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Free flexural vibration studies on laminated composite skew plates

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Abstract

This paper presents studies made on fundamental flexural frequencies of isotropic and laminated composite skew plates with simply supported and clamped boundary conditions. The fundamental flexural frequencies have been obtained using finite element, the accuracy of which is verified against literature values. The effects of the skew angle, aspect ratio and length-to-thickness-ratio on the fundamental frequency of isotropic skew plates are presented. In addition, the effect of parameters such as skew angle, fiber orientation angle, numbers of layers in the laminate and laminate sequence on the fundamental frequencies of antisymmetric composite laminates have also been presented. It is found that the CQUAD8 element yields better results than the CQUAD4 element in the said study of free flexural vibration. The frequencies are found to increase with the skew angle. When the number of layers in the laminate is large, the variation of frequency with the number of layers is not appreciable.

Keywords: Skew Plates, Antisymmetric Laminates, Free Flexural Vibration, Fundamental Frequency Coefficient, Frequency Ratio.

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1. Introduction

The skew plates find a wide range of application in civil, marine, aeronautical and mechanical engineering applications. They are often used in modern structures in spite of the mathematical difficulties involved in their study. The various applications of skew plates can be found in swept wings of aeroplanes, complex alignment problems in bridge design, ship hulls and parallelogram slabs in buildings. The exact solutions to skew plate vibration problems are rare, and those available in the literature are based on approximate methods. Over the past four decades, a lot of research has been carried out on the study of vibration characteristics of skew plates. Today the skew plate problem has been widely used by FEM as a benchmark check on the capability of a particular newly developed finite element.

The earlier studies on free vibration characteristics of skew plates are those of Barton (1951), Kaul and Cadambe (1956) and Hasegawa (1957), using Rayleigh-Ritz method. Hamada (1959) applied the Lagrangian-multiplier method to obtain the fundamental frequency of the rhombic skew plate. Claassen (1963) extended the work of Barton (1951) by adopting a Fourier sine series solution scheme in conjunction with the Rayleigh-Ritz method. Conway and Farnham (1965) employed the point matching method to study the free vibration of triangular, rhombic and parallelogram plates. The frequencies were calculated for different skew angles of simply supported and clamped boundary conditions. Laura and Grosson (1968) obtained fundamental frequencies for vibration of simply supported rhombic plates using conformal mapping and Galerkin's method, and compared their results with Conway and Farnham (1965). The difference increases with the skew angle. Monforton (1968) obtained fundamental frequencies of clamped rhombic plates by using FEM. The frequencies and mode shapes of clamped skew plates were studied by Durvasula (1969) using Galerkin's method. The deflection function was expressed as a double series of beam characteristic functions in terms of skew coordinates to satisfy zero deflection and normal slope on all the edges. Some interesting results were obtained and compared with the results of Kaul and Cadambe (1956), Hasegawa (1957) and Conway and Farnham (1965). Thangam Babu and

Reddy (1971) investigated the free vibration of orthotropic skew plates with two opposite edges simply supported and the other two edges free. Nair and Durvasula (1973) reported the frequencies of isotropic and orthotropic skew plates for simply supported, clamped, free edge boundary condition and for combination of the above three edge conditions. Srinivasan and Ramachandran (1975) employed a numerical method to study variations of frequencies and mode shapes of orthotropic skew plates. Kuttler and Sigillito (1980) have used trial function method to solve vibration problem of skew plates.

Mizusuwa et al. (1979,1980) and Mizusuwa and Kajita(1986,1987) employed the Rayleigh-Ritz method with B-spline functions to study the effect of skew angle and location of point supports on natural frequencies of isotropic skew plates. Liew and Lam (1990) used a set of 2-D orthogonal plate functions as an admissible deflection function to study the flexural vibration of skew plates using Rayleigh-Ritz method and obtained the results for four rhombic plates with different support conditions. Bardell (1992) adopted the hierarchical finite element method to determine natural frequencies and mode shapes of isotropic skew plates. Liew and Wang (1993) employed the Rayleigh-Ritz method and obtained results for four rhombic plates with different support conditions by changing internal support, skew angle and aspect ratio. Singh and Chakraverthy (1994) evaluated the first five frequencies for the transverse vibration of skew plates under different boundary conditions by using boundary characteristics orthogonal polynomials. McGee and Butalia (1994) studied the free vibration of thick and thin cantilever skew plates using C^0 continuous isoparametric quadrilateral elements

Though vast literature on the free vibration of isotropic and orthotropic skew plates is available, few studies on free vibration of composite skew plates have been made. Kamal and Durvasula (1986) studied the free vibration characteristics of composite laminates using the modified shear deformation layered composite theory by employing Rayleigh-Ritz energy approach. Malhotra *et al.* (1988) studied the rhombic orthotropic plates using a parallelogrammic orthotropic plate finite element for various boundary conditions and skew angles. Krishnan and Deshpande (1992) adopted DKT finite element to demonstrate the effect of fiber orientation angle, skew angle, aspect ratio and length to thickness ratio on the fundamental frequencies of single layer graphite/epoxy and glass/epoxy skew plates. Krishna Reddy (1999) used a general high precision triangular bending element to study free vibration of laminated skew plates. A consistent mass matrix in explicit form is used for the study. Singha and Ganapathi (2004) studied the large amplitude free flexural vibrations of laminated composite skew plates using FE approach. Garg *et.al.* (2006) have presented the free vibration studies on isotropic, orthotropic, and layered anisotropic composite and sandwich skew laminates using isoparametric finite element model.

This paper deals with the studies on fundamental flexural frequencies of isotropic and laminated composite skew plates using CQUAD4 and CQUAD8 finite elements. The accuracy of the elements has been verified with literature values. The effects of skew angle, fiber orientation angle, number of layers in the laminate, laminate sequence and boundary conditions on the fundamental frequencies of skew plate are investigated.

2. Convergence and Validation Studies

2.1 Convergence Study

The geometry of the skew plate with global and local coordinate systems is shown in the Figure 1 in which u and v are the displacement variables in x and y directions respectively. Since u and v are inclined to the skew edges, the displacement boundary conditions cannot be applied directly. In order to overcome this, a local coordinate system (x', y') normal and tangential to the skew edges is chosen. To obtain accurate and reliable results, it is necessary to study the convergence of the results so as to establish the optimum number of elements required in the finite element model. The convergence study has been performed on simply supported(S-S-S-S) (S3) (Jones, 1975) and clamped(C-C-C-C) (C2) (Jones, 1975) skew plates having aspect ratio (=a/b) of 1.0 and skew angles 0°, 15°, 30° and 45° using CQUAD4 (four-noded) and CQUAD8 (eight-noded isoparametric curved shell element) elements. The convergence details are furnished in Table 1.



Figure 1: Global and Local Coordinate Systems for the Skew Plate with Finite Element Mesh.

AUTHOURS			S-S	-S-S		C-C-C-C				
			Skew A	ngle (\mathcal{A})		Skew Angle (α)				
		0°	15°	30°	45°	0° 15° 30° 45°			45°	
DDESENIT(AVA)	CQUAD4	1.8856	2.0056	2.4387	3.5260	3.5680	3.7813	4.5350	6.3136	
PRESENT(4A4)	CQUAD8	1.9984	2.1262	2.6021	3.8474	3.6990	Skew A 15° 3.7813 3.9250 3.7374 3.8749 3.7617 3.8669 3.7862 3.8645 3.8645 3.8645 3.842 3.8645 3.8342 3.8645 3.8700 3.8720 3.8720	4.7589	6.8829	
DDESENT(6V6)	CQUAD4	1.9367	2.0534	2.4755	3.5293	3.5264	3.7374	4.4879	6.2998	
PRESENT(OAO)	CQUAD8	1.9988	2.1186	2.5653	3.7326	3.6489	Skew A 15° 3.7813 3.9250 3.7374 3.8749 3.7374 3.8749 3.7617 3.8669 3.7862 3.8651 3.8045 3.8645 3.8645 3.8645 3.8645 3.8645 3.8645 3.8645 3.8700 3.8720 3.8720	4.6853	6.6984	
DDECENT(QVQ)	CQUAD4	1.9598	2.0756	2.4949	3.5406	3.5480	3.7617	4.5247	6.3843	
PRESENT(OAO)	CQUAD8	1.9991	2.1166	2.5521	3.6839	3.6432	3.8669	4.6693	6.6576	
DDESENT(10V10)	CQUAD4	1.9724	2.0879	2.5057	3.5475	3.5702	3.7862	4.5587	6.4505	
PRESENT(IUXIU)	CQUAD8	1.9993	2.1158	2.5452	3.6560	3.6422	3.8651	4.6651	6.6452	
DDESENT(12V12)	CQUAD4	1.9801	2.0953	2.5120	3.5510	3.5869	3.8045	4.5835	6.4966	
FRESENT(12A12)	CQUAD8	1.9994	2.1153	2.5409	3.6375	3.6420	3.8646	4.6638	6.6408	
	CQUAD4	1.9849	2.1001	2.5159	3.5524	3.5990	3.8177	4.6012	6.5285	
$PRESENT(14\Lambda 14)$	CQUAD8	1.9995	2.1149	2.5379	3.6241	3.6419	3.8645	4.6634	6.6391	
DDESENT(16V16)	CQUAD4	1.9882	2.1032	2.5164	3.5527	3.6077	3.8272	4.6138	6.5511	
PRESENT(IOAIO)	CQUAD8	1.9995	2.1147	2.5357	3.6139	3.6419	3.8645	4.6632	6.6383	
DDECENT(10V10)	CQUAD4	1.9905	2.1055	2.5173	3.5523	3.6141	3.8342	4.6231	6.5675	
PRESENT(IOAIO)	CQUAD8	1.9996	2.1145	2.5341	3.6057	3.6419	3.8645	4.6632	6.6379	
DDESENT(20V20)	CQUAD4	1.9922	2.1070	2.5189	3.5516	3.6190	3.8395	4.6301	6.5798	
PRESENT(20A20)	CQUAD8	2.0000	2.1144	2.5327	3.5991	3.6420	3.860 3.7813 3.9250 3.7374 3.8749 3.7617 3.8669 3.7862 3.8665 3.8045 3.8045 3.8045 3.8645 3.8272 3.8645 3.8342 3.8645 3.8342 3.8645 3.8395 3.8645 3.8720 3.8720 3.8720 3.8720 3.8720 3.8720 3.8720	4.6632	6.6378	
Durvasula (1	1969)	2.0000	2.1100	2.5200	3.5300	3.6467 3.8700 4.6750		6.6800		
Liew &Lam((1990)	2.0000	2.1100	2.5400	3.5400	3.6360	3.8691	4.6698	6.6519	
Krishna Reddy	y(1999)	2.0000	2.1200	2.5300	3.5100	3.6380	3.8720	4.6720	6.6630	
A. K. Garg et al.(2006)		2.0000	2.1150	2.5240	3.6290	3.6480	3.8720	4.6800	6.6990	

Table 1: Convergence study for fundamental frequency coefficients (k_t) for isotropic skew plates (a/b=1, a/t=100)

2.2 Validation Check

The validation for the present elements is performed by comparing the values for the non-dimensional fundamental frequency coefficient (K_f) obtained in this work with those available in literature. The same is presented in Table 2 for simply supported(S-S-S-S) and in Table 3 for clamped(C-C-C-C) isotropic skew plates. It can be seen from Tables 2 and 3 that the results obtained using the present elements are in good agreement with the literature values. In a similar manner validation check is performed on square antisymmetric graphite/epoxy angle-ply laminates with different fiber angle (0° to 90°) and different number of layers while keeping the total thickness constant. The material properties used are: $E_l / E_t = 40$, $G_{lt} / E_t = 0.5$ and $v_{lt} = 0.25$. The values of fundamental frequencies are compared with those available in the literature. From Table 4 it can be seen that CQUAD8 element yields better results when compared with the CQUAD4 element. Hence CQUAD8 element is employed in the present work for further study.

AUT	HORS	Non-dimensional Fundamental Frequencies Coefficients(K_f)						
101	nons	Skew angle(α)						
		$0^{\rm o}$	15°	30°	45°			
Durvasula	(1969)	2.000	2.110	2.520	3.530			
Liew &La	n(1990)	2.000	2.110	2.540	3.540			
Liew et.al(1993)	2.000	-	2.530	-			
Krishnan e	t.al (1992)	2.000	2.120	2.520	3.450			
Krishna Re	eddy (1999)	2.000	2.120	2.530	3.510			
Garg A. K	et al.(2006)	2.000	2.115	2.524	3.629			
Present	CQUAD8	2.000	2.114	2.533	3.599			
	CQUAD4	1.992	2.107	2.519	3.551			

Table 2: Fundamental frequencies of isotropic simply supported(S-S-S-S) skew plates

AUTHORS		Non-dimensional Fundamental Frequencies Coefficients(<i>K_f</i>)						
		Skew angle(α)						
		0°	15°	30°	45°			
Hasegawa	u (1957)	3.647	3.879	4.678	-			
Hamada (1959)	-	3.868	4.669	6.645			
Conway (1965)	3.648	3.872	4.682	6.651			
Monfortor	n (1968)	3.646	3.870	4.673	6.670			
Durvasula	u (1969)	3.646	3.646 3.870		6.680			
Mizusuwa	a et. al(1979)	3.646	-	4.669	6.642			
Bardell(1992)		3.646	3.869	4.670	6.651			
Liew &Lam(1993)		3.636	3.869	4.669	6.652			
Singh et.al (1994)		-			6.720			
Krishna Reddy(1999)		3.638 3.872		4.672	6.663			
Garg et al.(2006)		3.648	3.872	4.680	6.699			
Dragant	CQUAD8	3.642	3.864	4.663	6.638			
Present	CQUAD4	3.619	3.840	4.630	6.580			

Table 3: Fundamental frequencies of isotropic clamped(C-C-C-C) skew plates

Table 4: Fundamental frequencies of simply supported square antisymmetric angle-ply laminates

Fiber	Number of Layers(NL)=2			Number of Layers(NL)=4				Number of Layers(NL)=6				
Angle	Jones	Reddy	Pre	sent	Jones	Reddy	Present		Jones	Reddy	Pre	sent
(θ)	(1975)	(1999)	CQUAD8	CQUAD4	(1975)	(1999)	CQUAD8	CQUAD4	(1975)	(1999)	CQUAD8	CQUAD4
0°	18.805	18.806	18.804	18.685	18.806	18.806	18.804	18.685	18.805	18.806	18.804	18.685
15°	14.646	14.646	14.645	14.528	19.431	19.431	19.430	19.278	20.192	20.193	20.190	20.033
30°	14.204	14.204	14.203	14.075	22.176	22.175	22.172	21.974	23.355	23.355	23.353	23.137
45°	14.638	14.638	14.637	14.506	23.258	23.258	23.256	23.028	24.827	24.828	24.825	24.072
60°	14.204	14.204	14.203	14.075	22.176	22.175	22.172	21.974	23.355	23.355	23.353	23.137
75°	14.646	14.646	14.645	14.528	19.431	19.431	19.430	19.278	20.192	20.193	20.190	20.033
90°	18.805	18.806	18.804	18.685	18.806	18.806	18.804	18.685	18.805	18.806	18.804	18.685

3 Results and Discussions

The results of the present work are presented in the form of non-dimensional fundamental frequency coefficient (K_f) as $K_f = \frac{\omega a^2}{\pi^2} \sqrt{\frac{\rho_1}{D}}$ for isotropic and $K_f = \omega a^2 \sqrt{\frac{\rho_1}{E_f t^3}}$ for laminated composites for the following cases:

- 1) Simply supported and clamped isotropic skew plates and
- 2) Simply supported and clamped antisymmetric angle and cross-ply laminates.

3.1 Simply Supported and Clamped Isotropic Skew Plates

Numerical studies have been made for a number of skew plates with different aspect ratios, skew angles, and length-thickness ratio with simply supported and clamped boundary conditions. The results obtained are tabulated in Table 5 in the form of nondimensional frequency coefficients (K_f). It is seen that K_f increases as the aspect ratio (a/b), skew angles (α) increases. It can also be observed that K_f increases as the skew angle α increases for constant values of a/b and a/t. K_f decreases as a/t decreases for constant a/b ratio and skew angle.

		Non-dimensional fundamental frequencies $coefficients(K_f)$										
а			Skew angle(α)									
a/b	a/t	0°	15°	30°	45°	0°	15°	30°	45°			
			S-S	-S-S			C-C	-C-C				
	1000	1.2477	1.3234	1.6108	2.3318	2.4901	2.6568	3.2619	4.7958			
	500	1.2476	1.3234	1.6106	2.3314	2.4900	2.6567	3.2618	4.7956			
0.5	100	1.2462	1.3218	1.6071	2.3228	2.4882	2.6545	3.2584	4.7881			
	50	1.2432	1.3187	1.6010	2.3090	2.4823	2.6840	3.2484	4.7664			
	20	1.2306	1.3058	1.5808	2.2695	2.4437	2.6039	3.1824	4.6264			
	1000	1.9968	2.1171	2.5376	3.6084	3.6460	3.8691	4.6700	6.6470			
	500	1.9964	2.1166	2.5364	3.6048	3.6459	3.8689	4.6697	6.6514			
1.0	100	1.9998	2.1144	2.5327	3.5991	3.6420	3.8645	4.6632	6.6378			
	50	1.9784	2.0872	2.4873	3.4691	3.6278	3.8488	4.6410	6.5947			
	20	1.9432	2.0489	2.4314	3.3651	3.5523	3.7640	4.5183	6.3513			
	1000	3.2437	3.4361	4.1391	5.9022	6.1589	6.5527	7.9756	11.5494			
	500	3.2450	3.4332	4.1349	5.8889	6.1559	6.5494	7.9714	11.5429			
1.5	100	3.2326	3.4199	4.1063	5.8130	6.1448	6.5368	7.9526	11.5033			
	50	3.2114	3.3967	4.0698	5.7397	6.1071	6.4944	7.8912	11.3782			
	20	3.1412	3.3184	3.9661	5.5631	5.5816	6.2515	7.5363	10.6588			
	1000	5.007	5.3157	6.4519	9.3249	9.9855	10.6534	13.0788	19.2264			
	500	4.9946	5.3012	6.4319	9.2913	9.9597	10.6258	13.0446	19.1749			
2.0	100	4.9751	5.2804	6.3943	9.2299	9.9303	10.5924	12.9942	19.0660			
	50	4.9412	5.2436	6.3413	9.1459	9.8347	10.4840	12.8336	18.7285			
	20	4.8162	5.1066	6.1596	8.8335	9.2814	9.8587	11.9133	16.8754			
	1000	7.2497	7.7058	9.4085	13.7191	14.9630	15.9898	19.7243	29.2190			
	500	7.2433	7.7079	9.4092	13.7138	14.9697	15.9967	19.7322	29.2281			
2.5	100	7.2158	7.6779	9.3604	13.6112	14.9035	15.9209	19.6168	28.9751			
	50	7.1636	7.6205	9.2826	13.4703	14.6926	15.6816	19.2586	28.2131			
	20	6.9539	7.3883	8.9606	9.6414	13.4964	14.3298	17.2780	24.2504			

Table 5: Free vibration of simply supported and clamped isotropic skew plates

3.2.1 Antisymmetric Cross-Ply Skew Laminates

The variations of fundamental frequency coefficient (K_f) with respect to skew angle(α), fiber orientation angle (θ) and number of layers(NL) for graphite/epoxy antisymmetric cross-ply skew laminates are presented in Figures 2 and 3 respectively for simply supported (S3) (Jones, 1975) and clamped boundary conditions(C2) (Jones, 1975). From Figures 2 and 3 it is seen that, as the skew angle increases, K_f increases. For a given skew angle, it is seen that K_f increases up to NL = 4 and remains constant thereafter.



Figure 2: Values of K_f for simply supported antisymmetric cross-ply skew laminates



Figure 3: Values of K_f for clamped antisymmetric cross-ply skew laminates.

3.2.2 Antisymmetric Angle-Ply Skew Laminates

(a) Simply Supported Boundary Condition

The variations of fundamental frequency coefficient (K_f) with skew $angle(\alpha)$, fiber orientation $angle(\theta)$ and number of layers(NL) for antisymmetric angle-ply skew laminates under S3 boundary conditions are presented in Figures 4-7. For skew angles up to 45 degrees and NL=2, K_f initially decreases with fibre orientation angle and later varies as shown in Figures 4-7. For NL≥4 and for all the skew angles, K_f increases and reaches a maximum at about $\theta=45^\circ,47^\circ,50^\circ$ and 70° , for skew angles = $0^\circ,15^\circ,30^\circ$ and 45° respectively. It is observed that the variation of K_f with θ is symmetric about $\theta=45^\circ$ for $\alpha=0^\circ$ and becomes asymmetric as α increases. The increase in K_f with an increase in NL beyond NL= 10 is not appreciable in all the cases. The Frequency coefficient ratio f_r defined as $f_r = \left[\left(K_f \right)_{\theta=0^\circ} / \left(K_f \right)_{\theta=90^\circ} \right]$, is unity for skew angle $\alpha = 0^\circ$ (square plate) and its values for other skew angles are presented in Table 6. A perusal of Figures 4-7 indicates that f_r decreases with the skew angle.



Figure 4: Values of K_f for simply supported antisymmetric angle-ply skew laminates $(\alpha=0^\circ, S3, a/b=1, Graphite/Epoxy).$



Figure 5: Values of K_f for simply supported antisymmetric angle-ply skew laminates (α =15°, S3, a/b=1, Graphite/Epoxy).



Figure 6: Values of K_f for Simply Supported Antisymmetric Angle-Ply Skew Laminates (α =30°, S3, a/b=1, Graphite/Epoxy).



Figure 7: Values of K_f for simply supported antisymmetric angle-ply skew laminates (α =45°, S3, a/b=1, Graphite/Epoxy).

(b) Clamped Boundary Condition

For antisymmetric angle-ply skew plates of $\alpha = 0^{\circ}, 15^{\circ}, 30^{\circ}$ and 45° , the variations of K_f with θ and NL are presented in Figures 8-11. The variation of K_f is symmetric at about $\theta = 45^{\circ}$ line for $\alpha = 0^{\circ}$ and becomes asymmetric for other skew angles. For NL = 2, K_f initially decreases and varies later as shown in Figures 8-11. The increase in K_f with NL for NL greater than 10 is not appreciable. The f_r values for various skew angles (α) are presented in Table 6.



Figure 8: Values of K_f for clamped supported antisymmetric angle-ply skew laminates ($\alpha=0^\circ$, C2, a/b=1, Graphite/Epoxy).



Figure 9: Values of K_f for clamped supported antisymmetric angle-ply skew laminates (α =15°, C2, a/b=1, Graphite/Epoxy)



Figure 10: Values of K_f for Clamped Supported Antisymmetric Angle-Ply Skew Laminates (α =30°, C2, a/b=1, Graphite/Epoxy).



Figure 11: Values of K_f for Clamped Supported Antisymmetric Angle-Ply Skew Laminates (α =45°, C2, a/b=1, Graphite/Epoxy).

Boundary		Skew A	angle (α)	
Conditions	0°	15°	30°	45°
S-S-S-S	1.000	0.9219	0.7405	0.5252
C-C-C-C	1.000	0.9365	0.7657	0.5433

Table 6: *f_r* Values for graphite/epoxy antisymmetric skew plates

4 Conclusions

- (i) The CQUAD8 element is seen to yield better results for the fundamental flexural frequencies of skew plates compared with the CQUAD4 element. Both the elements have been validated against available literature values.
- (ii) In the case of cross-ply antisymmetric skew plate, it is seen that K_f increases up to NL = 4 and remains constant thereafter for a given skew angle.
- (iii) In the case of simply supported and clamped antisymmetric skew plates, it is observed that the variation of K_f with θ is symmetric at about θ =45° for α =0° and becomes asymmetric as α increases. The increase in K_f with the number of layers beyond NL= 10 is not appreciable in all the cases.
- (iv) The frequencies increase with the skew angle of the laminate.
- (v) The value of f_r decreases as the skew angle increases.

Nomenclature:

- *a* : Plate length
- *b* : Plate width
- *t* : Plate thickness
- *NL* : Number of layers
- *E* : Modulus of elasticity

- E_1 : Young's modulus of the fiber in longitudinal direction
- E_t : Young's modulus of the fiber in transverse direction
- G_{lt} : Shear modulus
- *D* : Plate bending rigidity, $Et^3 / 12(1-v^2)$
- f_r : Frequency coefficient ratio
- K_f : Non-dimensional frequency coefficient
- α : Skew angle
- θ : Fiber orientation angle
- v : Poisson's ratio
- v_{lt} : Major Poisson's ratio
- ρ : Mass density
- ρ_1 : Mass density per unit area
- ω : Angular frequency

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