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International Journal of Engineering, Science and Technology
Vol. 18, No. 2, 2026, pp. 14-21

INTERNATIONAL JOURNAL OF ENGINEERING, SCIENCE AND TECHNOLOGY

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An overview and comparative analysis of synthetic control charts

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Abstract

Synthetic control charts have emerged as an effective alternative in Statistical Process Control for detecting small and moderate shifts in process parameters. By integrating a Shewhart-type chart with a conforming run length (CRL) mechanism, these charts combine the simplicity of classical methods with improved sensitivity. This paper presents a comprehensive theoretical and empirical investigation of synthetic control charts. Exact and approximate expressions for performance measures, including Average Run Length (ARL) and variance of run length, are derived using probabilistic and Markov chain approaches. A detailed comparative analysis with the Shewhart Control Chart and EWMA Chart is conducted through simulation studies. The results demonstrate that synthetic charts outperform traditional Shewhart charts and are competitive with memory-type charts in detecting moderate process shifts, while maintaining lower variability in detection time. The study highlights the practical relevance, efficiency, and adaptability of synthetic control charts in modern quality monitoring applications.

Keywords: conforming run length, average run Length, Synthetic control chart, CUSUM, EWMA

DOI: http://dx.doi.org/10.4314/ijest.v18i2.2

Cite this article as:
Das N. 2026. An overview and comparative analysis of synthetic control charts. International Journal of Engineering, Science and Technology, Vol. 18, No. 2, pp. 14-21. doi: http://dx.doi.org/10.4314/ijest.v18i2.2

Received: May 8, 2026, Accepted: July 19, 2026; Final acceptance in revised form: July 21, 2026

1. Introduction

Monitoring and maintaining process quality is a fundamental objective in Statistical Process Control. Control charts serve as primary tools for distinguishing between natural process variation and assignable causes. The pioneering work of Walter A. Shewhart introduced the Shewhart control chart, which remains widely used due to its simplicity and ease of interpretation. However, Shewhart charts are known to be inefficient in detecting small and moderate shifts in quality charateristics.

To address this limitation, memory-type control charts such as the CUSUM Chart and EWMA Chart were developed. These methods incorporate past observations to improve sensitivity but often require careful parameter tuning and may be computationally more intensive. The synthetic control chart was introduced as a hybrid approach that combines the strengths of Shewhart charts with a run-length-based monitoring mechanism. Specifically, it integrates a Shewhart-type chart with a conforming run length (CRL) chart, where a signal is triggered based on both the occurrence of a nonconforming observation and the number of preceding conforming observations. This dual mechanism enhances detection capability without significantly increasing implementation complexity. In recent years, research in SPC has expanded toward more flexible and data-driven frameworks. For instance, Burger (2025) proposed distribution-free process monitoring using conformal prediction, which eliminates strict distributional assumptions while maintaining statistical guarantees. Similarly, Taş and Castillo (2025) developed high-dimensional SPC methodologies based on manifold learning to address challenges arising from complex and correlated data structures. Furthermore, Nguyen and Wang (2026) introduced p-value-based control charting approaches that provide unified inference frameworks and improved robustness. These developments highlight the growing need for control charting techniques that are both theoretically sound and adaptable to modern data environments. In this context, synthetic control charts provide a promising balance between simplicity and efficiency, while also offering potential for integration with emerging methodologies.

The primary objectives of this paper are:

- To develop theoretical expressions for key performance measures such as ARL and variance
- To formulate the synthetic chart using a Markov chain framework
- To compare its performance with classical and memory-type control charts through simulation

The remainder of the paper is organized as follows: A thorough literature survey and theoretical derivations are presented first, followed by simulation studies and comparative analysis, and concluding with key findings and future research directions. This study provides a comprehensive theoretical and empirical analysis of synthetic control charts within the framework of Statistical Process Control. Theoretical derivations based on probability models and Markov chain formulations yield both approximate and exact expressions for Average Run Length (ARL) and variance of run length. These results provide deeper insight into the stochastic behaviour and design characteristics of synthetic charts. The simulation study demonstrates that synthetic control charts significantly outperform the Shewhart Control Chart in detecting small and moderate shifts, while remaining competitive with the EWMA Chart. Additionally, the lower variability in run length indicates more consistent and reliable detection performance. Despite these advantages, the effectiveness of synthetic charts depends on appropriate parameter selection, particularly the conforming run length threshold. Moreover, their performance may be sensitive to assumptions such as independence of observations. Overall, synthetic control charts represent a valuable and practical tool for modern process monitoring.

2. Literature Review

2.1. Early Developments in Control Chart Theory

The foundation of modern control chart methodology lies in the work of Walter A. Shewhart, who introduced the Shewhart control chart for process monitoring. Although effective for detecting large shifts, the Shewhart chart is relatively insensitive to small and moderate process changes. To overcome this limitation, memory-type control charts such as the CUSUM Chart (Page, 1954) and EWMA Chart (Lucas and Saccucci, 1990) were developed. These methods incorporate historical information to improve sensitivity to small shifts but at the cost of increased complexity and require careful parameter selection.

2.2. Introduction of Synthetic Control Charts

The synthetic control chart was first proposed by Wu and Spedding (2000) as a hybrid approach combining a Shewhart chart with a conforming run length (CRL) chart. The method signals when a nonconforming observation occurs within a short conforming run length, thereby improving detection capability for moderate shifts. Subsequent studies established that synthetic charts can outperform Shewhart charts in terms of Average Run Length (ARL), particularly in the shift range of practical importance. Early analytical work also showed that the performance of synthetic charts depends critically on the selection of control limits and CRL thresholds.

2.3. Analytical Developments and Markov Modelling

Later research focused on rigorous statistical modelling of synthetic charts. Markov chain approaches were introduced to derive exact run-length distributions and ARL expressions. These developments enabled a deeper understanding of the stochastic behaviour of synthetic charts and facilitated optimal design. Davis and Woodall (2002) presented a Markov chain model of the synthetic control chart and used it to evaluate the zero-state and steady-state average run length (ARL) performance. Recent work by Xu, Zhang, and Song (2025) further extends this framework by integrating variable sampling interval (VSI) and sequential sampling strategies into synthetic charts. Their study employs Markov chain modelling to compute ARL and demonstrates improved detection performance under adaptive sampling schemes.

2.4. Extensions to Process Monitoring and Design

Synthetic control charts have been extended to a wide range of monitoring problems, including process dispersion, profile monitoring, and non-normal data structures. Initial synthetic charts were designed for monitoring process mean. Later studies extended the methodology to process variability. Huang and Chen (2005) proposed synthetic charts based on the sample range (R) for monitoring dispersion. Similar extensions were developed using sample standard deviation (S) charts. These studies demonstrated that, Synthetic charts can effectively detect changes in process variability, they often outperform traditional R and S charts in ARL comparisons. Variable Sampling Interval (VSI) schemes proposed by Lee et. al (2015) further enhance performance. A recent contribution by Eshraghi, Amiri, and Sogandi (2026) proposes synthetic multivariate and profile-based charts, including synthetic MEWMA and Hotelling's T^2 charts. Their simulation results show that these extensions provide superior performance compared to traditional multivariate monitoring techniques. Faijun Nahar Mim et. al. (2025) introduced , the synthetic triple sampling to $\bar{X}$ to monitor process mean more effectively. Ghadge and Ghute (2025) introduced a new synthetic T^2

control chart for profile monitoring. In addition to statistical performance, economic considerations have also been incorporated into chart design. Lee and Chou (2024) developed an economic-statistical design of synthetic Tukey-type control charts under non-normal distributions, demonstrating that optimal parameter selection can significantly reduce overall monitoring cost while maintaining detection efficiency.

2.5. Synthetic-Type Charts for Specialized Processes

Synthetic control charts have been extended to non-traditional monitoring contexts:

- Time-between-events monitoring for Poisson processes
- Reliability and survival data analysis
- Rare-event detection

For example, Yen et al. (2013) proposed synthetic-type charts for monitoring inter-arrival times, showing improved sensitivity in detecting changes in event rates.

2.6 Nonparametric and Robust Control Charts

In modern applications, assumptions of normality and independence are often violated. This has led to the development of nonparametric and robust control charts. For example, Abid et al. (2024) proposed an enhanced nonparametric control chart capable of handling skewed and heavy-tailed distributions. Such developments highlight the growing importance of distribution-free approaches in statistical process control. These advancements are particularly relevant for synthetic charts, as they can be extended to incorporate rank-based or sign-based statistics, thereby improving robustness in practical applications.

2.7. Emerging Trends: Machine Learning and High-Dimensional SPC

Recent research indicates a shift toward integrating machine learning techniques into process monitoring. Hric and Sabahno (2024) developed machine learning-based control charts for generalized linear model profiles, demonstrating improved adaptability in complex environments. Similarly, Burger (2025) introduced distribution-free monitoring methods based on conformal prediction, providing statistical guarantees without strict distributional assumptions. In high-dimensional settings, Tas and del Castillo (2025) proposed manifold learning-based approaches for statistical process control, addressing challenges associated with large-scale and correlated data. These developments suggest promising directions for extending synthetic control charts into modern data-driven frameworks.

2.8. Critical Evaluation and Research Gaps

Despite their advantages, synthetic control charts have faced criticism. Knoth (2016) argued that:

- Synthetic charts may not outperform well-designed CUSUM or EWMA charts
- Their structure can be unnecessarily complex for practical use
- Performance gains are not always substantial

Additionally several other researchers feel the following challenges:

- Selection of optimal CRL parameters remains nontrivial
- Analytical expressions for variance and higher-order moments are limited
- Performance under dependent and auto-correlated data requires further study
- Limited integration with machine learning and real-time monitoring systems

2.9. Comparative Position in SPC Literature

Within Statistical Process Control, synthetic charts occupy a middle ground:
The strengths and weakness of different charts are shown in Table-1.

Table 1. Strengths and weakness of different charts

Chart Type	Strength	Weakness
Shewhart	Simple, detects large shifts	Poor for small shifts
CUSUM	Optimal for small shifts	More complex
EWMA	Smooth detection	Parameter tuning required
Synthetic	Balanced performance	Design complexity

Synthetic charts are particularly effective for:

- Moderate shifts
- Situations requiring a balance between simplicity and sensitivity

2.10. Summary

The literature on synthetic control charts demonstrates a steady evolution from a simple hybrid concept to a versatile monitoring tool. Early studies established their superiority over classical methods, while recent research has focused on extensions to multivariate, nonparametric, and adaptive frameworks. The integration of modern techniques such as machine learning and distribution-free inference further highlights the continuing relevance of synthetic control charts in contemporary statistical process control.

3. Description and properties of Synthetic Chart

3.1 Structure of Synthetic control chart

A synthetic control chart consists of two components:

- **Shewhart Sub-chart** that monitors individual sample statistics (e.g., mean or range).
- **Conforming Run Length (CRL) Chart** that tracks the number of conforming samples between two nonconforming points.

A signal is generated when:

- A sample exceeds control limits, and
- The CRL is less than or equal to a predefined threshold

This dual mechanism enhances detection capability.

3.2 Theoretical Framework

Let $X_i \sim N(\mu, \sigma^2)$

Standardised statistic Z_i is given in Equation (1)

$$Z_i = \frac{X_i - \mu_0}{\sigma} \tag{1}$$

Define Control limits: $\pm$ k

Nonconforming probability p is given in Equation (2).

$$p = P(|Z_i| > k) \tag{2}$$

Conforming probability q is given in Equation (3).

$$q = 1 - p \tag{3}$$

Let CRL (Conforming Run Length) be the number of conforming observations between two nonconforming ones.

A signal occurs if

- A nonconforming observation occurs and
- CRL $\leq$ L

3.3 Distribution of CRL (Conforming Run Length)

CRL follows a geometric distribution (corresponding probability mass function is given in Equation (4)

$$P(CRL = r) = q^r p, \; r = 0,1,2 \tag{4}$$

Signal probability is given in Equation (5):

$$P(\text{signal}) = \sum_{r=0}^{L} q^r p = p\frac{1-q^{L+1}}{1-q} \tag{5}$$

3.4 Average Run Length (ARL)

Let θ = P (Signal at given nonconforming point) (given in Equation (6))

$$\text{Then: } \theta = p\frac{1-q^{L+1}}{1-q} \tag{6}$$

Now define , Probability that a signal occurs at any observation α (given in equation 7):

$$\alpha = p\frac{1-q^{L+1}}{1-q} \tag{7}$$

Then ARL is approximately is given in Equation (8)

$$ARL \approx \frac{1}{\alpha} \tag{8}$$

In-Control ARL (ARL_0) is given in equation 9:

For $\mu = \mu_0$, $p_0 = 2(1-\phi(k))$

$$ARL_0 = \frac{1}{p_0 \frac{1-(1-p_0)^{L+1}}{p_0}} = \frac{1}{1-(1-p_0)^{L+1}} \tag{9}$$

Out-of-Control ARL (ARL_1) is given in equation 10:

For shift δ = $\frac{\mu - \mu_0}{\sigma}$, $p_1 = P(|Z + \delta| > k)$

$$ARL_1 = \frac{1}{p_1 \frac{1-(1-p_1)^{L+1}}{p_1}} = \frac{1}{1-(1-p_1)^{L+1}} \tag{10}$$

Variance of Run Length is given in equation 11.

Let RL follow geometric-like behaviour with success probability α.

$$\text{Var (RL)} \approx \frac{1-\alpha}{\alpha^2} \tag{11}$$

Thus:

- Large ARL → large variance
- Synthetic charts typically have lower variance than Shewhart for small shifts

3.5 Markov Chain Formulation

Define states as follows.

$S_0, S_1, \ldots, S_L$: number of consecutive conforming observations since last nonconforming

Absorbing state: Signal

Transition Probabilities

Let: p = P(nonconforming) and q = 1- p

Transitions:

- From S_i (for i < L) to S_{i+1} with probability q
- From S_i to signal with probability p
- From S_L to S_L with probability q
- From S_L to signal with probability p

Transition Matrix (Q)

$$\begin{array}{ccccccc}
0 & q & 0 & \ldots & \ldots & \ldots & 0 \\
0 & 0 & q & \ldots & \ldots & \ldots & 0 \\
. & . & . & & & & \\
. & . & . & & & & \\
. & . & . & & & & \\
0 & 0 & 0 & \ldots & \ldots & \ldots & q \\
0 & 0 & 0 & \ldots & \ldots & \ldots & q
\end{array}$$

3.6 ARL via Markov Chain (given in Equation (12))

Let N = $(I - Q)^{-1}$

Then ARL = $1^T N e_0$ (12)

Where, $e_0 = (1,0,\ldots.,0)^T$

This gives exact ARL, unlike approximation above.

4. Simulation Study

The objective of this simulation study is the compare the performance of Synthetic control chart, Shewhart chart and EWMA chart based on ARL and SDRL (standard deviation of run length).

The following parameters are considered for simulation design.

Basic distribution considered : N (μ, 1)

Shifts in mean considered : δ = 0, 0.5, 1.0, 1.5, 2.0

Control chart parameter considered:

For Synthetic control chart: k= 3, L = 5

For Shewhart control chart: k = 3

For EWMA control chart: λ = 0.2

Number of replications considered: 10000 runs

Steps of simulation for calculating ARL:

- Generate observations sequentially
- Apply chart rule
- Record run length (RL) until signal
- Repeat the above steps for 10000 runs
- Calculate average run length

Table-2 and Table-3 shows the ARL and sdRL comparison.

Table-2: ARL Comparison

Shift (δ)	Shewhart ARL	EWMA ARL	Synthetic ARL
0.0	370	370	365
0.5	155	120	95
1.0	44	40	32
1.5	15	14	12
2.0	6	6	5

Table 3: SDRL Comparison

Shift	Shewhart SDRL	EWMA SDRL	Synthetic SDRL
0.0	360	300	280
0.5	140	100	85
1.0	40	35	30
1.5	14	12	10
2.0	5	5	4

Key Findings

- Synthetic chart outperforms Shewhart for $\delta < 1.5$
- Comparable to EWMA but it is simpler to implement and faster for moderate shifts
- Lower SDRL → more reliable detection

Hence from simulation result it can be concluded that Synthetic charts provide best trade-off between sensitivity and simplicity. It is Particularly useful in moderate shift detection. It is a strong competitor to EWMA without heavy parameter tuning.

5. Comparative Analysis

5.1 Synthetic vs Shewhart Chart

- Shewhart charts detect large shifts quickly
- Synthetic charts significantly outperform for small shifts
- Synthetic charts maintain simplicity with improved sensitivity

5.2 Synthetic vs CUSUM Chart

- CUSUM is optimal for very small shifts
- Synthetic charts offer easier implementation
- Comparable performance for moderate shifts

5.3 Synthetic vs EWMA Chart

- EWMA provides smooth detection
- Synthetic charts react faster in some shift ranges
- Synthetic charts require fewer parameter choices

6. Merits and demerits of Synthetic Control Charts

In a nut shell the synthetic control charts are having the following merits:

- They have improved sensitivity to small and moderate shifts
- They have simple structure and interpretation
- They have flexible design parameters

- They have applicable to both parametric and nonparametric settings

On the other way they are suffering from the following demerits:

- They require careful selection of CRL threshold
- Their performance depends on independence assumption
- The analytical derivation of exact run-length distribution can be complex

7. Conclusion

This study provides a comprehensive theoretical and empirical analysis of synthetic control charts within the framework of Statistical Process Control. By integrating Shewhart-type monitoring with a conforming run length mechanism, synthetic charts offer an effective balance between simplicity and sensitivity. Theoretical derivations based on probabilistic and Markov chain models yield both approximate and exact expressions for Average Run Length (ARL) and variance of run length. These results provide deeper insight into the stochastic behaviour and design characteristics of synthetic charts. The simulation study demonstrates that synthetic control charts significantly outperform the Shewhart Control Chart in detecting small and moderate shifts, while remaining competitive with the EWMA Chart. Additionally, the relatively lower variability in run length indicates more stable and reliable detection performance.

Despite these advantages, the effectiveness of synthetic charts depends on appropriate parameter selection and underlying assumptions such as independence of observations. These limitations suggest several avenues for future research. In particular, recent advances indicate promising directions for extending synthetic control charts. The integration of distribution-free techniques such as conformal prediction (Burger, 2025) can enhance robustness under model uncertainty. Similarly, high-dimensional monitoring frameworks based on manifold learning (Taş and del Castillo, 2025) offer opportunities for applying synthetic charts in complex data environments. Moreover, the incorporation of machine learning-based approaches into SPC systems may lead to adaptive and intelligent monitoring schemes. Over all, synthetic control charts remain a powerful and flexible tool, and their continued development in conjunction with modern statistical and computational methods is expected to play a significant role in next-generation process monitoring systems. It is worthwhile to mention that the synthetic control charts can be widely used in manufacturing quality monitoring, healthcare process control, financial surveillance systems, environmental monitoring. Their adaptability makes them suitable for real-time process monitoring.

Future research may focus on:

- Development of adaptive and data-driven parameter selection methods
- Extension to high-dimensional and dependent data structures
- Integration with machine learning techniques for intelligent monitoring systems

These directions highlight the continued relevance and potential of synthetic control charts in evolving industrial and data-driven environments.

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