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Magnetic field effect on a three-dimensional mixed convective flow with mass transfer along an infinite vertical porous plate

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Abstract

An analytical solution to the problem of the MHD free and forced convection three dimensional flow of an incompressible viscous electrically conducting fluid with mass transfer along a vertical porous plate with transverse sinusoidal suction velocity is presented. A uniform magnetic field is assumed to be applied transversely to the direction of the free stream. The expressions for skin friction at the plate in the direction of the main flow and the rate of heat transfer and mass transfer from the plate to the fluid are obtained in non-dimensional form. The amplitudes of the perturbed parts of these fields and the skin friction at the plate are presented in graphs and the effects of different physical parameters like Hartmann number M, Reynolds number R and the Schmidt number S on these fields are discussed and the results obtained are physically interpreted.

Keywords: Viscous , incompressible, electrically conducting, sinusoidal suction

1. Introduction

Many natural phenomena and technological problems are susceptible to MHD analysis. Geophysics encounters MHD characteristics in the interactions of conducting fluids and magnetic fields. Engineers employ MHD principle, in the design of heat exchangers pumps and flow meters, in space vehicle propulsion, thermal protection, braking, control and re-entry, in creating novel power generating systems etc. From technological point of view, MHD convection flow problems are also very significant in the fields of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. Model studies of the above phenomena of MHD convection have been made by many. Some of them are Sanyal and Bhattacharya (1992), Ferraro and Plumpton (1966), Cramer and Pai (1973) and Nikodijevic *et al.* (2009). On the other hand, along with free convection currents, caused by the temperature difference, the flow is also affected by the difference in concentrations on material constitutions. Many investigators have studied the phenomena of MHD free convection and mass transfer flow of whom the names of Acharya *et al.* (2000), Bejan and Khair (1985), Babu and Prasad Rao (2006), Raptis and Kafousias (1982), Singh and Singh (2000) as well as Singh *et al*. (2007), etc. are worth mentioning.

Investigations of the problems of laminar flow control are being done by many researchers due to its importance in the field of aeronautical engineering in view of its application to reduce drag and hence the vehicle power requirement by a substantial amount. The development of this subject has been compiled by (Lachman 1961). Theoretical and experimental investigations indicate that the transition from laminar to turbulent flow which causes the drag co-efficient to increase may be prevented by suction of the fluid, by the application of transverse magnetic field and by heat and mass transfer from the boundary layer to the wall. To obtain any desired reduction in the drag by increasing suction alone is uneconomical as the energy consumptions of the suction pumps will be more. Therefore the method of “cooling of the wall” in controlling the laminar flow together with the application of suction has become more useful and hence received the attention of many workers.

The effect of the flow caused by the periodic suction velocity perpendicular to the main flow when the difference between the wall temperature and free stream temperature gives rise to buoyancy force in the direction of the free stream on heat transfer characteristics has been investigated by Singh *et al*. (1978). Ahmed and Sarma (1997) have extended the work of Singh *et al*. (1978) to the case when the medium is porous. Gupta and Johari (2001) have analyzed the effects of magnetic field on the three-dimensional forced flow of an incompressible viscous fluid past a porous plate. Singh and Sharma (2001) have studied the effect of the porosity of the porous medium on the three-dimensional Couette flow and heat transfer. The same authors (Singh and Sharma, 2002) have also studied the effect of the periodic permeability on the free convective flow of a viscous incompressible fluid through a highly porous medium. The effect of transverse sinusoidal injection velocity distribution on the three dimensional free convective Couette flow of a viscous incompressible fluid in slip flow regime under the influence of heat source has been studied by Jain and Gupta (2006). Ahmed *et al*. (2006) have obtained an analytical solution to the problem of the three-dimensional free convective flow of an incompressible viscous fluid past a porous vertical plate with the transverse sinusoidal suction velocity taking into account the presence of species concentration. Singh (1991) has studied the effect of a uniform transverse magnetic field on the free convection flow of an electrically conducting viscous incompressible fluid past an infinite vertical porous plate with sinusoidal suction velocity and uniform free stream. As the present author is aware till now no attempt has been made to study the effect of a transverse magnetic field on a mixed convective flow of an incompressible viscous electrically conducting fluid with mass transfer along a vertical porous plate with transverse sinusoidal suction velocity taking into account the effect of Ohmic and viscous dissipations together. Such an attempt has been made in the present work. Though the flow geometry of the present work and that of the work of Singh (1991) are common, yet this paper is not a routine extension of the paper (Singh, 1991). Both papers differ in several aspects such as the forms of suction velocities and the combinations of dimensionless substitutions.

2. Basic equations

The equations governing the steady motion of an incompressible viscous electrically conducting fluid in presence of a magnetic field are –

the equation of continuity (law of conservation of mass) :

$$\text{div}\ \vec{q} = 0 \tag{1}$$

the Gauss's law of magnetism (law of conservation of magnetic flux) :

$$\text{div}\ \vec{B} = 0 \tag{2}$$

the momentum equation (law of conservation of momentum) :

$$\left(\vec{q}.\vec{\nabla}\right)\vec{q} = -\frac{1}{\rho}\vec{\nabla}p + \frac{\vec{J}\times\vec{B}}{\rho} + \upsilon\nabla^2\vec{q} + \vec{g} \tag{3}$$

the energy equation (law of conservation of energy) :

$$\rho C_p\left[\left(\vec{q}.\vec{\nabla}\right)\overline{T}\right] = k\nabla^2\overline{T} + \phi + \frac{\vec{J}^2}{\sigma} \tag{4}$$

the species continuity equation (law of conservation of species) :

$$\left(\vec{q}.\vec{\nabla}\right)\overline{C} = D\nabla^2\overline{C} \tag{5}$$

the Ohm's law (Current density and electric field relation) :

$$\vec{J} = \sigma\left[\vec{E}_0 + \vec{q}\times\vec{B}\right] \tag{6}$$

All physical quantities are defined in the Nomenclature.

We now consider the steady free and forced convection flow of an incompressible viscous electrically conducting fluid taking into account the species concentration past a vertical porous plate with transverse sinusoidal suction velocity as mentioned earlier by making the following assumptions.

(i) All the fluid properties except the density in the buoyancy force term are constants.
(ii) A magnetic field of uniform strength B_0 is applied transversely to the direction of the free stream.
(iii) The magnetic Reynolds number Rm is small so that the induced magnetic field can be neglected.
(iv) The level of species concentration in the fluid is very low so that the Soret and Dufour effects can be neglected.
(v) $\overline{T}_w > \overline{T}_\infty$ and $\overline{C}_w > \overline{C}_\infty$

We introduce a coordinate system $\left(\overline{x},\overline{y},\overline{z}\right)$ with X-axis vertically upwards along the plate, Y-axis perpendicular to it and directed into the fluid region and Z-axis along the width of the plate.

Let $q = \hat{i}\overline{u} + \hat{j}\overline{v} + \hat{k}\overline{w}$ be the fluid velocity at the point $\left(\overline{x},\overline{y},\overline{z}\right)$ and $\vec{B} = B_0\hat{j}$ be the applied magnetic field.

The transverse sinusoidal suction velocity is taken as follows:

$$\overline{v}_w\left(\overline{z}\right) = -V_0\left[1+\varepsilon\, Cos\frac{\pi\overline{z}}{L}\right] \tag{7}$$

which consists of basic steady distribution V_0 with superimposed weak distribution $\varepsilon V_0 Cos\frac{\pi z}{L}$ confined in the boundary layer only. Here negative sign indicates that the direction of the suction velocity is towards the plate. . This suction velocity $\bar{v}_w\left(\bar{z}\right)$ is applied transversely to the plate and weak distribution $\varepsilon V_0 Cos\frac{\pi z}{L}$ will have no role in the outer edge of the boundary layer. Due to application of suction at the surface, the fluid particles at the edge of the boundary layer will have a tendency to get displaced towards the plate surface. Therefore $\bar{v} \to -V_0$ at $\bar{y} \to \infty$. This phenomenon is clearly supported by the equation of continuity.

The velocity ,temperature and concentration fields are independent of $\bar{x}$, because an asymptotic flow has been considered but the flow itself is three dimensional due to cross flow.

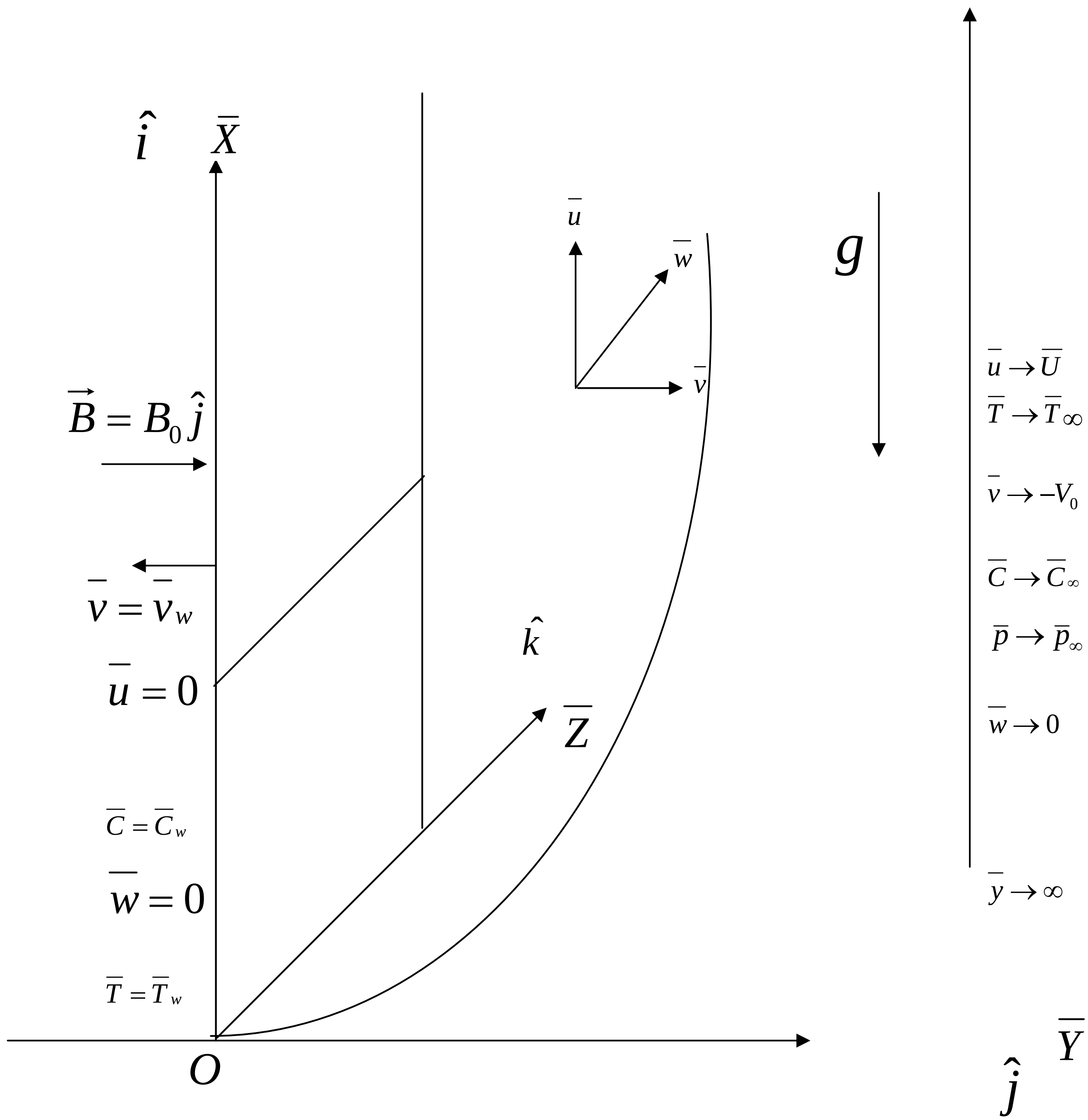

Fig. 1 The flow configuration

With the foregoing assumptions and under the usual boundary layer and Boussinesq approximations, the equations (1), (3), (4) and (5) reduce to:

Equation of continuity $$\frac{\partial \bar{v}}{\partial \bar{y}}+\frac{\partial \bar{w}}{\partial \bar{z}}=0 \qquad (8)$$

x-component of momentum equation $$\rho\left(\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{u}}{\partial \bar{z}}\right)=-\frac{\partial p}{\partial \bar{x}}-\rho g+\mu\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right)-\sigma B_0^2\bar{u} \qquad (9)$$

At the outer edge of the boundary layer the parallel component $\bar{u}=\bar{U}$, the free stream velocity. Since there is no large velocity gradient here, the viscous term in the equation (9) vanishes for small μ and hence for the outer flow, we have

$$0=-\frac{\partial p_\infty}{\partial \bar{x}}-\rho_\infty g-\sigma B_0^2\bar{U} \qquad (10)$$

It is emphasized by (Schlichting 1950) that in case of hot vertical plate, the pressure in each horizontal plane is equal to the gravitational pressure. That is $p=p_\infty$. Hence (10) reduces to

$$0=-\frac{\partial p}{\partial \bar{x}}-\rho_\infty g-\sigma B_0^2\bar{U} \qquad (11)$$

By eliminating the pressure term from the equations (9) and (11), we obtain

$$\rho\left(\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{u}}{\partial \bar{z}}\right)=\left(\rho_\infty-\rho\right)g+\mu\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right)+\sigma B_0^2\left(\bar{U}-\bar{u}\right) \qquad (12)$$

The Boussinesq approximation gives

$$\rho_\infty-\rho=\rho_\infty\beta\left(\bar{T}-\bar{T}_\infty\right)+\rho_\infty\bar{\beta}\left(\bar{C}-\bar{C}_\infty\right) \qquad (13)$$

On using (13) in the equation (12) and noting that ρ_∞ is approximately equal to ρ , we obtain the momentum equations as follows:

$\bar{x}$ -component: $$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{u}}{\partial \bar{z}}=g\beta(\bar{T}-\bar{T}_\infty)+g\bar{\beta}\left(\bar{C}-\bar{C}_\infty\right)+\upsilon\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right)+\frac{\sigma B_0^2}{\rho}\left(\bar{U}-\bar{u}\right) \qquad (14)$$

$\bar{y}$ -component: $$\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{v}}{\partial \bar{z}}=-\frac{1}{\rho}\frac{\partial p}{\partial \bar{y}}+\upsilon\left(\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{v}}{\partial z^2}\right) \qquad (15)$$

$\bar{z}$ -component: $$\bar{v}\frac{\partial \bar{w}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{w}}{\partial \bar{z}}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{z}}+\upsilon\left(\frac{\partial^2 \bar{w}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right)-\frac{\sigma B_0^2\bar{w}}{\rho} \qquad (16)$$

Energy equation:

$$\begin{aligned}\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{T}}{\partial \bar{z}}&=\alpha\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right)\\&+\frac{\upsilon}{C_p}\left[\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2+\left(\frac{\partial \bar{u}}{\partial \bar{z}}\right)^2+\left(\frac{\partial \bar{v}}{\partial \bar{z}}+\frac{\partial \bar{w}}{\partial \bar{y}}\right)^2+2\left\{\left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)^2+\left(\frac{\partial \bar{w}}{\partial \bar{z}}\right)^2\right\}\right]\\&+\frac{\sigma B_0^2}{\rho C_p}\left[\left(\bar{U}-\bar{u}\right)^2+w^2\right]\end{aligned} \qquad (17)$$

Species continuity equation:

$$\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{C}}{\partial \bar{z}}=D\left(\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{C}}{\partial \bar{z}^2}\right) \qquad (18)$$

The symbols involved have been defined in the Nomenclature.

The relevant boundary conditions are:

$$\bar{y}=0:\ \bar{u}=0,\ \bar{v}=\bar{v}_w,\ \bar{w}=0,\ \bar{T}=\bar{T}_w,\ \bar{C}=\bar{C}_w, \tag{19}$$

$$\bar{y}\to\infty:\ \bar{u}=\bar{U},\ \bar{v}=-V_0,\ \bar{w}=0,\ \bar{T}=\bar{T}_\infty,\ \bar{C}=C_{\bar{\infty}},\ \bar{p}=\bar{p}_\infty \tag{20}$$

We introduce the following non-dimensional quantities:

$$y=\frac{\bar{y}}{L},\ z=\frac{\bar{z}}{L},\ u=\frac{\bar{u}}{V_0},\ v=\frac{\bar{v}}{V_0},\ U=\frac{\bar{U}}{V_0},\ w=\frac{\bar{w}}{V_0}$$

$$\theta=\frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty},\ \phi=\frac{\bar{C}-\bar{C}_\infty}{\bar{C}_w-\bar{C}_\infty},\ Pr=\frac{\upsilon}{\alpha},\ Sc=\frac{\upsilon}{D},\ Gr=\frac{Lg\bar{\beta}\left(\bar{T}_w-\bar{T}_\infty\right)}{V_0^2},\ p=\frac{\bar{p}}{\rho\left(\frac{\upsilon}{L}\right)^2}$$

$$Gm=\frac{Lg\bar{\beta}\left(\bar{C}_w-\bar{C}_\infty\right)}{V_0^2},\ E=\frac{V_0^2}{C_p\left(\bar{T}_w-\bar{T}_\infty\right)},\ M=\frac{\sigma B_0^2\upsilon}{\rho V_0^2},\ \mathrm{Re}=\frac{V_0L}{\upsilon},\ p_\infty=\frac{\bar{p}_\infty}{\rho\left(\frac{\upsilon}{L}\right)^2}$$

The non-dimensional forms of (8), (14), (15), (16), (17) and (18) are:

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{21}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=Gr\,\theta+Gm\,\phi+\frac{1}{Re}\left[\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right]+MRe\left(U-u\right) \tag{22}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{Re^2}\frac{\partial p}{\partial y}+\frac{1}{Re}\left[\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right] \tag{23}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{Re^2}\frac{\partial p}{\partial z}+\frac{1}{Re}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-MRe\,w \tag{24}$$

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=-\frac{1}{PrRe}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right)+\frac{E}{Re}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right]+\frac{2E}{Re}\left\{\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right\}$$
$$+\frac{E}{Re}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)^2+MERe\left\{\left(U-u\right)^2+w^2\right\} \tag{25}$$

$$v\frac{\partial \phi}{\partial y}+w\frac{\partial \phi}{\partial z}=\frac{1}{ScR\mathrm{e}}\left(\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 \phi}{\partial y^2}\right) \tag{26}$$

With relevant boundary conditions:

$$y=0:\ u=0,\ v=-\left(1+\varepsilon\,Cos\,\pi\,z\right),\ w=0\ \theta=1,\ \phi=1, \tag{27}$$

$$y\to\infty:\ u=U,\ v=-1,\ w=0,\ \theta=0,\ \phi=0,\ p=p_\infty \tag{28}$$

3. Method of solution

We assume the solutions of the equations (21) to (26) to be of the form:

$$u=u_0(y)+\varepsilon u_1(y,z)+0\left(\varepsilon^2\right) \tag{29}$$

$$v=v_0(y)+\varepsilon v_1(y,z)+0\left(\varepsilon^2\right) \tag{30}$$

$$w=w_0(y)+\varepsilon w_1(y,z)+0\left(\varepsilon^2\right) \tag{31}$$

$$p=p_0(y)+\varepsilon p_1(y,z)+0\left(\varepsilon^2\right) \tag{32}$$

$$\theta=\theta_0(y)+\varepsilon \theta_1(y,z)+0\left(\varepsilon^2\right) \tag{33}$$

$$\phi=\phi_0(y)+\varepsilon \phi_1(y,z)+0\left(\varepsilon^2\right) \tag{34}$$

with $p_0 = p_\infty,\ \ w_0 = 0$

Substituting these in the equations (21) to (26) and equating the co-efficient of same degree terms and neglecting ε^2, we get the following sets of the differential equations.

Zeroth-order equations:

$$\frac{dv_0}{dy} = 0 \tag{35}$$

$$v_0\frac{du_0}{dy} = Gr\,\theta_0 + Gm\,\phi_0 + \frac{1}{Re}\frac{d^2u_0}{dy^2} + MRe(U - u_0) \tag{36}$$

$$v_0\frac{d\theta_0}{dy} = \frac{1}{PrRe}\frac{d^2\theta_0}{dy^2} + \frac{2E}{Re}v_0'^2 + \frac{E}{Re}u_0'^2 + MERe(U - u_0)^2 \tag{37}$$

$$v_0\frac{d\phi_0}{dy} = \frac{1}{Sc\,Re}\frac{d^2\phi_0}{dy^2} \tag{38}$$

First order equations:

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{39}$$

$$-\frac{\partial u_1}{\partial y} + v_1\frac{\partial u_0}{\partial y} = Gr\,\theta_1 + Gm\,\phi_1 + \frac{1}{Re}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - MRe\,u_1 \tag{40}$$

$$-\frac{\partial v_1}{\partial y} = -\frac{1}{Re^2}\frac{\partial p_1}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) \tag{41}$$

$$-\frac{\partial \theta_1}{\partial y} + v_1\frac{d\theta_0}{dy} = \frac{1}{PrRe}\left[\frac{\partial^2\theta_1}{\partial y^2} + \frac{\partial^2\theta_1}{\partial z^2}\right] + \frac{2E}{Re}\frac{du_0}{dy}\frac{du_1}{dy} + \frac{4E}{Re}\frac{dv_0}{dy}\frac{\partial v_1}{\partial y} + 2MERe(U - u_0)u_1 \tag{42}$$

$$-\frac{\partial w_1}{\partial y} = -\frac{1}{Re^2}\frac{\partial p_1}{\partial z} + \frac{1}{Re}\left[\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right] - MRew_1 \tag{43}$$

$$-\frac{\partial \phi_1}{\partial y} + v_1\frac{d\phi_0}{dy} = \frac{1}{ScRe}\left(\frac{\partial^2\phi_1}{\partial y^2} + \frac{\partial^2\phi_1}{\partial z^2}\right) \tag{44}$$

With conditions:

$$y = 0:\ u_0 = 0,\ v_0 = -1,\ \theta_0 = 1,\ \phi_0 = 1, u_1 = 0,\ v_1 = -Cos\,\pi\, z,$$
$$w_1 = 0,\ \theta_1 = 0,\ \phi_1 = 0 \tag{45}$$

$$y \to \infty: u_0 = U,\ v_0 = -1,\ \theta_0 = 0,\ \phi_1 = 0, u_1 = 0,$$
$$v_1 = 0,\ \ w_1 = 0,\ \ p_1 = 0\ \theta_1 = 0,\ \phi_1 = 0 \tag{46}$$

The solutions of the equations (35) and (38) subject to boundary conditions (45) and (46) are respectively

$$v_0 = -1 \tag{47}$$

$$\phi_0 = e^{-Sc\,\mathrm{Re}\,y} \tag{48}$$

In order to solve the coupled equations (36) and (37) under the above boundary conditions, we note that E<1 for all the incompressible fluids and it is assumed that the solutions of these equations to be of the form:

$$u_0(y) = u_{00}(y) + Eu_{01}(y) + 0(E^2) \tag{49}$$

$$\theta_0(y) = \theta_{00}(y) + E\theta_{01}(y) + 0(E^2) \tag{50}$$

Substituting from (49) and (50) in the equations (36) and (37) and equating the co-efficients of the same degree terms and neglecting the term of $0(E^2)$, the following differential equations with corresponding boundary conditions are derived.

$$u_{00}'' + Re\,u_{00}' - MRe^2 u_{00} = -MRe^2 U - Gr\,Re\theta_{00} - GmRe\,e^{-ScRey} \tag{51}$$

$$u''_{01} + Reu'_{01} - MRe^2 u_{01} = -Gr\,\theta_{01} \tag{52}$$

$$\theta_{00}'' + PrRe\theta_{00}' = 0 \tag{53}$$

$$\theta''_{01} + PrRe\theta'_{01} = -Pru'^2_{00} - MPrRe^2\left(U - u_{00}\right)^2 \tag{54}$$

Subject to the boundary conditions:

$$y = 0: \quad u_{00} = 0,\ u_{01} = 0,\ \theta_{00} = 1,\ \theta_{01} = 0 \tag{55}$$

$$y \to \infty: \quad u_{00} = U,\ u_{01} = 0,\ \theta_{00} = 0,\ \theta_{01} = 0 \tag{56}$$

The solutions of these equations under the boundary conditions (55) and (56) are as follows:

$$\theta_{00} = e^{-PrRey} \tag{57}$$

$$u_{00} = U + A_1 e^{-Pr\,Re\,y} + A_2 e^{-ScRe\,y} + A_3 e^{-\lambda Re\,y} \tag{58}$$

$$\begin{aligned}\theta_{01} &= E_0 e^{-Pr\ Rey} + E_1 e^{-2Pr\,Rey} + E_2 e^{-2ScRey} + E_3 e^{-2\lambda Rey} \\ &\quad + E_4 e^{-Re(Pr+Sc)y} + E_5 e^{-Re(\lambda+Sc)y} + E_6 e^{-Re(\lambda+Pr)y}\end{aligned} \tag{59}$$

$$\begin{aligned}u_{01} = Gr\Big[& F_0 e^{-\lambda\,Re\,y} - F_1 e^{-Pr\,Re\,y} - F_2 e^{-2\,Pr\,Re\,y} - F_3 e^{-2ScRe\,y} \\ & -F_4 e^{-2\lambda\,Rey} - F_5 e^{-Re(Pr+Sc)y} \\ & -F_6 e^{-Re(\lambda+Sc)y} - F_7 e^{-(\lambda+Pr)Re\,y}\Big]\end{aligned} \tag{60}$$

where

$$\lambda = \frac{1+\sqrt{1+4M}}{2},\ A_1 = \frac{Gr}{Re\left(Pr^2 - Pr - M\right)},\ A_2 = \frac{Gm}{Re\left(Sc^2 - Sc - M\right)},\ A_3 = -A_1 - A_2 - U,\ B_1 = \frac{A_1^2}{2},$$

$$B_2 = \frac{A_2^2 Sc}{4Sc - 2Pr},\ B_3 = \frac{A_3^2\lambda}{4\lambda - 2Pr},\ B_6 = \frac{2A_1A_3Pr}{\lambda(\lambda+Pr)},\ B_4 = \frac{2A_1A_2PrSc}{Sc(Pr+Sc)},\ B_5 = \frac{2A_2A_3\lambda Sc}{(\lambda+Sc)(\lambda+Sc-Pr)}$$

$$,D_1 = \frac{A_1^2}{2Re^2\,Pr^2},\ D_2 = \frac{A_2^2}{2ScRe^2(2Sc-Pr)},\ D_3 = \frac{A_3^2}{2\lambda Re^2(2\lambda - Pr)},\ D_4 = \frac{2A_1A_2}{ReSc^2(Pr+Sc)},\ D_5$$

$$= \frac{2A_2A_3}{Re^2(\lambda+Sc)(\lambda+Sc-Pr)},\ D_6 = \frac{2A_1A_3}{\lambda Re^2(\lambda+Pr)},\ E_1 = -MPr\,Re^2D_1 - PrB_1,\ E_2 = -PrB_2 - MPr\,Re^2D_2$$

$$E_3 = -PrB_3 - MPr\,Re^2D_3,\ E_4 = -PrB_4 - MPrRe^2D_4,\ E_5 = -PrB_5 - MPrRe^2D_5,\ E_6 = -PrB_6 - MPr\,Re^2D_6,$$

$$E_0 = -(E_1+E_2+E_3+E_4+E_5+E_6),\ F_1 = \frac{E_0}{Re^2\left(Pr^2 - Pr - M\right)},\ F_2 = \frac{E_1}{\left(4Pr^2 - 2\mathrm{Pr} - M\right)Re^2},\ F_4$$

$$= \frac{E_3}{\left(4\lambda^2 - 2\lambda - M\right)Re^2},\ F_5 = \frac{E_4}{Re^2\left\{(Pr+Sc)^2 - (Pr+Sc) - M\right\}},\ F_3 = \frac{E_2}{Re^2\left\{4Sc^2 - 2Sc - M\right\}}$$

$$F_6 = \frac{E_5}{\left\{(\lambda+Sc)^2 - (\lambda+Sc) - M\right\}Re^2},\ F_7 = \frac{E_6}{\left\{(\lambda+Pr)^2 - (\lambda+Pr) - M\right\}Re^2},\ F_0 = \sum_{k=1}^{7} Fk$$

4. Cross flow solution

We shall first consider the equations (39), (41) and (42) for $v_1(y,z)$, $w_1(y,z)$ and $p_1(y,z)$ which are independent of main flow component u_1, temperature field θ_1 and concentration field ϕ_1.The suction velocity.

$v_w = -(1+\varepsilon Cos\pi z)$ consists of a basic uniform distribution –1 with superimposed weak sinusoidal distribution $\varepsilon Cos\pi z$. Hence the velocity components $v, w\, and\ p$ are also separated into mean and small sinusoidal components $v_1, w_1\, and\ p_1$. We assume $v_1, w_1\, and\ p_1$ to be of the following form:

$$v_1 = -\pi v_{11}(y) Cos\pi z \tag{61}$$

$$w_1 = v'_{11}(y) Sin\pi z \tag{62}$$

$$p_1 = Re^2 p_{11}(y) Cos\pi z \tag{63}$$

On substitution of (61), (62) and (63), the equation (39) is satisfied and the equations (41) and (42) reduce to the following differential equations:

$$v''_{11} + Rev'_{11} - \pi^2 v_{11} = -\frac{\text{Re}\, p'_{11}}{\pi} \tag{64}$$

$$v'''_{11} + Rev''_{11} - \left(\pi^2 + MRe^2\right) v'_{11} = -Re\pi\, p_{11} \tag{65}$$

The relevant boundary conditions for these equations are:

$$y = 0:\ v_{11} = \frac{1}{\pi},\ v'_{11} = 0 \tag{66}$$

$$y \to \infty:\ v_{11} = 0,\ v'_{11} = 0 \tag{67}$$

The solution of the equations (64) and (65) subject to the boundary conditions (66) and (67) is

$$v_{11} = \frac{1}{\pi(\bar{\xi} - \xi)}\left[\bar{\xi} e^{-\xi y} - \xi e^{-\bar{\xi} y}\right] \tag{68}$$

Where

$$\xi = \frac{Re\lambda + \sqrt{Re^2\lambda^2 + 4\pi^2}}{2}, \bar{\xi} = \frac{Re\bar{\lambda} + \sqrt{Re^2\bar{\lambda}^2 + 4\pi^2}}{2}\ \bar{\lambda} = \frac{1 - \sqrt{1+4M}}{2},\ \lambda = \frac{1+\sqrt{1+4M}}{2}$$

Hence the solutions for the velocity components v_1, w_1 and pressure p_1 are as follows:

$$v_1 = \frac{1}{(\xi - \bar{\xi})}\left[\bar{\xi} e^{-\xi y} - \xi e^{-\bar{\xi} y}\right] Cos\pi z \tag{69}$$

$$w_1 = \frac{\xi\bar{\xi}}{\pi(\bar{\xi} - \xi)}\left[e^{-\bar{\xi} y} - e^{-\xi y}\right] Sin\pi z \tag{70}$$

$$P_1 = \frac{Re\xi\bar{\xi}}{\pi^2(\bar{\xi} - \xi)}\left[\bar{\xi}_1 e^{-\bar{\xi} y} - \xi_1 e^{-\xi y}\right] \tag{71}$$

Where, $\bar{\xi}_1 = \pi^2 + MRe^2 + Re\bar{\xi} - \bar{\xi}^2, \xi_1 = \pi^2 + MRe^2 + Re\xi - \xi^2$

5. Solutions for flow, concentration, and temperature field

We shall now consider the equations (40), (43) and (44). The solutions for the velocity component u, concentration field ϕ and temperature field θ are also separated into mean and sinusoidal components u_1, θ_1, ϕ_1. To reduce the partial differential equations (40), (43) and (44) into ordinary differential equations, we consider for the following forms for u_1, θ_1, ϕ_1.

$$u_1 = u_{11}(y) Cos\pi z \tag{72}$$

$$\theta_1 = \theta_{11}(y) Cos\pi z \tag{73}$$

$$\phi_1 = \phi_{11}(y) Cos\pi z \tag{74}$$

Using the expressions for $v_1, u_1, \theta_1, \phi_1$ in (40), (43), (44), we get the following ordinary differential equations

$$u''_{11} + Reu'_{11} - \left(\pi^2 + MRe^2\right) u_{11} = -\pi Rev_{11} u'_0 - Gr\, Re\theta_{11} - Gm\, Re\,\phi_{11} \tag{75}$$

$$\theta_{11}'' + PrRe\theta_{11}' - \pi^2\theta_{11} = -\pi PrRev_{11}\theta_0' - 2EPr\,u_0'u_{11}' + 2EMRe^2Pr\left(U - u_0\right)u_{11} \tag{76}$$

$$\phi_{11}'' + ScRe\phi_{11}' - \pi^2\phi_{11} = -ScRe\pi\, v_{11}\phi_0' \tag{77}$$

with the boundary conditions

$$y = 0:\; u_{11} = 0, \theta_{11} = 0, \phi_{11} = 0 \tag{78}$$

$$y \to \infty : u_{11} = 0, \theta_{11} = 0, \phi_{11} = 0 \tag{79}$$

The solution of the equation (77) subject to the boundary conditions (78) and (79) is

$$\phi_{11} = H_0 e^{-ay} + H_1 e^{-(\xi + ScRe)y} + H_2 e^{-\left(\bar{\xi} + ScRe\right)y} \tag{80}$$

Where

$$a = \frac{ScRe + \sqrt{Sc^2\,Re^2 + 4\pi^2}}{2}\;, H_1 = \frac{Sc^2\,Re^2\bar{\xi}}{\left(\bar{\xi} - \xi\right)\left(\xi^2 + Sc\,Re\xi - \pi^2\right)}$$

$$H_2 = \frac{-Sc^2\,Re^2\xi}{\left(\bar{\xi} - \xi\right)\left(\bar{\xi}^2 + ScRe\bar{\xi} - \pi^2\right)}\;, H_0 = -\left(H_1 + H_2\right)$$

Now in order to solve the coupled equations (75) and (76), the solutions of these two equations are assumed to be of the form:

$$u_{11}(y) = f_0(y) + Ef_1(y) + 0\left(E^2\right) \tag{81}$$

$$\theta_{11}(y) = \psi_0(y) + E\,\psi_1(y) + 0\left(E^2\right) \tag{82}$$

Substituting these in the equations (75) and (76) and equating the coefficients of similar terms and neglects E^2, we get the following differential equations:

$$f_0'' + Ref_0' - \left(\pi^2 + MRe^2\right) f_0 = -\pi Rev_{11}u_{00}' - GrRe\psi_0 - GmRe\phi_{11} \tag{83}$$

$$f_1'' + Ref_1' - \left(\pi^2 + MRe^2\right) f_1 = -GrRe\psi_1 - \pi Rev_{11}u_{01}' \tag{84}$$

$$\psi_0'' + PrRe\psi_0' - \pi^2\psi_0 = -\pi PrRev_{11}\theta_{00}' \tag{85}$$

$$\psi_1'' + PrRe\,\psi_1' - \pi^2\psi_1 = -2Pru_{00}'\,f_0' + 2MRe^2Pr\left(U - u_{00}\right) f_0 - \pi PrRev_{11}\theta_{01}' \tag{86}$$

with boundary conditions:

$$y = 0:\; f_0 = 0,\;\; f_1 = 0,\;\; \psi_0 = 0,\;\; \psi_1 = 0 \tag{87}$$

$$y \to \infty:\; f_0 = 0,\;\; f_1 = 0,\;\; \psi_0 = 0,\;\; \psi_1 = 0 \tag{88}$$

The equations (83) to (86) are solved subject to the boundary conditions (87) and (88), but not presented here for the sake of brevity.

6. Skin friction and heat and mass flux.

The non-dimensional skin-friction in the direction of the free stream at the wall *y = 0* is given by

$$\tau = \frac{-\mu\left.\dfrac{\partial\bar{u}}{\partial\bar{y}}\right]_{\bar{y}=0}}{\rho v_0 \bar{U}} = -\frac{1}{Re}\left.\frac{\partial u}{\partial y}\right]_{y=0} = -\frac{1}{Re}\left[u_0'\left(0\right) + \varepsilon u_{11}'\left(0\right)Cos\pi z\right]$$

$$= \tau_0 + \varepsilon Q_1\left(Pr, Sc, Re, Gr, Gm, E, M\right)Cos\pi\, z \tag{89}$$

Where $Q_1 = -\dfrac{u_{11}'\left(0\right)}{Re}\,,\; \tau_0 = -\dfrac{u_0'\left(0\right)}{Re}$

The heat flux from the plate to the fluid in terms of Nusselt number is given by

$$Nu = -\frac{k}{\rho v_0 C_p\left(\bar{T}_w - \bar{T}_\infty\right)}\left(\frac{\partial\bar{T}}{\partial\bar{y}}\right)_{\bar{y}=0} = -\frac{1}{Pr\,Re}\left.\frac{\partial\theta}{\partial y}\right]_{y=0}$$

$$= Nu_0 + \varepsilon Q_2\left(Pr, Re, M, E, Gr, Gm, Sc\right)Cos\pi\, z \tag{90}$$

where $Nu_0 = -\frac{1}{PrRe}\theta_0'(0), \quad Q_2 = -\frac{\theta_{11}'(0)}{Pr\,Re}$

The mass flux at the wall *y = 0* in terms of Sherwood number Sh is given by

$$Sh = -\frac{D}{V_0\left(\bar{C}_w - \bar{C}_\infty\right)}\left(\frac{\partial \bar{C}}{\partial \bar{y}}\right)_{\bar{y}=0} = \frac{-1}{ScRe}\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = \frac{-1}{ScRe}\left[\phi_0'(0) + \varepsilon\phi_{11}'(0)Cos\pi z\right]$$

$$= 1 + \varepsilon Q_3(Sc, Re, M)Cos\pi z \tag{91}$$

where $Q_3 = -\frac{\phi_{11}'(0)}{ScRe}$

7. Discussion of results

In order to get the physical insight into the problem, the numerical values for τ, Q_1, Q_2 and Q_3 which are respectively the skin friction and the amplitudes of the first order skin friction, first order Nusselt number and first order Sherwood number at the plate are obtained for different values of the physical parameters involved in the problem and these are demonstrated in graphs. The investigation is restricted to Prandtl number Pr equal to 0.7 which corresponds to air . The value of each of G and G_m has been chosen as 10. The Schmidt number *S* are chosen in such a way that they represent the diffusing chemical species of common interest in air and water (for example S = 0.24 for H_2, S = .60 for H_2O, S = 0.78 for NH_3 and S = 1 for CO_2). That is in the present investigation the air is considered as the diffusing medium (solvent) and H_2, H_2O, NH_3 and CO_2 as diffusing species (solutes). The free stream velocity U is taken to be equal to 1 and E is selected to be 0.05. The values of the other physical parameters namely M and Re are chosen arbitrarily.

Figures 2 and 3 depict the variation of the skin friction τ at the plate under the influences Re, M, and Sc. It is observed from Figure 2 that an increase in M leads to a decrease in the magnitude $|\tau|$ of the skin friction. That is there is a reduction in the viscous drag (shearing stress) on the plate due to the application of the transverse magnetic field. This result has a good agreement with the physical realities. Because the application of a transverse magnetic field to a flow of an electrically conducting fluid has a retarding effect to the fluid motion and hence the increase of the velocity gradient in the direction normal to the plate is prevented due to imposition of magnetic field for which the sharing stress at the plate is reduced.

It is inferred from Figure 3 that an increase in Schmidt number results in a decrease in $|\tau|$. That is the mass diffusion causes the viscous drag on the plate to increase and it clearly supports the physical situation as the mass diffusion accelerates the fluid motion for which the velocity gradient at the plate increases causing growth in drag on the plate. It is also seen from these two figures that $|\tau|$, the magnitude of the skin friction at the plate becomes very large for small Reynolds number whereas there is a fall in $|\tau|$ for large R. In other words the frictional force on the plate becomes high for high viscosity. This result is clearly supported by the Newton's law of viscosity. There is a clear indication from the Figures 2 and 3 the magnetic field as well as mass diffusion ceases to affect the sharing stress at the plate for low viscosity or large suction.

The change of behaviour of Q_1, the amplitude of the first order skin friction at the plate against the Reynolds number R under the effects of M and S is presented in Figures 4 and 5. . Figure 4 shows that the magnitude of Q_1 decreases due to application of the magnetic field. It is seen that Q_1 is negative for small and moderate values of R and it takes its positive values for large R. That is the direction of the first order shearing stress at z=0 for large R is opposite to that of the first order skin friction at z=0 for small and moderate R. Fig.5 clearly shows that an increase in the Schmidt number S leads to a decrease in the magnitude of Q_1. There is an indication from figure 5 that the magnitude of Q_1 first decreases as R increases for small R and then it slowly and steadily increases as R.

The variation of the amplitude Q_2 of the first order Nusselt number Nu_1 is demonstrated in Figure 6. It is observed from Figure 6 that Q_2 decreases under effect of the magnetic field. The same figure also shows that Q_2 is diminished by the frictional property of the fluid. In other words the rate of first order heat transfer (for z=0) from the plate to the fluid drops due to application of the transverse magnetic field or due to small suction..

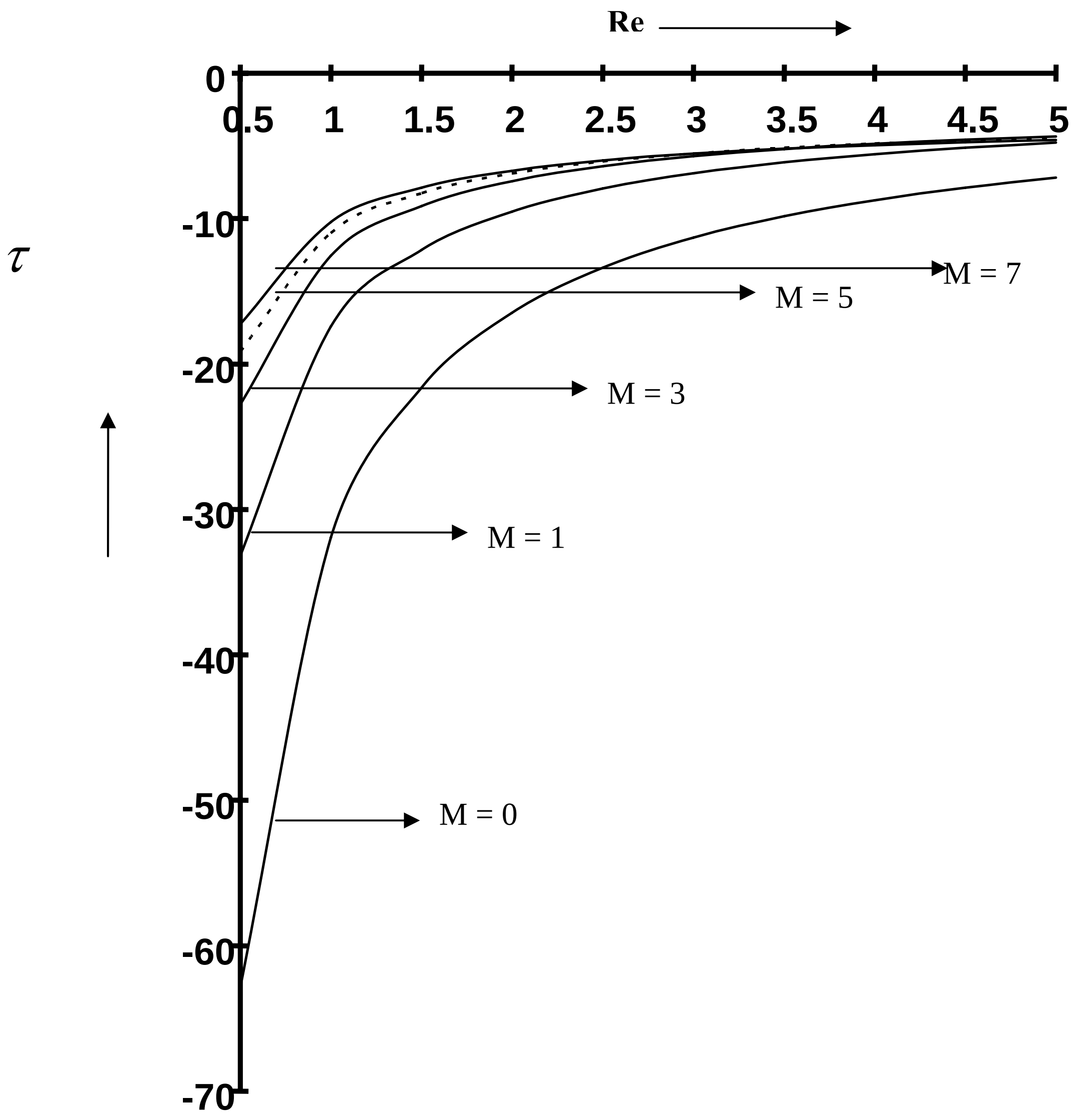

Fig .2: Skin friction τ versus Re for Pr = 0.7, Sc = 0.60, Gr=10, G m= 10, U = 1, E = 0.05

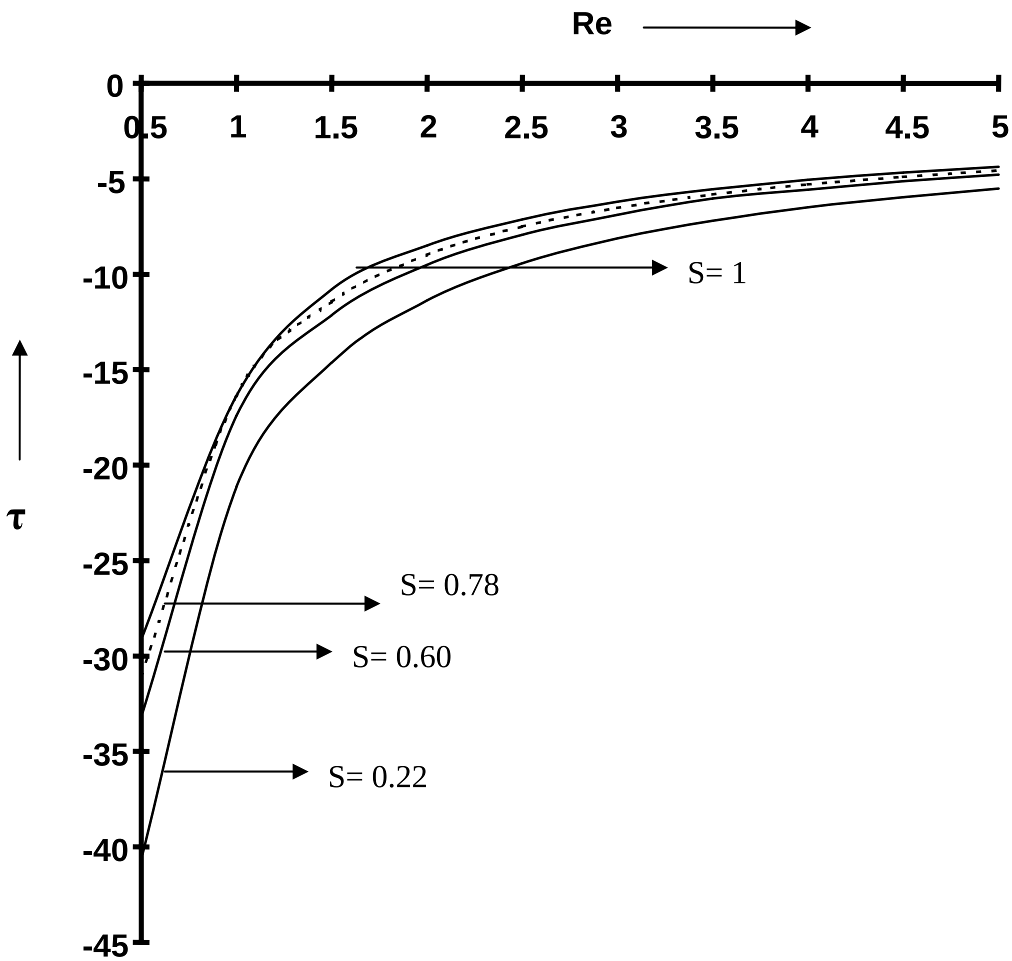

Fig. 3: Skin friction τ versus Re for Pr =0.7, M = 1, Gr=10, Gm = 10, U = 1, E = 0.05

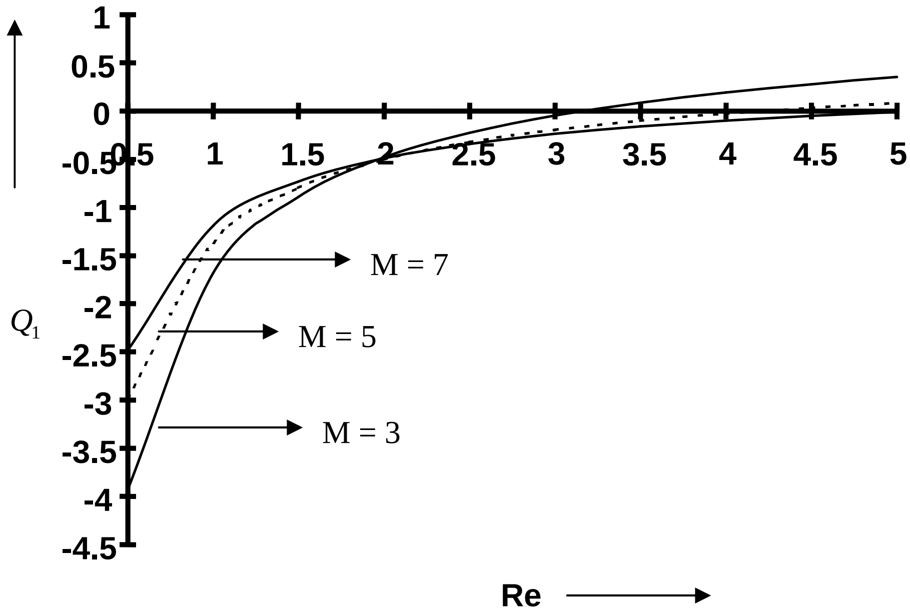

Fig.4: The amplitude Q_1 of the first order skin friction τ_1 versus Re for P =0.7, S = 0.60, G=10, G_m = 10, U = 1, E = 0.05

The effects of the Reynolds number R, the Hartmann number M and the Schmidt number S on Q_3, the amplitude of the first order Sherwood number are shown in figures 7 and 8. These two figures show that there is a steady fall in $|Q_3|$ when M and R are increased whereas $|Q_3|$ steadily increases for increasing Schmidt number.

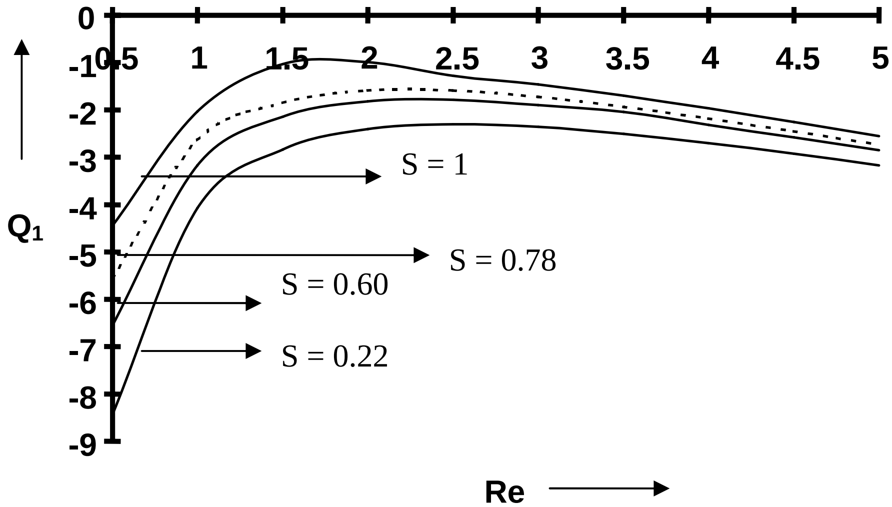

Fig.5: The amplitude Q_1 of the first order skin friction τ_1 versus Re for P =0.7, M = 1, G=10, G_m = 10, U = 1, E = 0.05

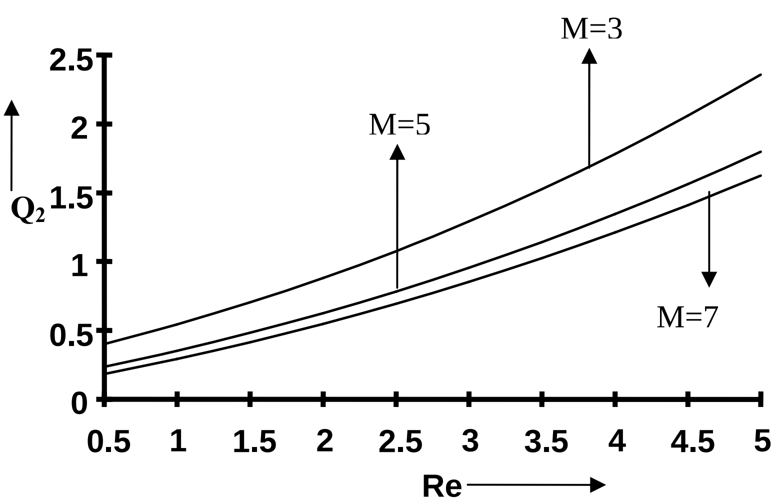

Fig.6: The amplitude Q_2 of the first order Nusselt number Nu_1 versus Re for P =0.7, S = 0.60, G=10, G_m = 10, U = 1, E = 0.05

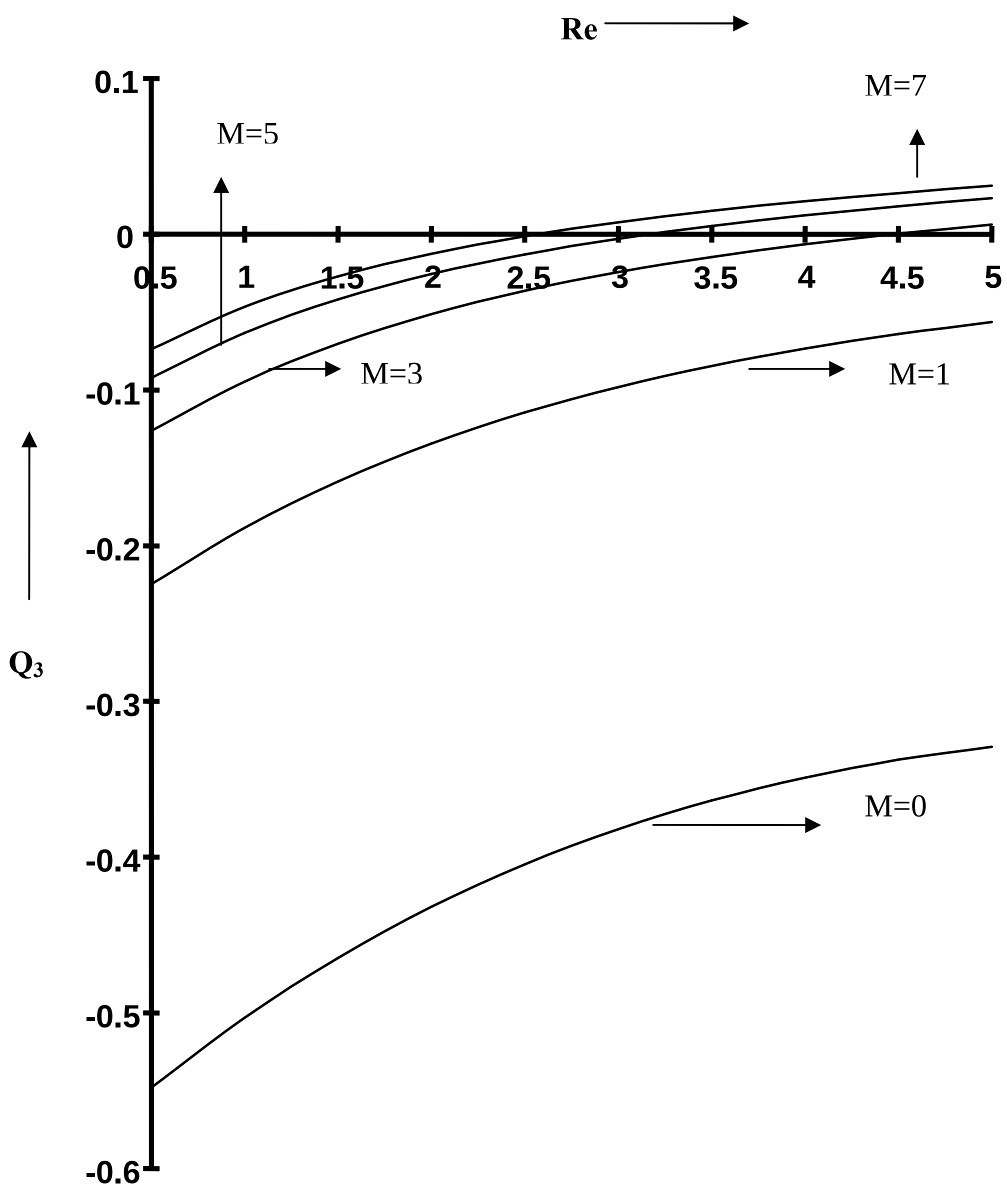

Fig.7: The amplitude Q_3 of the first order Sherwood number Sh_1 versus Re for S=0.60, G = 10, G_m = 10, U = 1, E = 0.05

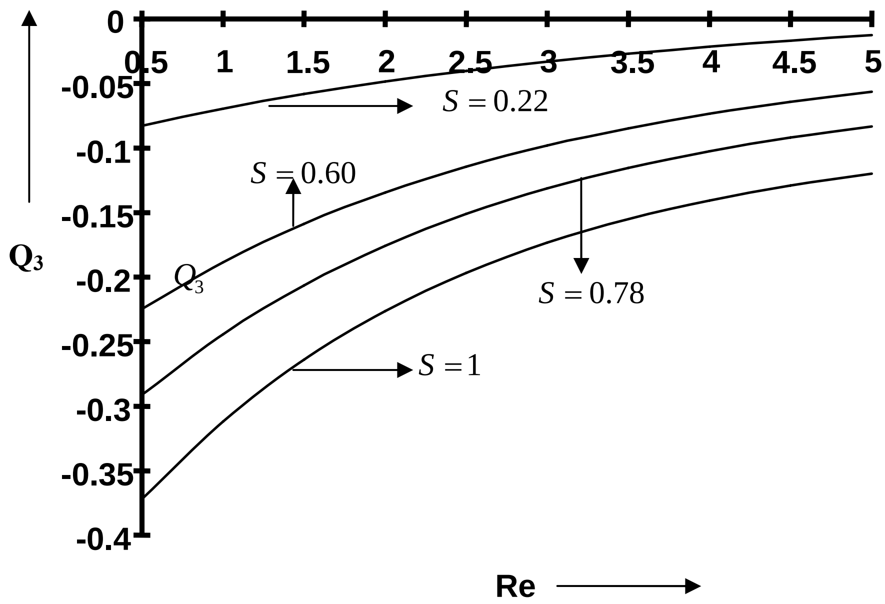

Fig. 8: The amplitude Q_3 of the first order Sherwood number versus Re for M =1, G = 10, G_m = 10, U = 1, E = 0.05

That is to say that low viscosity or magnetic field effect or an increase in mass diffusivity leads to a steady drop in the first order mass flux (for z=0) from plate to the fluid. The same figures further indicate that Q_3 is not affected by the Prandtl number Pr .

7.1 Comparison of results

To compare the results of our paper with those cited in the references, we choose the paper by Gupta and Johari (2001)

Table 1: The variation of the numerical values of the first order skin friction F_1 at the plate against the Hartmann number M and the velocity ratio α for the paper by Gupta and Johari (2001) for Re=1

α	M =0	M =2	M =4
0.5	1.6999	1.6967	1.6939
1.0	3.6796	3.6655	3.6524
1.5	5.9675	5.9341	5.9032

From the above table it is observed that the first skin friction F_1 at the plate decreases as the Hartmann number M increases for Re=1.In the present paper also it is seen that for Re=1, the magnitude of the first order skin friction at the plate decreases as M increases (fig.4).Thus we see that the results concerning first order skin friction under effect of the parameter M are in a good agreement for both the papers. It may be mentioned that in the velocity ratio does not appear in the present work. It is adjusted in the boundary conditions of the flow problem under consideration.

8. Conclusions

The results obtained from our investigation lead to the following conclusions:

i) The transverse magnetic field or low viscosity or high Schmidt number causes a reduction to the viscous drag on the plate.

ii) The transition from laminar to turbulent flow may be prevented up to a certain extent due to the application of a transverse magnetic field.

iii) The application of the transverse magnetic field or a reduction in mass diffusivity for small R causes the magnitude of amplitude of the perturbed part of the skin friction at the plate to fall.

iv) The rate of first order heat transfer (for z=0) from the plate to the fluid drops due to application of the transverse magnetic field or due to small suction..

v) The low viscosity or magnetic field effect or an increase in mass diffusivity leads a steady drop in the first order mass flux(for z=0) from plate to the fluid.

Nomenclature

$\vec{B}$ [-] magnetic induction vector
B_0 [Tesla] intensity of the applied magnetic field
C_p [$J/kg\,K$] specific heat at constant pressure
$\overline{C}$ [kmol/m ^{3}] species concentration
$\overline{C}_\infty$ [kmol/m ^{3}] concentration of the fluid at infinity
$\overline{C}_w$ [kmol/m ^{3}] concentration of the fluid at the plate
D [m ^{2} s^{-1}] coefficient of chemical molecular mass diffusivity
E [-] Eckert number
$\vec{E}_0$ [-] electric field
Gr [-] Grashof number for heat transfer
Gm [-] Grashof number for mass transfer
g [m s^{-2}] acceleration due to gravity
$\hat{i}, \hat{j}, \hat{k}$ [-] unit vectors along the co-ordinate axes
$\vec{J}$ [-] electric current density
$\vec{J} \times \vec{B}$ [-] Lorentz force per unit volume
$\dfrac{\vec{J}^2}{\sigma}$ [W m^{-3}] Ohmic dissipation per unit volume
k [W/mK] thermal conductivity
L [m] wave length of the periodic suction velocity
M [-] Hartmann number
$\Pr$ [-] Prandtl number
p [N m^{-2}] pressure
p_∞ [N m^{-2}] gravitational pressure
$\vec{q}$ [-] velocity vector
Re [-] Reynolds number
Sc [-] Schmidt number
$\vec{T}$ [K] temperature
$\overline{T}_\infty$ [K] temperature of the fluid at infinity
$\overline{T}_w$ [K] temperature of the fluid at the plate
$\overline{U}$ [m s^{-1}] free stream velocity
$(\overline{u}, \overline{v}, \overline{w})$ [m s^{-1}] components of $\vec{q}$
u [-] dimensionless velocity in x-direction
v [-] dimensionless velocity in y-direction
$\overline{v}_w$ [m s^{-1}] suction velocity

V_0 [m s^{-1}] mean suction velocity
w = dimensionless velocity in z-direction
$(\bar{x}, \bar{y}, \bar{z})$ [m] Cartesian coordinates
y, z [-] dimensionless co-ordinates perpendicular to the free stream velocity

Greek Symbols

α [m^2 K^{-1}] thermal diffusivity
β [K^{-1}] co-efficient of volume expansion for thermal expansion
$\bar{\beta}$ [m^3 /k mol] the volumetric co-efficient of expansion with concentration
ε [-] small reference parameter ($\varepsilon << 1$)
ϕ [-] dimensionless concentration
φ [W m^{-3}] viscous dissipation per unit volume
θ [-] dimensionless temperature
ρ [kg/ m^3] density of the fluid
ρ_∞ [kg/ m^3] density of the fluid in the free stream
σ [Ω^{-1} m^{-1}] electrical conductivity
υ [m^2 s^{-1}] kinematic viscosity

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