Response of RLC network circuit with steady source via rohit transform

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Abstract

The electric network circuits are designed by using the elements like resistor $R$, inductor $L$, and capacitor $C$. There are a number of techniques: exact, approximate, and purely numerical available for analyzing the $RLC$ network circuits. Since the application of numerical method becomes more complex, computationally intensive, or needs complicated symbolic computations, there is a need to seek the help of integral transform methods for analyzing the $RLC$ network circuits. Integral transform methods provide effective ways for solving a variety of problems arising in basic sciences and engineering. In this paper, a new integral transform Rohit transform is discussed for obtaining the response of a series $RLC$ electric network circuit connected to a steady voltage source, and a parallel $RLC$ electric network circuit connected to a steady current source. The response of a series $RLC$ network circuit connected to a steady voltage source via the application of Rohit transform will provide an expression for the electric current, and that of a parallel $RLC$ network circuit connected to a steady current source will provide an expression for the voltage across the parallel $RLC$ electric network circuit. The nature of the response of such series (or parallel) network circuits is determined by the values of $R$, $L$, and $C$ of the electric network circuit. The Rohit transform will come out to be a powerful technique for analyzing such series or parallel electric network circuits with steady voltage or current sources.

Keywords: Rohit transform, Series and Parallel RLC Network Circuits, Response

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1. Introduction

A series or parallel $RLC$ network circuit consists of three basic electric elements- an inductor having inductance $L$, a capacitor having capacitance $C$, and a resistor having resistance $R$ and is widely used as a tuning or resonant circuit in the radio and television sets, and also in oscillatory circuits. There is a number of techniques available for analyzing the $RLC$ network circuits like calculus method (Chitode and Jalnekar, 2007), matrix method (Gupta et al, 2018), residue theorem approach (Gupta et al, 2020), convolution method (Gupta et al, 2019) and (Gupta et al, 2020). Since the application of these methods becomes more complex, computationally intensive, or needs complicated symbolic computations, there is a need to seek the help of integral
transform methods for analyzing the RLC network circuits. Integral transforms like Laplace transform (Murray, 1965), Mohand transform (Gupta et al, 2020 and Verma et al, 2020), Gupta transform (Gupta et al, 2020), and etc. provide effective ways for solving a variety of problems arising in basic sciences and engineering. (Gupta et al, 2021) and (Verma et al, 2020). The Rohit Transform is a new integral transform which has been proposed by the author Rohit Gupta in the recently in the year 2020 and generally, it has been applied to boundary value problems in most of the science and engineering disciplines such as for the analysis of radioactive decay problem, damped oscillator (Gupta et al, 2020) for the analysis of uniform infinite fin (Pandita et al, 2020); for solving the Schrodinger equation (Gupta et al, 2020); for the Analysis of RLC circuits with exponential excitation sources (Gupta et al, 2022); for the Analysis of Basic Series Inverter (Anamika et al, 2020); for the analysis of Electric Network Circuits with Sinusoidal Potential Sources (Talwar et al, 2020); for the analysis of one-way streamline flow between parallel plates (Gupta et al, 2022), and etc. This paper presents the application of a new integral transform Rohit transform for the analysis of a series RLC network circuit connected to a steady voltage source, and a parallel RLC network circuit connected to a steady current source. The response obtained by Rohit transform is the same as is obtained from the other integral transforms or approaches. This paper has brought up the new integral transform Rohit transform as a powerful technique for analyzing such series or parallel electric network circuits with steady voltage or current sources.

The Rohit transform of $g(y)$, $y \geq 0$ is denoted by $G(r)$ and is given by

$$G(r) = r^3 \int_0^\infty e^{-ry} g(y)dy,$$

provided the integral is convergent, where $r$ may be a real or complex parameter (Gupta et al, 2020).

According to the definition of Rohit Transform (RT),

$$R(g(y)) = r^3 \int_0^\infty e^{-ry} g(y)dy,$$

then

1. $R\{1\} = r^3 \int_0^\infty e^{-ry} dy = -r^2 (e^{-\infty} - e^{-0}) = -r^2 (0 - 1) = r^2$

Hence $R\{1\} = r^2$

2. $R\{y^n\} = r^3 \int_0^\infty e^{-ry} y^n dy = r^3 \int_0^\infty e^{-r\frac{z}{r}} \left(\frac{z}{r}\right)^n \frac{dz}{r} = r^2 \int_0^\infty e^{-z} (z)^n dz$

Applying the definition of the gamma function,

$$R\{y^n\} = \frac{n!}{r^{n+2}}$$

Hence $R\{y^n\} = \frac{n!}{r^{n+2}}$

3. $R\{\sin by\} = r^3 \int_0^\infty e^{-ry} \sin by dy = r^3 \int_0^\infty e^{-ry} \left(\frac{e^{ib}y - e^{-ib}y}{2i}\right) dy = r^3 \int_0^\infty \left(\frac{e^{-ry}(e^{ib}y - e^{-ib}y)}{2i}\right) dy = -\frac{r^3}{2(r-b)} (e^{-\infty} - e^{-0}) + \frac{r^3}{2(r+b)} (e^{-\infty} - e^{-0}) = \frac{r^3}{2(r-b)} - \frac{r^3}{2(r+b)} = \frac{br^3}{r^2-b^2}$

Hence $R\{\sin by\} = \frac{br^3}{r^2-b^2}$

4. $R\{\sinh by\} = r^3 \int_0^\infty e^{-ry} \sinh by dy = r^3 \int_0^\infty e^{-ry} \left(\frac{e^{ib}y + e^{-ib}y}{2}\right) dy = r^3 \int_0^\infty \left(\frac{e^{-ry}(e^{ib}y + e^{-ib}y)}{2}\right) dy = -\frac{r^3}{2(r-b)} (e^{-\infty} - e^{-0}) - \frac{r^3}{2(r+b)} (e^{-\infty} - e^{-0}) = \frac{r^3}{2(r-b)} + \frac{r^3}{2(r+b)} = \frac{br^3}{r^2-b^2}$

Hence $R\{\sinh by\} = \frac{br^3}{r^2-b^2}$

5. $R\{\cos by\} = r^3 \int_0^\infty e^{-ry} \cos by dy = r^3 \int_0^\infty e^{-ry} \left(\frac{e^{ib}y + e^{-ib}y}{2}\right) dy = r^3 \int_0^\infty \left(\frac{e^{-ry}(e^{ib}y + e^{-ib}y)}{2}\right) dy = -\frac{r^3}{2(r-b)} (e^{-\infty} - e^{-0}) - \frac{r^3}{2(r+b)} (e^{-\infty} - e^{-0}) = \frac{r^3}{2(r-b)} + \frac{r^3}{2(r+b)} = \frac{br^3}{r^2-b^2}$

Hence $R\{\cos by\} = \frac{br^3}{r^2-b^2}$

6. $R\{\cosh by\} = r^3 \int_0^\infty e^{-ry} \cosh by dy = r^3 \int_0^\infty \left(\frac{e^{ib}y + e^{-ib}y}{2}\right) dy = \left[\frac{e^{ib}y + e^{-ib}y}{2}\right]_0^\infty = \frac{r^3}{2(r-b)} (e^{-\infty} - e^{-0}) - \frac{r^3}{2(r+b)} (e^{-\infty} - e^{-0}) = \frac{r^3}{2(r-b)} + \frac{r^3}{2(r+b)} = \frac{br^3}{r^2-b^2}$

Hence $R\{\cosh by\} = \frac{br^3}{r^2-b^2}$

7. $R\{e^{by}\} = r^3 \int_0^\infty e^{-ry} e^{by} dy = r^3 \int_0^\infty (e^{-ry}(e^{by})) dy = -\frac{r^3}{r-b} (e^{-\infty} - e^{-0}) = \frac{r^3}{r-b}$

Hence $R\{e^{by}\} = \frac{r^3}{r-b}$

8. $R\{u(y-b)\} = r^3 \int_0^\infty e^{-ry} u(y-b) dy$

Since the unit step function $u(y-b)$, $b \geq 0$, is defined as $u(y-b) = 0$ when $y < b$ and $u(y-b) = 1$ when $y \geq b$, therefore, the above integral can be rewritten as
\[ R\{u(y - b)\} = r^3 \int_0^\infty e^{-ry} \, dy = \frac{r^3}{r} \left( e^{-\infty} - e^{-br} \right) = r^2 e^{-br} \]

Hence \( R\{u(y - b)\} = r^2 e^{-br} \)

9. \( R\{\delta(y)\} = r^3 \int_0^\infty e^{-ry} \, dy \)

Since the Dirac delta (Impulse) function \( \delta(y) \), is defined as \( \delta(y) = 0 \) when \( y \neq 0 \) and \( \int_0^\infty g(y) \, \delta(y) \, dy = g(0) \), therefore, the above integral can be rewritten as

\[ R\{\delta(y)\} = r^3 e^{-r(0)} = r^3 \]

Hence \( R\{\delta(y)\} = r^3 \)

Hence we found that the Rohit Transform (RT) of some elementary functions are

- \( R\{1\} = r^2, \ r > 0 \)
- \( R\{y^n\} = \frac{n!}{r^{n-2}}, \ where \ n = 0,1,2,3 \ldots \)
- \( R\{e^{by}\} = \frac{r^3}{r-b}, \ r > b \)
- \( R\{sinby\} = \frac{b^r}{r^2+b^2}, \ r > 0 \)
- \( R\{sinhby\} = \frac{b^r}{r^2-b^2}, \ r > |b| \)
- \( R\{cosby\} = \frac{r^4}{r^2+b^2}, \ r > 0 \)
- \( R\{coshby\} = \frac{r^4}{r^2-b^2}, \ r > |b| \)
- \( R\{u(y - b)\} = r^2 e^{-br} \)
- \( R\{\delta(y)\} = r^3 \)

Let \( g(y) \) is continuous function and is piecewise continuous on any interval, then the Rohit Transform (RT) of first derivative of \( g(y) \) i.e. \( R\{g'(y)\} \) is given by

\[ R\{g'(y)\} = r^3 \int_0^\infty e^{-ry} g'(y) \, dy \]

Integrating by parts and applying limits, we get

\[ R\{g'(y)\} = r^3 \{-g(0) - \int_0^\infty e^{-ry} g(y) \, dy\} = r^3\{-g(0) + r \int_0^\infty e^{-ry} g(y) \, dy\} = r^3\{-g(0) + rR\{g(y)\}\} = rG(r) - r^3g(0) \]

Hence

\[ R\{g'(y)\} = rG(r) - r^3g(0) \]

Therefore, on replacing \( g(y) \) by \( y \) and \( g'(y) \) by \( g''(y) \), we have

\[ R\{g''(y)\} = rR\{g'(y)\} - r^3g(0) = r\{rR\{g(y)\} - r^3g(0)\} - r^3g'(0) = r^2r\{g(y)\} - r^4g(0) - r^3g'(0) \]

Hence

\[ R\{g''(y)\} = r^2G(r) - r^4g(0) - r^3g'(0) \]

Similarly,

\[ R\{g'''(y)\} = r^3G(r) - r^5g(0) - r^4g'(0) - r^3g''(0), \text{ and so on.} \]

2. Material and Method

Case I: Analysis of a series RLC network circuit with a steady voltage source

The governing differential equation for analyzing a series RLC network circuit connected to a steady voltage source of potential \( V_o \) (Chitode and Jalnekar, 2007) and (Gupta et al, 2018) is given by

\[ \frac{d}{dt}I(t) + \frac{1}{L}I(t) + \frac{q(t)}{C} = V_o \ldots (1) \]

Differentiating both sides with respect to \( t \) and simplifying we get, we get

\[ \frac{d}{dt}I(t) + \frac{R}{L}I(t) + \frac{1}{LC}I(t) = 0 \ldots (2) \]

Here, \( I(t) \) is the instantaneous current through the series RLC network circuit.

To solve equation (2) by Rohit transform, the relevant initial conditions are as follows (Gupta et al, 2019):
Taking inverse Rohit transform of equation (2), we get
\[ q^2 I(q) - q^4 I(0) - q^3 I(0) + \frac{R}{L} \{ q I(q) - q^3 I(0) \} + \frac{1}{LC} I(q) = 0 \quad \text{(3)} \]

Applying boundary conditions: \( I(0) = 0 \) and \( I(0) = \frac{V_0}{L} \), equation (3) becomes,
\[ q^2 I(q) - q^3 \frac{V_0}{L} + \frac{R}{L} q I(q) + \frac{1}{LC} I(q) = 0 \]
Or
\[ I(q) q^2 + \frac{R}{L} q + \frac{1}{LC} = q^3 \frac{V_0}{L} \]
Or
\[ I(q) = \frac{V_0}{L} \left[ \frac{q^3}{q^2 + \frac{R}{L} q + \frac{1}{LC}} \right] \quad \text{.... (4)} \]

For convenience, let \( 2\delta = \frac{R}{L} \) and \( \omega = \frac{1}{\sqrt{LC}} \), then equation (4) becomes
\[ I(q) = \frac{V_0}{L} \left[ \frac{q^3}{q^2 + 2\delta q + \omega^2} \right] \]

or
\[ I(q) = \frac{V_0}{L} \left[ \frac{q^3}{(q + \delta)^2 - (\sqrt{\delta^2 - \omega^2})^2} \right] \]

or
\[ I(q) = \frac{V_0}{L} \left[ \frac{q^3}{(q + \delta + \sqrt{\delta^2 - \omega^2})(q + \delta - \sqrt{\delta^2 - \omega^2})} \right] \quad \text{.... (5)} \]

Again, for convenience, let us substitute
\( \delta + \sqrt{\delta^2 - \omega^2} = \beta_1 \) and \( \delta - \sqrt{\delta^2 - \omega^2} = \beta_2 \)

such that \( \beta_1 - \beta_2 = 2\sqrt{\delta^2 - \omega^2} \), then equation (5) can be rewritten as
\[ I(q) = \frac{V_0}{L} \left[ \frac{q^3}{(q + \beta_1)(q + \beta_2)} \right] \quad \text{.... (6)} \]

This equation (6) can be rewritten as
\[ I(q) = \frac{V_0}{L} \left[ \frac{q^3}{(q + \beta_1)(\beta_2 - \beta_1)} - \frac{q^3}{(\beta_2 - \beta_1)} \right] \quad \text{.... (7)} \]

Taking inverse Rohit transform of equation (7), we can write
\[ I(t) = \frac{V_0}{L} \left[ \frac{e^{-\beta_1 t} - e^{-\beta_2 t}}{\beta_2 - \beta_1} \right] \quad \text{.... (8)} \]

Or
\[ I(t) = \frac{V_0}{L} e^{-\delta t} \frac{e^{\sqrt{\delta^2 - \omega^2} t} - e^{-\sqrt{\delta^2 - \omega^2} t}}{2\sqrt{\delta^2 - \omega^2}} \quad \text{.... (8)} \]

Or
\[ I(t) = \frac{V_0}{L} e^{-\delta t} \frac{e^{\sqrt{\delta^2 - \omega^2} t} - e^{-\sqrt{\delta^2 - \omega^2} t}}{\sqrt{\delta^2 - \omega^2}} \quad \text{.... (8)} \]

Or
\[ I(t) = \frac{V_0}{L} e^{-\frac{R}{2L} t} \left\{ \exp\left( \frac{R}{2L} t - \frac{1}{LC} t \right) - \exp\left( -\frac{R}{2L} t - \frac{1}{LC} t \right) \right\} \quad \text{.... (9)} \]

This equation (9) provides an expression for the electric current flowing through a series R L C network circuit connected to a steady voltage source. It is clear from the equation that the nature of the response of the network circuit depends on quantity \( \sqrt{\frac{R^2}{2L}} = \frac{1}{LC} \), whether it is real, zero, or imaginary. The value of the quantity \( \sqrt{\frac{R^2}{2L}} = \frac{1}{LC} \), in turn, depends on the values of elements R, L, and C of the network circuit.
Case II: Analysis of a parallel RLC network circuit with a steady current source

The governing differential equation for analyzing a parallel RLC network circuit connected to a steady current source providing steady current $I_0$ (Chitode and Jalnekar, 2007) and (Gupta et al, 2018) is given by

$$\frac{V(t)}{R} + \frac{1}{L} \int V(t) dt + C \frac{dV}{dt} = I_0 \quad (10)$$

Differentiate both sides of equation (10) with respect to $t$ and simplifying, we get,

$$\frac{dV}{dt} + \frac{1}{RC} V(t) + \frac{1}{LC} V(t) = 0 \quad (11)$$

To solve equation (11) by convolution method, the relevant initial conditions (Gupta et al, 2019) are as follows:

(i) At $t = 0$, $V(0) = 0$.

(ii) Since at $t = 0$, $V(0) = 0$, therefore, equation (1) gives $\dot{V}(0) = \frac{I_0}{C}$.

Taking Rohit transform of equation (11), we get

$$q^2\tilde{V}(q) - q^3V(0) - q^3\tilde{V}(0) + \frac{1}{RC} \{ q \tilde{V}(q) - q^3V(0) \} + \frac{1}{LC} \tilde{V}(q) = 0 \quad (12)$$

Applying boundary conditions:

$$V(t) = \frac{I_0}{C}$$

We substitute

$$\tilde{V}(q) = \frac{I_0}{C}$$

The governing differential equation for analyzing a parallel RLC network circuit with a steady current source providing steady current $I_0$ (Chitode and Jalnekar, 2007) and (Gupta et al, 2018) is given by

$$\tilde{V}(q) = \frac{1}{q^2 + \frac{1}{RC} q + \frac{1}{LC}} \tilde{V}(q)$$

For convenience, let $2a = \frac{1}{RC}$ and $\omega' = \frac{1}{LC}$, then equation (13) becomes

$$\tilde{V}(q) = \frac{I_0}{C} \left[ \frac{q^3}{q^2 + 2aq + \omega'^2} \right]$$

Taking inverse Rohit transform this equation, we can write

$$V(t) = \frac{I_0}{C} \left[ e^{-a_1 t} - e^{-a_2 t} \right]$$

This equation can be rewritten as

$$\tilde{V}(q) = \frac{I_0}{C} \left[ \frac{q^3}{(q + a_1)(a_2 - a_1) - (a_2 - a_1)(q + a_2)} \right]$$

Taking inverse Rohit transform this equation, we can write

$$V(t) = \frac{I_0}{C} \left[ e^{-a_1 t} - e^{-a_2 t} \right]$$

Or

$$V(t) = \frac{I_0}{C} \left[ e^{-a_1 t} - e^{-a_2 t} \right]$$

Or

$$V(t) = \frac{I_0}{C} \left[ e^{-a_1 t} - e^{-a_2 t} \right]$$

Or

$$V(t) = \frac{I_0}{C} \left[ e^{-a_1 t} - e^{-a_2 t} \right]$$
V(t) = \frac{1}{c} e^{-at} \frac{e^{\sqrt{a^2-\omega^2}t} - e^{-\sqrt{a^2-\omega^2}t}}{2\sqrt{a^2-\omega^2}}

Or

V(t) = \frac{1}{c} e^{-\frac{1}{2RC}t} \left\{ \exp\left[ \frac{1}{2RC} \sqrt{\frac{1}{2RC}^2 - \frac{1}{LC}} t \right] - \exp\left[ -\frac{1}{2RC} \sqrt{\frac{1}{2RC}^2 - \frac{1}{LC}} t \right] \right\} \ldots (16)

This equation (16) provides an expression for the voltage across a parallel R L C network circuit connected to a steady current source. It is clear from the equation that the nature of response the parallel R L C network circuit depends on quantity $\sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$, whether it is real, zero or imaginary. The value of the quantity $\sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$, in turn, depends on the values of elements R, L and C of the network circuit.

3. Discussion

The Rohit transform has been applied successfully to a series RLC network circuit with a steady voltage source, and a parallel RLC network circuit with a steady current source for obtaining their responses. It is clear from the above discussion that the nature of electric current flowing through a series RLC network circuit connected to a steady voltage source i.e. that the nature of the response of the network circuit depends on quantity $\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$, whether it is real, zero, or imaginary. The value of the quantity $\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$, in turn, depends on the values of elements R, L and C of the network circuit. Also, it is clear from the above discussion the nature of the voltage across a parallel R L C network circuit connected to a steady current source i.e. the nature of response the parallel R L C network circuit depends on quantity $\sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$, whether it is real, zero or imaginary.

The value of the quantity $\sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$, in turn, depends on the values of elements R, L and C of the network circuit. The response obtained by Rohit transform is the same as is obtained from the other integral transforms or approaches. (Chitode and Jalnekar, 2007), (Gupta et al, 2018) and (Gupta et al, 2019).

4. Conclusions

This paper has brought up the new integral transform Rohit transform as a powerful technique for analyzing series or parallel electric network circuits with steady voltage or current sources. We concluded that the response of the series (or parallel) network circuit is determined by the values of L, C, and R of the network circuit, which can be made oscillatory or non-oscillatory by selecting the suitable values of L, C, and R. It is also concluded that the Rohit transform is an effective and simple technique for obtaining the response of series RLC network circuit with steady voltage source; parallel RLC network circuit with steady current source. Further, it would be applied to the series RLC network circuit with exponential voltage source; parallel RLC network circuit with exponential current source; series RLC network circuit with sinusoidal voltage source; parallel RLC network circuit with sinusoidal current source; and for the analysis of heat transfer through fins.

References


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