigma}_x$ ) of laminated plate under uniformly distributed load



Figure 9. Effect of shear deformation and material orthotropy on Normal stress  $(\overline{\sigma}_y)$  of laminated plate under uniformly distributed load



Figure 10. Effect of shear deformation and material orthotropy on shear stress  $(\bar{\tau}_{xy})$  of laminated plate under uniformly distributed load



Fig 11: variation of central deflection (mm) for modulus ratio 2 and side to thickness ratio 10



Fig 12: variation of stress (N/mm<sup>2</sup>) in X-direction for modulus ratio 2 and side to thickness ratio 10



Fig 13: variation of stress (N/mm<sup>2</sup>) in Y-direction for modulus ratio 2 and side to thickness ratio 10



Fig 14: variation of stress (N/mm<sup>2</sup>) in XY-direction for modulus ratio 2 and side to thickness ratio 10

# 5. Conclusions

A number of analyses are performed in this design study, using a finite element model of the plate for various side-to-thickness ratios, aspect ratios and modulus ratios. The model was developed using linear layered structural shell elements in ANSYS 10.0. From the results of sixteen ply simply supported symmetric laminated composite plate; it was observed that, the deflections are larger for smaller modulus ratios and aspect ratios, the degree of orthotropy has less influence on the deflections for large ratios of  $E_1/E_2$ , the effect of shear deformation is to decrease the deflections and increase the stresses with the increase of modulus ratios and side-to-thickness ratios.

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