

Distribution system reliability evaluation using credibility theory

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Abstract

This paper describes a new method of using credibility theory to evaluate distribution system reliability. The advantage of this method lies in its ability to account for both objective and subjective uncertainty by integrating stochastic and fuzzy approaches. Equipment failures are modeled as random events, while the uncertainty in failure indices is modeled using credibility theory. In this manner, distribution system reliability is evaluated by simultaneously considering objective and subjective uncertainty. In this paper, a hybrid algorithm based on fuzzy simulation and Failure Mode and Effect Analysis (FMEA) is applied to determine fuzzy reliability indices of distribution system. This approach can obtain fuzzy expected values and their variances of reliability indices, and the credibilities of reliability indices meeting specified targets. The method is demonstrated using an IEEE test system comprising sixty seven nodes and thirty eight load points.

Keywords: Distribution system, reliability evaluation, credibility theory, fuzzy reliability indices, fuzzy simulation

1. Introduction

Distribution system reliability indices quantify, in various ways, the ability of a distribution system to meet customer demand at all its load points. Due to the larger impact of failures in the bulk power system, the development of predictive reliability approaches has received more attention from the research community. However, distribution system components are relatively more numerous and they are exposed to a more complex set of uncertain events. As distribution systems grow in complexity, with the deployment of distributed generation and storage, and of distribution automation equipment, it is important to develop more complex methods for predictive reliability analysis of distribution systems so as to suitably address these uncertainties.

Recent research efforts in distribution system reliability evaluation have focused on the following: (1) consideration of the impact of distributed generation (Atwa and Saadany, 2009; Bae and Kim, 2007) and microgrids (Bae and Kim, 2008; Kennedy, 2009); (2) incorporating weather effects (Billinton and Acharya, 2005; Billinton and Acharya, 2006); (3) time-varying failure rate (Retterath *et al.*, 2004; Moon *et al.*, 2004). Since all of these factors are indeterminate, some research has considered them collectively (Dong *et al.*, 2009; Wang *et al.*, 2002).

Uncertainty factors can be classified into two categories: stochastic factors and fuzzy factors. Usually, randomness is related to objective uncertainty factors that have statistical characteristic, while fuzziness is commonly used to define factors that have subjective uncertainty (Wang *et al.*, 2007). Reliability evaluation methods in which only randomness is taken into account often fails to include the effect of fuzzy factors which can be significant. Models with both stochastic and fuzzy factors are often more appropriate; however, modeling this dual uncertainty can be challenging. Some recent efforts have modeled these uncertainties: trapezoidal fuzzy numbers have been used to express uncertainties in Lei *et al.*, 2005; Yuan *et al.*, 2007 have used interval algorithm to deal with the uncertainty of component data to calculate the interval reliability indices.

Most of fuzzy methods for reliability evaluation of distribution system are based on fuzzy set theory. In recent years researchers have found that credibility theory, which broadens the scope of fuzzy set theory, is an effective tool for representing fuzzy events, and have developed a theoretical framework similar to that of probability theory (Liu, 2004). Credibility theory has been successfully applied in power system analysis fields, such as fuzzy power flow (Wang *et al.*, 2007) and fuzzy risk assessment (Feng *et al.*, 2008).

This paper focuses on the reliability evaluation of distribution system under uncertain environment and proposes an approach based on credibility theory. The approach intensively analyzes the fuzzy factors in evaluation process and defines fuzzy reliability counterparts of traditional indices. Using this approach, a hybrid algorithm based on FMEA and fuzzy simulation is developed and implemented. The algorithm is demonstrated using the distribution system connected to bus 4 of the IEEE test system commonly referred to the RBTS (Allan *et al*, 1991). The case study includes the computation of fuzzy expected values and variances of reliability indices, as well as ranges of these indices, which could be useful in analysis and decision making processes.

2. Credibility Theory

Definition 1: Let $\tilde{\xi}$ be a fuzzy variable; then the *possibility*, the *necessity*, and the *credibility* of the fuzzy event A are expressed as follows:

$$\text{Pos}\{A\} = \sup_{x \in A} \mu_{\tilde{\xi}}(x) \tag{1}$$

$$\text{Nec}\{A\} = 1 - \sup_{x \in A^c} \mu_{\tilde{\xi}}(x) \tag{2}$$

$$\text{Cr}\{A\} = \frac{1}{2}(\text{Pos}\{A\} + \text{Nec}\{A\}) \tag{3}$$

where $\mu_{\tilde{\xi}}(x)$ is membership function of $\tilde{\xi}$; A^c is the complementary event of A .

In traditional fuzzy set theory, possibility measure is considered to be a parallel concept of probability measure. But it does not possess self-duality: when $\text{Pos}\{A\} = 1$, fuzzy event A is not always true; when $\text{Nec}\{A\} = 0$, A may also be true. In order to solve this problem, reference (Liu and Liu, 2002) put forward the credibility measure and established credibility theory in 2004. In this theory, credibility measure which corresponds to probability measure in probability theory possesses self-duality and monotonicity – when $\text{Cr}\{A\} = 1$, fuzzy event A is inevitably true, otherwise it is false.

Definition 2 (Liu and Peng, 2005): Let $\tilde{\xi}$ be a fuzzy variable. Then the expected value of ξ is defined by

$$E[\tilde{\xi}] = \int_0^{+\infty} \text{Cr}\{\tilde{\xi} \geq r\} dr - \int_{-\infty}^0 \text{Cr}\{\tilde{\xi} < r\} dr \tag{4}$$

provided that at least one of the two integrals is finite.

Definition 3 (Liu and Peng, 2005): Let $\tilde{\xi}$ be a fuzzy variable with finite expected value $E[\tilde{\xi}]$. Then the variance of ξ is defined as

$$V[\tilde{\xi}] = E[(\tilde{\xi} - E[\tilde{\xi}])^2] \tag{5}$$

3. The Definition of Interrelated Fuzzy Indices

Equipment failure rate and its repair time are two basic parameters used in the reliability evaluation of distribution systems. They are statistical values with a lot of subjectivity and are also used to evaluate equipment of similar types. For this reason, this approach adopts trapezoidal fuzzy number which affords comparatively broad adaptability in engineering area.

3.1 Modeling of fuzzy factors

1) Failure rate $\tilde{\lambda}_i$

The equipment failure rate is generally used to quantify the probability of failure of a specific piece of equipment. Although equipment status in the future is not predictable, the failure rate of this equipment can be obtained from tests or experience. Also, $\tilde{\lambda}_i$ can be modeled by trapezoidal fuzzy number $\tilde{\lambda}_i = (\lambda_i^1, \lambda_i^2, \lambda_i^3, \lambda_i^4)$, in which $[\lambda_i^1, \lambda_i^4]$ is the failure rate interval of specific equipment i and $[\lambda_i^2, \lambda_i^3]$ is the most possible limits. The membership function of $\tilde{\lambda}_i$ is defined as below:

$$\mu_{\tilde{\lambda}_i}(x) = \begin{cases} \frac{x - \lambda_i^1}{\lambda_i^2 - \lambda_i^1}, & \lambda_i^1 \leq x \leq \lambda_i^2 \\ 1, & \lambda_i^2 < x < \lambda_i^3 \\ \frac{\lambda_i^4 - x}{\lambda_i^4 - \lambda_i^3}, & \lambda_i^3 \leq x \leq \lambda_i^4 \\ 0, & x < \lambda_i^1 \text{ or } x > \lambda_i^4 \end{cases} \tag{6}$$

2) Repair time \tilde{r}_i

Since the insufficiency of information will result the inaccuracies of forecasting failure time, for the equipment i , trapezoidal fuzzy number $\tilde{r}_i = (r_i^1, r_i^2, r_i^3, r_i^4)$ is used to describe repair time because this number is more reasonable than point value, and its membership function $\mu_{\tilde{r}_i}$ is shown in **Figure 1**.

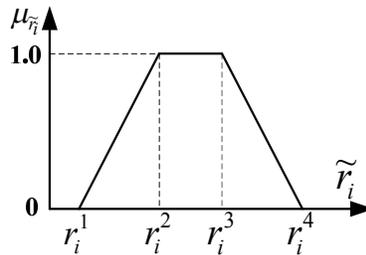


Figure 1. Membership function of repair time

3.2 Fuzzy reliability indices

Distribution networks are often radial and consist of series connections of lines, transformers, isolating switches, fuses, etc. Service continuity requires each equipment between power resource and load to be online. Therefore, the effect of the series system failure rate $\tilde{\lambda}_s$, repair time \tilde{r}_s and time to failure (up time) \tilde{T}_s are taken into account as follows:

$$\tilde{\lambda}_s = \sum \tilde{\lambda}_i \tag{7}$$

$$\tilde{r}_s = \frac{\sum \tilde{\lambda}_i \tilde{r}_i}{\sum \tilde{\lambda}_i} \tag{8}$$

$$\tilde{T}_s = \frac{\tilde{\lambda}_s}{\tilde{\lambda}_s + 1/\tilde{r}_s} \approx \sum \tilde{\lambda}_i \tilde{r}_i \tag{9}$$

where $\tilde{\lambda}_i, \tilde{r}_i$ are failure rate and repair time of equipment i respectively. Subscript s expresses equivalent series network from power sending end to the receiving end.

The definition of fuzzy reliability indices of distribution system is given as follows, based on Billinton and Allan, 1996. Comparing with regular standard, those fuzzy indices indicate the impact of fuzzy parameters on the reliability indices.

1) Fuzzy Energy Not Supplied (ENS)

$$ENS = \sum d_s \tilde{T}_s \text{ (MWh/yr)} \tag{10}$$

where d_s is the load of the equivalent series network. As $\tilde{\lambda}_i$ and \tilde{r}_i are fuzzy variables, ENS is also a fuzzy index reflecting the influence of parameter ambiguity on reliability index.

2) System Average Interruption Frequency Index (SAIFI)

$$SAIFI = \frac{\sum \tilde{\lambda}_s N_s}{\sum N_s} \text{ (int/cus-yr)} \tag{11}$$

where N_s is the number of users on load node s .

3) System Average Interruption Duration Index (SAIDI)

$$SAIDI = \frac{\sum \tilde{T}_s N_s}{\sum N_s} \text{ (hr/cus-yr)} \tag{12}$$

4) Customer Average Interruption Duration Index (CAIDI)

$$CAIDI = \frac{\sum \tilde{T}_s N_s}{\sum \tilde{\lambda}_s N_s} \text{ (hr/int)} \tag{13}$$

5) Average Service Availability Index (ASAI)

$$ASAI = \frac{\sum 8760 \times N_s - \sum \tilde{T}_s N_s}{\sum 8760 \times N_s} \quad (14)$$

6) Average Energy Not Supplied (AENS)

$$AENS = \frac{\sum \tilde{T}_s d_s}{\sum N_s} \text{ (MWh/cus-yr)} \quad (15)$$

4. Solution Procedure

This approach adopts Failure mode and effect analysis (FMEA) (Billinton and Allan, 1996) to evaluate reliability. The entire process based on fuzzy sampling can be divided into three main parts and it is described as follows.

4.1 Distribution system analysis

First, the branch set connected by closed switches is defined as Sub-Power Supply System (SPSS). Based on the status of switches, the distribution system can be divided into multiple SPSS, which are connected with tie switches. For example, SPSS corresponding to feeder F1 in **Figure 2** is composed of $\{1, 2, \dots, 12\}$.

Secondly, based on the definition of SPSS, the power supply area of each feeder is analyzed by depth-first search strategy. The searching process begins from power source and ends in load point or tie-line.

Finally, two matrices are defined and generated: one is network layer matrix which is used to reflect the level of branches and their relationships; and the other is loading point information matrix which contains information of each load point, such as number, active load values, number of customers, branch numbers and feeder numbers.

4.2 Reliability evaluation process

The procedure for evaluating fuzzy reliability indices consists of the following steps.

Step 1: Data input:

Set $v_i = 0$ ($i = 1, 2, \dots, 6$) and fuzzy simulation time N .

Step 2: Fuzzy sampling:

Randomly generating $\lambda^k = (\lambda'_1, \lambda'_2, \dots, \lambda'_n)$ from ε level cut sets of fuzzy vector $\tilde{\lambda}$. The membership degree of λ^k can be obtained by following equation:

$$v_{\lambda}^k = \mu_{\tilde{\lambda}_1}(\lambda'_1) \cap \mu_{\tilde{\lambda}_2}(\lambda'_2) \cap \dots \cap \mu_{\tilde{\lambda}_n}(\lambda'_n) \quad (16)$$

where ε is a positive number small enough; n is equipment number; $k = 1, 2, \dots, N$.

Similarly, fuzzy random value of \tilde{r} is r^k , and its membership degree is v_r^k .

Step 3: Calculate reliability indices of load point using (7), (8) and (9). Input data required are network layer matrix, load point information matrix and fuzzy sampling values.

The network reliability is assessed as well as reliability index $f_i^k(\lambda^k, r^k)$ using (10)–(15).

Step 4: Calculate $f_i^k(\lambda^k, r^k)$, and its membership degree $v^k = v_d^k \cap v_{\lambda}^k$.

Step 5: Repeat steps 2–4 for a total of N times;

Step 6: Calculate a_i and b_i by the following equations, where a_i and b_i are both non-negative real numbers.

$$a_i = f_i^1(\lambda^1, r^1) \cap f_i^2(\lambda^2, r^2) \dots \cap f_i^N(\lambda^N, r^N) \quad (17)$$

$$b_i = f_i^1(\lambda^1, r^1) \cup f_i^2(\lambda^2, r^2) \dots \cup f_i^N(\lambda^N, r^N) \quad (18)$$

Step 7: Generate τ_i from $[a_i, b_i]$ randomly.

Step 8: Set $v_i \leftarrow v_i + \text{Cr}\{f_i(\tilde{\lambda}, \tilde{r}) \geq \tau_i\}$.

$\text{Cr}\{f_i(\tilde{\lambda}, \tilde{r}) \geq \tau_i\}$ are approximately calculated using (19).

$$\text{Cr}\{f_i(\tilde{\lambda}, \tilde{r}) \geq \tau_i\} = \frac{1}{2} \left(\max_{1 \leq k \leq N} \{v^k | f_i^k(\lambda^k, r^k) \geq \tau_i\} + \min_{1 \leq k \leq N} \{1 - v^k | f_i^k(\lambda^k, r^k) < \tau_i\} \right) \quad (19)$$

Step 9: Repeat steps 7,8 for a total of N' times; N' is a given positive integer.

Step 10: $E[f_i(\tilde{\lambda}, \tilde{r})] = a_i + v_i \cdot (b_i - a_i) / N'$, ($i = 1, 2, \dots, 6$).

Fuzzy reliability indices $V[f_i(\tilde{\lambda}, \tilde{r})]$ can be used to reflect the rate of deviation between fuzzy sampling values and fuzzy expected value, just as the definition of variance in probability theory. The deriving of variance of fuzzy variable is basically the same as the above process; the difference lies in the calculation of fuzzy sampling value of variance $g_i^k(\lambda^k, r^k)$ which can be obtained from (20).

$$g_i^k(\lambda^k, r^k) = (f_i^k(\lambda^k, r^k) - E[f_i(\tilde{\lambda}, \tilde{r})])^2 \tag{20}$$

4.3 Credibility calculation

Sometimes it may be necessary, during planning or operation, to use credibility of reliability indices meeting some stipulated levels or guarantees; this credibility can be obtained by following equation:

$$Cr\{f_i(\tilde{\lambda}, \tilde{r}) \leq F_i\} = \frac{1}{2} \left(\max_{1 \leq k \leq N} \{v^k | f_i^k(\lambda^k, r^k) \leq F_i\} + \min_{1 \leq k \leq N} \{1 - v^k | f_i^k(\lambda^k, r^k) > F_i\} \right) \tag{21}$$

where F_i ($i = 1, 2, \dots, 6$) is default value which is also the value expected.

5. Simulation Results

The IEEE-RBTS Bus 4 system (Allan *et al.*, 1991) is used to demonstrate the feasibility of the proposed model and algorithm. A one-line diagram of the system is shown in **Figure 2**. This system contains 67 nodes (including 38 load nodes), 71 lines and 26 switches. The main feeders are LGJ-240, trunk lines are LGJ-120, and branch lines are LGJ-50. Parameters used in the algorithm are as follows: $N = 5000$; $N^* = 2000$. The equipment failure rate and repair time are fuzzified by using the multiplying factors [0.80, 0.95, 1.05, 1.2].

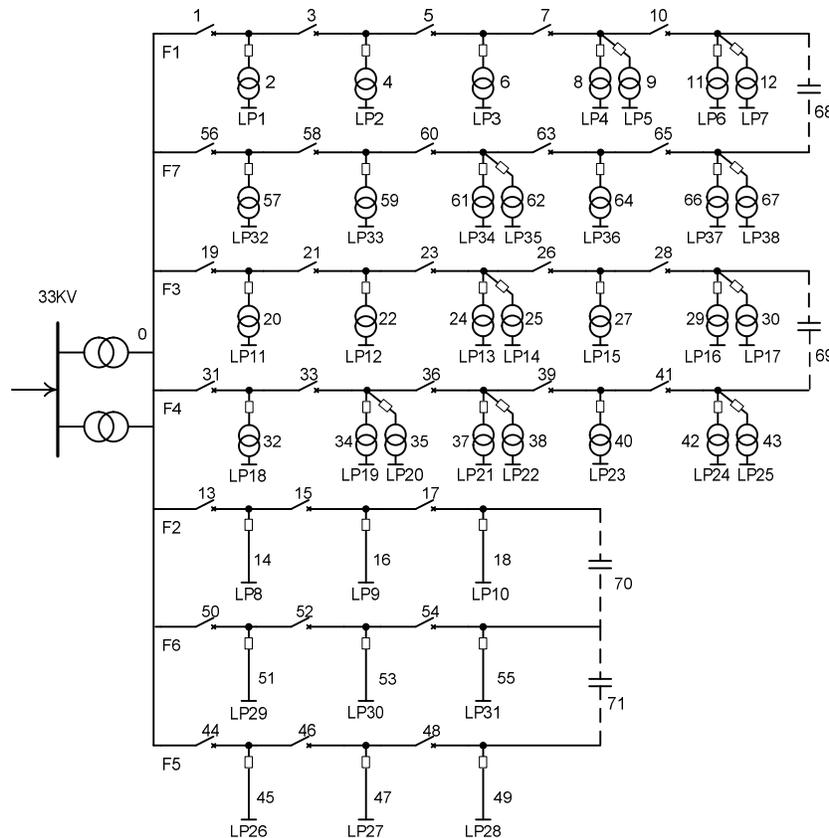


Figure 2. Distribution system for RBTS Bus 4

This work analyzes the scenario that includes disconnects, fuses and alternative supply. The fuzzy expected value of reliability indices are shown in the column three of **Table 1**. The result derived from the approach in this work compare well with results reported in Allan and Billinton, 1991. Fuzzy reliability indices of feeder are shown in **Table 2**, and the fuzzy expected values and variances of three main reliability indices of each load points are shown in **Figure 3**, **Figure 4** and **Figure 5**. The stars denote the fuzzy expected values while the circles denote the traditional reliability indices.

Table 1. Reliability indices of the system

Reliability indices	Result reported in Allan and Billinton, 1991	Result obtained using proposed method	Deviation (%)
SAIFI (int/cus-yr)	0.300	0.2993	0.2331
SAIDI (hr/cus-yr)	3.47	3.4694	0.0184
CAIDI (hr/int)	11.56	11.6412	0.7025
ASAI	0.9996	0.9996	0.0000
ENS (MWh/yr)	54.293	54.3161	0.0425
AENS (MWh/cus-yr)	11.36	11.3655	0.0481

Table 2. Feeder fuzzy reliability indices

Feeders	SAIFI (int./cus.yr)	SAIDI (hr/cus.yr)	CAIDI (hr/int.)	ASAI	ENS (MWh/yr)	AENS (MWh/cus.yr)
F1	0.3027	3.4707	11.6110	0.9996	12.1958	11.0857
F2	0.1888	0.3805	2.0120	1.0000	1.3277	443.8710
F3	0.2933	3.4698	12.0096	0.9996	12.0032	11.1142
F4	0.3070	3.4766	11.4866	0.9996	13.9398	10.7227
F5	0.1860	0.3763	2.0332	1.0000	1.1309	376.6532
F6	0.1940	0.3657	1.8882	1.0000	1.2704	423.0530
F7	0.2950	3.4722	11.8993	0.9996	12.4705	9.6672

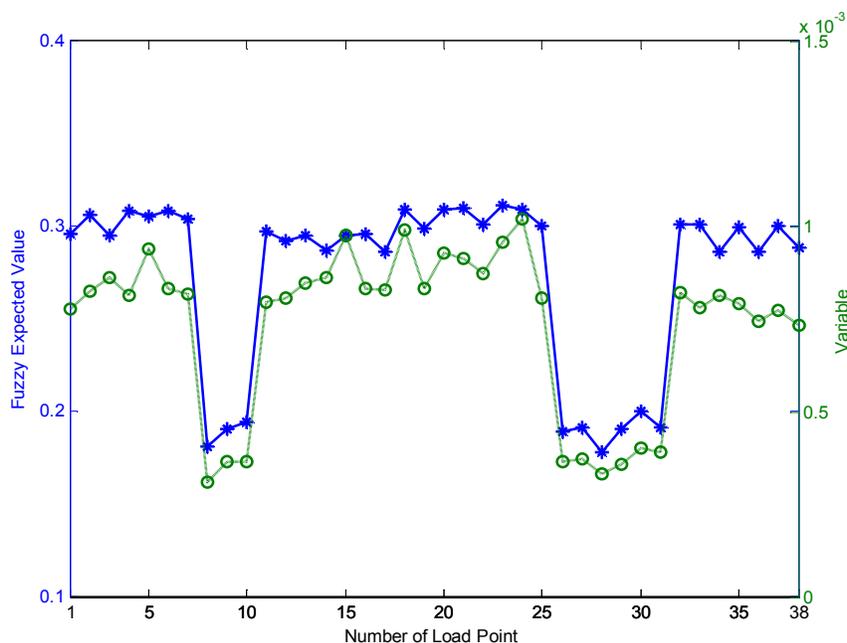


Figure 3. Failure rates of load points

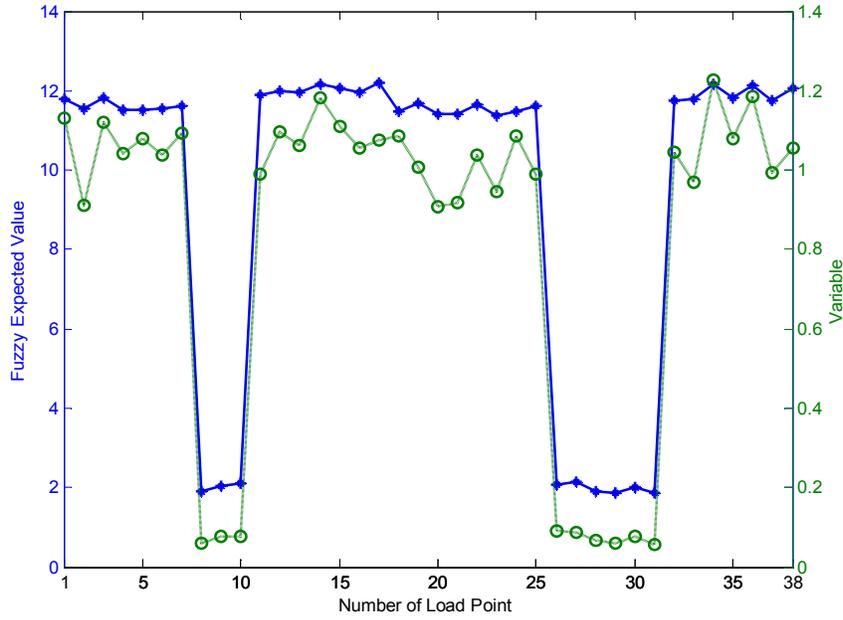


Figure 4. The average interruption durations of load points

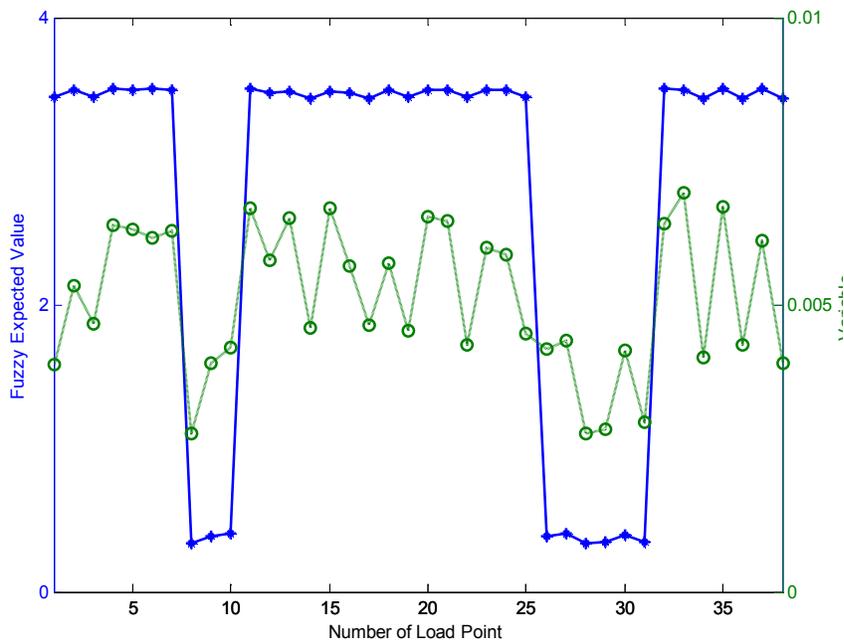


Figure 5. The failure time of load points

The fuzzy expected value $E[f_{ENS}(\tilde{\lambda}, \tilde{r})]$ and its variable $V[f_{ENS}(\tilde{\lambda}, \tilde{r})]$ of ENS is 54.3161 MWh/yr and 0.2676 respectively. If we set 0.005 as unit interval, the range of fuzzy sampling value of ENS can be divided into several intervals. Defining m_j as the frequency that sampling value will be inside band j and $\frac{m_j}{N}$ as the subsection frequency function, the relationship between $\frac{m_j}{N}$ and sampling value ENS is shown in **Figure 6**. It can be seen that most of sampling values assemble around $E[f_{ENS}(\tilde{\lambda}, \tilde{r})]$. In order to reflect the degree of dispersion, unbiased variance σ is defined by square root of variance $V[f_{ENS}(\tilde{\lambda}, \tilde{r})]$. Within the interval $[53.7988, 54.8334]$ ($[E - \sigma, E + \sigma]$), the total percent of frequency is 0.9936.

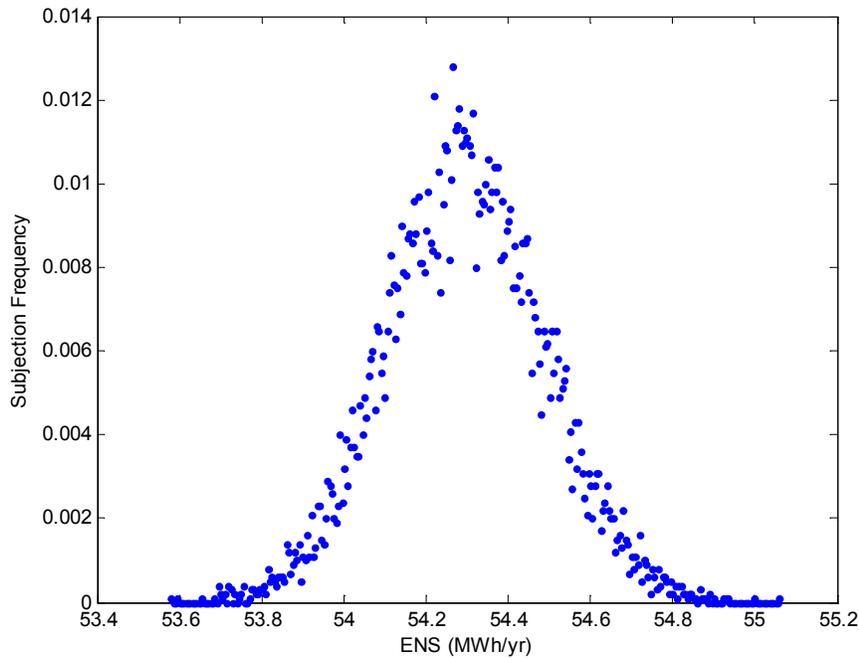


Figure 6. Frequency distribution of simulation results of ENS

In order to evaluate the influence of uncertain parameters on final results, three cases are designed for comparison: Case 1, using parameters given; Case 2, fuzzifying multiplying factor is [0.70, 0.95, 1.05, 1.3]; Case 3, uses a triangular membership function, with fuzzifying multiplying factor is [0.8, 1.0, 1.2]. The results are shown in Table 3. It can be found that the influence of uncertainty on fuzzy variance will be much more than fuzzy expected value, through comparing Case 1 and Case 2. Triangle fuzzy number which is also widely used in engineering is adopted in Case 3.

Table 3. Fuzzy expected value and variance of system reliability indices

Indices		SAIFI (int./cus.yr)	SAIDI (hr/cus.yr)	CAIDI (hr/int.)	ASAI	ENS (MWh/yr)	AENS (MWh/cus.yr)
Case1	$E[f_i]$	0.2993	3.4694	11.6412	0.9996	54.3161	11.3655
	$V[f_i]$	0.0002	0.0007	0.3224	0.0000	0.2676	0.0117
Case2	$E[f_i]$	0.3008	3.4706	11.6695	0.9996	54.3579	11.3744
	$V[f_i]$	0.0006	0.0018	0.8139	0.0000	0.6190	0.0271
Case3	$E[f_i]$	0.3007	3.4706	11.5725	0.9996	54.3344	11.3693
	$V[f_i]$	0.0003	0.0008	0.3408	0.0000	0.3311	0.0145

In our approach, equipment failure rate and repair time are uncertain, so reliability indices will also not assured. Thus, credibility is also a good option in addition to fuzzy expected value of reliability indices. We design another two cases: Case 4, $F_1 = 0.32$, $F_2 = 3.5$, $F_3 = 12.3$, $F_4 = 0.9996$, $F_5 = 58$, $F_6 = 12.15$; Case 5, assigning the reference result in column two of Table 2 to F_i ($i=1,2,\dots,6$) respectively. The simulation results are shown in Table 4. It can be seen that the higher the demand on the reliability indices, the lower credibility will be. The relationship between $Cr\{f_{SAIFI} < F_{SAIFI}\}$ and F_{SAIFI} is shown in Figure 7 and the curve shape is dependent on the fuzzy expression of parameters. The relationships of another five reliability indices are shown in Figure 8, Figure 9, Figure 10, Figure 11 and Figure 12 respectively.

Table 4. Credibility of reliability indices

Indices	Case4	Case5
SAIFI	$Cr\{f_1 < 0.32\} = 0.9773$	$Cr\{f_1 < 0.3\} = 0.0844$
SAIDI	$Cr\{f_2 < 3.5\} = 0.9904$	$Cr\{f_2 < 3.47\} = 0.0333$
CAIDI	$Cr\{f_3 < 12.3\} = 0.9900$	$Cr\{f_3 < 11.56\} = 0.9667$
ASAI	$Cr\{f_4 > 0.9996\} = 0.9990$	$Cr\{f_4 > 0.9996\} = 0.9990$
ENS	$Cr\{f_5 < 58\} = 1$	$Cr\{f_5 < 54.293\} = 0.9684$
AENS	$Cr\{f_6 < 12.15\} = 1$	$Cr\{f_6 < 11.36\} = 0.9684$

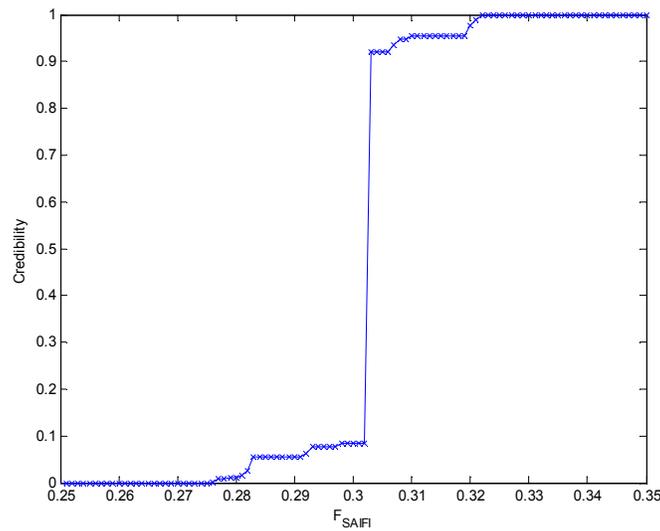


Figure 7. The relationship between $Cr\{f_{SAIFI} < F_{SAIFI}\}$ and F_{SAIFI}

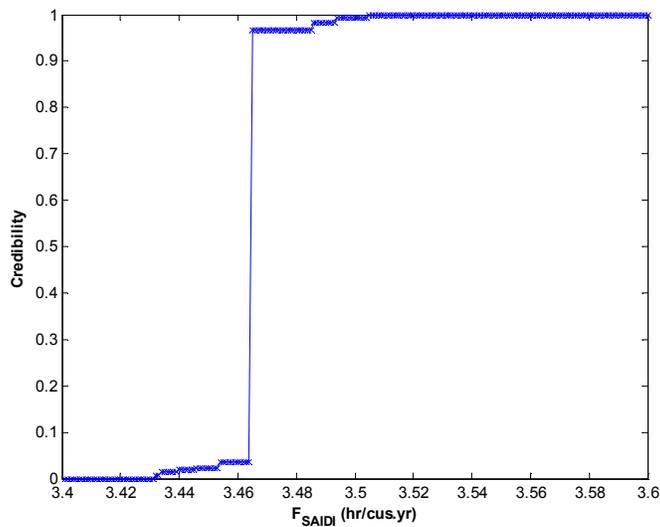


Figure 8. The relationship between $Cr\{f_{SAIDI} < F_{SAIDI}\}$ and F_{SAIDI}

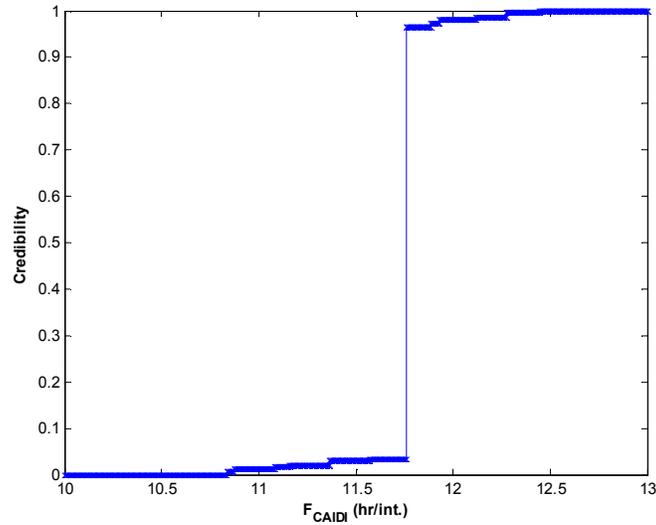


Figure 9. The relationship between $Cr\{f_{CAIDI} < F_{CAIDI}\}$ and F_{CAIDI}

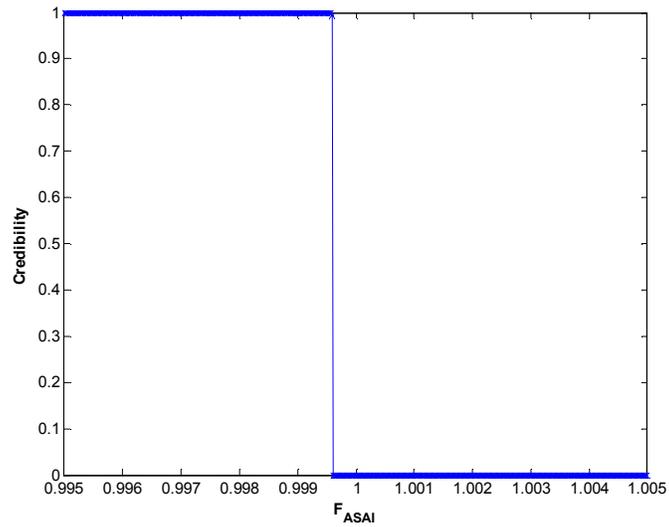


Figure 10. The relationship between $Cr\{f_{ASAT} > F_{ASAT}\}$ and F_{ASAT}

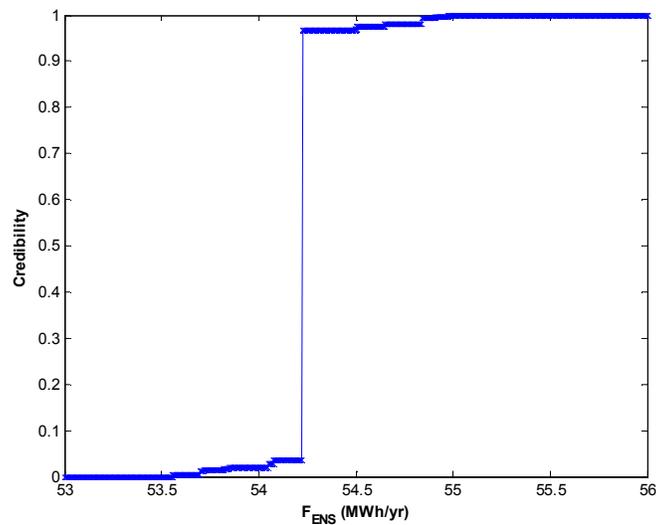


Figure 11. The relationship between $Cr\{f_{ENS} < F_{ENS}\}$ and F_{ENS}

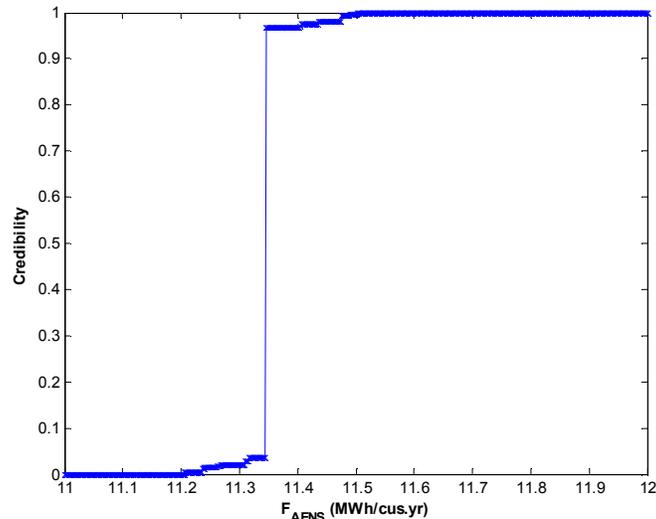


Figure 12. The relationship between $Cr\{f_{AENS} < F_{AENS}\}$ and F_{AENS}

6. Conclusion

This paper employed credibility theory to evaluate the reliability of a distribution network. This method is able to model both objective and subjective uncertainties. Compared with traditional methods, the proposed method provides more information about the indices by showing how the indices are dispersed, rather than merely providing their expected values. This affords a better sense of realizable reliability targets, during both planning and operation. The illustrative cases have also demonstrated the feasibility and value of the approach. It is surmised that this approach based on credibility theory will afford broad application in the area of distribution system reliability evaluation.

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