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Effect of point source and heterogeneity on the propagation of magnetoelastic shear wave in a monoclinic medium

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Abstract

This paper stands to investigate the possibility of propagation of SH waves due to a point source in a magnetoelastic monoclinic layer lying over a heterogeneous monoclinic half-space. The heterogeneity is caused by consideration of quadratic variation in rigidity. The methodology employed combines an efficient derivation for Green's functions based on algebraic transformations with the perturbation approach. Dispersion equation has been obtained in the closed form. The dispersion curves are compared for different values of magnetoelastic coupling parameters and inhomogeneity parameters. It is found that as heterogeneity parameters and magnetoelastic coupling parameters increases, the phase velocity increases for both isotropic and monoclinic cases but the increase is more prominent in the monoclinic case. In the isotropic case, when heterogeneity and magnetic field are absent, the dispersion equation is matched with the classical SH wave equation

Keywords: Shear wave, monoclinic, magnetoelastic, dispersion equation, seismic wave.

1. Introduction

Seismic waves may be of devastating force near their sources, like all waves, they lose energy as they travel through the Earth. The mantle appears hard and solid to seismic waves, but is believed to exhibit a softer, plastic behaviour over long geological time intervals. So, in the perspective of real earth, the study of seismic wave behaviour due to point source is of considerable interest. The basic characteristic and Geophysical studies about earth structure motivate to study the propagation of horizontally polarised shear waves (SH waves) in the medium of monoclinic. In the context of its application such study is of equal importance in various branches, like: Earth Science, Seismology, Geophysics etc.

Keeping in the mind the importance of monoclinic medium, many attempt have been made to better understand the propagation pattern of the seismic waves in the medium of monoclinic type. Chattopadhyay and Bandyopadhyay (1986) discussed the propagation of shear waves in an infinite monoclinic crystal plate as well as in infinite anisotropic non-homogeneous monoclinic plate. Kalyani et al. (2008) studied the propagation of SH waves in the plane of mirror symmetry of a monoclinic multilayered medium with displacement normal to the plane. Chattopadhyay et al. (1994) studied the propagation of a crack due to shear waves in a non-homogeneous medium of monoclinic. Subsequently, Singh and Tomar (2007) studied the propagation of quasi-P wave at the interface between two monoclinic half-spaces.

Geologists have long been aware of the Earth's dynamic condition. Several hypotheses have attempted to explain the underlying mechanisms. In the late nineteenth and early twentieth centuries geological orthodoxy favoured the hypothesis of a contracting Earth. The external disturbance gives rise to wave motions propagating away from the disturbed region. In seismology the problem of the source mechanism consists in relating observed seismic waves to the parameters that describes the source. In the Earth, neglecting the force of gravity, body forces in the equation of motion may be used to represent the processes that generate earthquakes. In general, these forces are functions of the spatial coordinates and time, may be different for each earthquake and are defined only inside a certain volume. A type of body forces of great importance in the solution of many problems of elastodynamics is that formed by a unit impulsive force in space and time with an arbitrary direction; this point action or impulse is usually described by the Dirac delta function. Thus the solutions of equations of motion represent the elastic displacement due to a unit impulse force in space and time. For this reason, the Green's function called the response of the medium to an impulsive excitation. The form of this function depends on the characteristics of the medium, its elastic coefficients, and its density. In a finite medium, it depends also on the shape of the volume and the boundary conditions on its surface. For each medium there is a different Green's function that defines how this medium reacts mechanically to an impulsive excitation force and is, therefore, a proper characteristic of each medium. Green's function plays an important role in the solution of numerous problems in the mechanics and physics of solids. There are many articles in various journals on the application of Green's function to seismological problems which are very much useful for both, researchers and practitioners with backgrounds in different branches of science. However, no extensive, detailed treatment of this subject has been available up to the present. The complete problem of Green's function corresponds to an impulsive force in an arbitrary direction (Aki and Richards, 1980). The propagation of Love type waves from a point source in either homogeneous or inhomogeneous elastic media has been considered by many authors (viz. De Hoop 1995; Brekhovskikh and Godin 1992; Vrettos 1991, 1998; Singh 1969; Deresiewiez 1962; Ewing et al., 1957 etc.). The propagation of Love waves due to point source in a homogeneous layer overlying a semi-homogeneous substratum has been discussed by Sezawa (1935). Chattopadhyay and Maugin (1993) studied the Magneto-elastic surface shear waves due to a momentary point source. Sato (1952) studied the propagation of SH waves in a double superficial layer over heterogeneous medium by taking variation in rigidity. Bhattacharya (1969) described the possibility of the propagation of love type waves in an intermediate heterogeneous layer lying between two semi-infinite isotropic homogeneous elastic layers. Chattopadhyay and Kar (1981) discussed the Love waves due to a point source in an isotropic elastic medium under initial stress. Covert (1958) indicated a method for finding the Green's function for composite bodies. Chattopadhyay et al (1986) studied the dispersion equation of Love waves in a porous layer. They used the Green's function technique to obtain the dispersion equation. Watanabe and Payton (2002) discussed the Green's function for SH waves in a cylindrically monoclinic material. He derived the closed form expression for Green's function for a few limited values of anisotropic parameters and shown the contours of the displacement amplitude for the time harmonic wave. Manolis and Bagtzoglou (1992) described a numerical comparative study of wave propagation in inhomogeneous and random media. He employed the Green's function approach for waves propagating from a point source, while techniques to account for the presence of boundaries are also discussed. Kausel and Park (2004) used a sub-structuring technique to obtain the impulse response in the wave number-time domain for a layered half-space. Manolis and Shaw (1995) developed the fundamental Green's function for the case of scalar wave propagation in a stochastic heterogeneous medium.

This paper stands to investigate the possibility of propagation of SH waves due to a point source in a magnetoelastic monoclinic layer lying over a heterogeneous monoclinic half-space. The heterogeneity is caused by consideration of quadratic variation in rigidity. The methodology employed combines an efficient derivation for Green's functions based on algebraic transformations with the perturbation approach. Dispersion equation has been obtained in the closed form. It is found that as heterogeneity parameters and magnetoelastic coupling parameters increases, the phase velocity increases for both isotropic and monoclinic cases but the increase is more prominent in the monoclinic case. In the isotropic case, when heterogeneity and magnetic field are absent, the dispersion equation is matched with the classical SH wave equation

2. Formulation and solution of the problem

We have considered a magnetoelastic monoclinic layer of thickness H lying over a heterogeneous monoclinic half-space. The z-axis has been taken along the propagation of waves and y-axis is positive vertically downwards as shown in Fig 1. The source of disturbance S is taken at the point of intersection of the interface of separation and y-axis. At first, we need to find the equation governing the propagation of SH wave in magnetoelastic monoclinic crustal layer. The strain-displacement relations for monoclinic medium are

$$S_1 = \frac{\partial u}{\partial x},\ S_2 = \frac{\partial v}{\partial y},\ S_3 = \frac{\partial w}{\partial z},\ S_4 = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z},\ S_5 = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x},\ S_6 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \tag{1}$$

where u, v, w are displacement components in the direction x, y, z respectively, and $S_i\ (i = 1, 2, ..., 6)$ are the strain components

Also, the stress-strain relation for a rotated y-cut quartz plate, which exhibits monoclinic symmetry with x being the diagonal axis are

$$\begin{aligned}
T_1 &= C_{11}S_1 + C_{12}S_2 + C_{13}S_3 + C_{14}S_4,\\
T_2 &= C_{12}S_1 + C_{22}S_2 + C_{23}S_3 + C_{24}S_4,\\
T_3 &= C_{13}S_1 + C_{23}S_2 + C_{33}S_3 + C_{34}S_4,\\
T_4 &= C_{14}S_1 + C_{24}S_2 + C_{34}S_3 + C_{44}S_4,\\
T_5 &= C_{55}S_5 + C_{56}S_6,\\
T_6 &= C_{56}S_5 + C_{66}S_6
\end{aligned} \tag{2}$$

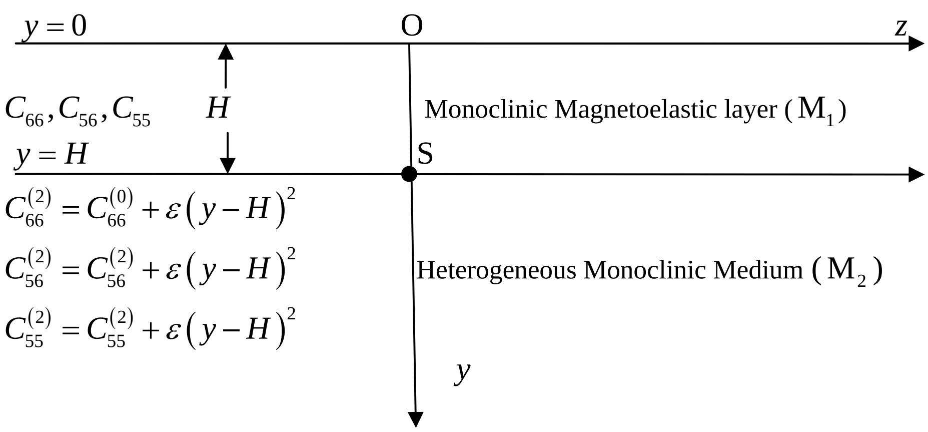

Figure 1: Geometry of the problem

where $T_i\,(i=1,2,...,6)$ are the stress components and $C_{ij} = C_{ji}\,(i=1,2,...,6)$ are the elastic constants. Equations governing the propagation of small elastic disturbances in a perfectly conducting monoclinic medium having electromagnetic force $\mathbf{J}\times\mathbf{B}$ (the Lorentz force, $\mathbf{J}$ being the electric current density and $\mathbf{B}$ being the magnetic induction vector) as the only body forces are

$$\frac{\partial T_1}{\partial x} + \frac{\partial T_6}{\partial y} + \frac{\partial T_5}{\partial z} + (J\times B)_x = \rho\frac{\partial^2 u}{\partial t^2},$$
$$\frac{\partial T_6}{\partial x} + \frac{\partial T_2}{\partial y} + \frac{\partial T_4}{\partial z} + (J\times B)_y = \rho\frac{\partial^2 v}{\partial t^2}, \qquad (3)$$
$$\frac{\partial T_5}{\partial x} + \frac{\partial T_4}{\partial y} + \frac{\partial T_3}{\partial z} + (J\times B)_z = \rho\frac{\partial^2 w}{\partial t^2}$$

where ρ is the density of the layer.

For SH wave propagating in the z- direction and causing displacement in the x- direction only, we shall assume that

$$u = u(y,z,t),\ v = w = 0 \text{ and } \frac{\partial}{\partial x} \equiv 0. \qquad (4)$$

Using Eqs. (1) and (4), the stress-strain relation (2) becomes

$$T_1 = T_2 = T_3 = T_4 = 0,$$
$$T_5 = C_{55}\frac{\partial u}{\partial z} + C_{56}\frac{\partial u}{\partial y}, \qquad (5)$$
$$T_6 = C_{56}\frac{\partial u}{\partial z} + C_{66}\frac{\partial u}{\partial y}.$$

Using Eq. (5) in Eq. (3), the only non-vanishing equation we have

$$C_{66}\frac{\partial^2 u}{\partial y^2} + 2C_{56}\frac{\partial^2 u}{\partial y \partial z} + C_{55}\frac{\partial^2 u}{\partial z^2} + (J\times B)_x = \rho\frac{\partial^2 u}{\partial t^2}. \qquad (6)$$

The well known Maxwell's equations governing the electromagnetic field are

$$\nabla.\boldsymbol{B} = 0, \nabla\times\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \nabla\times\boldsymbol{H} = \boldsymbol{J} \text{ with } \boldsymbol{B} = \mu_e \boldsymbol{H}, \boldsymbol{J} = \sigma\left(\boldsymbol{E} + \frac{\partial \boldsymbol{u}}{\partial t}\times\boldsymbol{B}\right) \qquad (7)$$

where $\boldsymbol{E}$ is the induced electric field, $\boldsymbol{J}$ is the current density vector and magnetic field $\boldsymbol{H}$ includes both primary and induced magnetic fields. μ_e and σ are the induced permeability and conduction coefficient respectively.

The linearized Maxwell's stress tensor $\left(\tau_{ij}^0\right)^{M_x}$ due to the magnetic field is given by $\left(\tau_{ij}^0\right)^{M_x} = \mu_e\left(H_i h_j + H_j h_i - H_k h_k \delta_{ij}\right)$.

Let $\boldsymbol{H} = (H_1, H_2, H_3)$, $\boldsymbol{u} = (u, v, w)$ and $h_i = (h_1, h_2, h_3)$ where h_i is the change in the magnetic field. In writing the above equations, we have neglected the displacement current.
From Eq. (7), we get

$$\nabla^2 \boldsymbol{H} = \mu_e \sigma \left\{ \frac{\partial \boldsymbol{H}}{\partial t} + \nabla \times \left(\frac{\partial \boldsymbol{u}}{\partial t} \times \boldsymbol{H} \right) \right\}. \tag{8}$$

In component form, Eq. (8) can be written as

$$\frac{\partial H_1}{\partial t} = \frac{1}{\mu_e \sigma} \nabla^2 H_1 + \frac{\partial \left(H_2 \frac{\partial u}{\partial t} \right)}{\partial y} + \frac{\partial \left(H_3 \frac{\partial u}{\partial t} \right)}{\partial z},$$
$$\frac{\partial H_2}{\partial t} = \frac{1}{\mu_e \sigma} \nabla^2 H_2, \tag{9}$$
$$\frac{\partial H_3}{\partial t} = \frac{1}{\mu_e \sigma} \nabla^2 H_3.$$

For perfectly conducting medium (i.e. $\sigma \to \infty$), the Eqs. (9) become

$$\frac{\partial H_2}{\partial t} = \frac{\partial H_3}{\partial t} = 0, \tag{10}$$

and

$$\frac{\partial H_1}{\partial t} = \frac{\partial \left(H_2 \frac{\partial u}{\partial t} \right)}{\partial y} + \frac{\partial \left(H_3 \frac{\partial u}{\partial t} \right)}{\partial z}. \tag{11}$$

It is clear from Eq. (10) that there is no perturbation in H_2 and H_3, however from Eq. (11) there may be perturbation in H_1. Therefore, taking small perturbation, say h_1 in H_1, we have
$H_1 = H_{01} + h_1$, $H_2 = H_{02}$ and $H_3 = H_{03}$, where (H_{01}, H_{02}, H_{03}) are components of the initial magnetic field $\boldsymbol{H}_0$.
We can write $\boldsymbol{H}_0 = (0, H_0 \sin\phi, H_0 \cos\phi)$, where $H_0 = |\boldsymbol{H}_0|$ and ϕ is the angle at which the wave crosses the magnetic field. Thus we have

$$\boldsymbol{H} = (h_1, H_0 \sin\phi, H_0 \cos\phi). \tag{12}$$

We shall take initial value of h_1 as $h_1 = 0$. Using Eq. (12) in Eq. (11), we get

$$\frac{\partial h_1}{\partial t} = \frac{\partial \left(H_0 \sin\phi \frac{\partial u}{\partial t} \right)}{\partial y} + \frac{\partial \left(H_0 \cos\phi \frac{\partial u}{\partial t} \right)}{\partial z}. \tag{13}$$

Integrating with respect to t, we get

$$h_1 = H_0 \sin\phi \frac{\partial u}{\partial y} + H_0 \cos\phi \frac{\partial u}{\partial z}. \tag{14}$$

Considering $\nabla \left(\frac{H^2}{2} \right) = -(\nabla \times \boldsymbol{H}) \times \boldsymbol{H} + (\boldsymbol{H}.\nabla) \boldsymbol{H}$ and Eqs (7), we get

$$\boldsymbol{J} \times \boldsymbol{B} = \mu_e \left\{ -\nabla \left(\frac{H^2}{2} \right) + (\boldsymbol{H}.\nabla) \boldsymbol{H} \right\}. \tag{15}$$

In the component form Eq. (15) can be written as

$$(\boldsymbol{J}\times\boldsymbol{B})_y=(\boldsymbol{J}\times\boldsymbol{B})_z=0$$

and (16)

$$(\boldsymbol{J}\times\boldsymbol{B})_x=\mu_e H_0{}^2\left(\sin^2\phi\frac{\partial^2 u}{\partial y^2}+\sin 2\phi\frac{\partial^2 u}{\partial y\partial z}+\cos^2\phi\frac{\partial^2 u}{\partial z^2}\right).$$

Using Eqs. (6) and (16), we find the equation of motion for the magnetoelastic monoclinic medium in the form

$$M_{66}\frac{\partial^2 u}{\partial y^2}+2M_{56}\frac{\partial^2 u}{\partial y\partial z}+M_{55}\frac{\partial^2 u}{\partial z^2}=\rho\frac{\partial^2 u}{\partial t^2}. \tag{17}$$

where

$$M_{66}=C_{66}\left(1+m_H\sin^2\phi\right),$$

$$M_{55}=C_{66}\left(\frac{C_{55}}{C_{66}}+m_H\cos^2\phi\right), \tag{18}$$

$$M_{56}=C_{66}\left(\frac{C_{56}}{C_{66}}+m_H\cos\phi\sin\phi\right)$$

where $m_H=\dfrac{\mu_e H_0{}^2}{C_{66}}$ is monoclinic-magnetoelastic coupling parameter.

If $\sigma_1(r,t)$ be the force density distribution in the upper layer due to the point source, the equation of motion for SH wave propagation along z-axis becomes as

$$\frac{\partial^2 u_1}{\partial y^2}+\frac{M_{55}}{M_{66}}\frac{\partial^2 u_1}{\partial z^2}+\frac{2M_{56}}{M_{66}}\frac{\partial^2 u_1}{\partial y\partial z}-\frac{\rho}{M_{66}}\frac{\partial^2 u_1}{\partial t^2}=\frac{4\pi\sigma_1(r,t)}{M_{66}}, \tag{19}$$

where r is the distance from the origin, where the force is applied to a point of coordinates and t is the time.

Considering $u_1(y,z,t)=U_1(y,z)e^{i\omega t}$ and $\sigma_1(r,t)=\sigma_1(r)e^{i\omega t}$ in eq.(17), we obtain

$$\frac{\partial^2 U_1}{\partial y^2}+\frac{M_{55}}{M_{66}}\frac{\partial^2 U_1}{\partial z^2}+\frac{2M_{56}}{M_{66}}\frac{\partial^2 U_1}{\partial y\partial z}+\frac{\rho\omega^2}{M_{66}}U_1=\frac{4\pi\sigma_1(r)}{M_{66}}, \tag{20}$$

where $\omega=kc$ is the angular frequency, k the wave number and c is the phase velocity. Here the disturbances caused by the impulsive force $\sigma_1(r)$ may be represented in terms of Dirac-delta function at the source point as

$$\sigma_1(r)=\delta(z)\delta(y-H).$$

Therefore the equation of motion for the upper magnetoelastic monoclinic layer with an impulsive point source is

$$\frac{\partial^2 U_1}{\partial y^2}+\frac{M_{55}}{M_{66}}\frac{\partial^2 U_1}{\partial z^2}+\frac{2M_{56}}{M_{66}}\frac{\partial^2 U_1}{\partial y\partial z}+\frac{\rho\omega^2}{M_{66}}U_1=\frac{4\pi\delta(z)\delta(y-H)}{M_{66}}, \tag{21}$$

Defining the Fourier transform $\overline{U}_r(\xi,y)$ of $U_r(z,y)$ as

$$\overline{U}_r(\xi,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}U_r(z,y)e^{i\xi z}dz \tag{22}$$

Then the inverse transform can be given as

$$U_r(z,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\overline{U}_r(\xi,y)e^{-i\xi z}d\xi. \tag{23}$$

Now taking the Fourier transform of eq. (21), we obtain

$$\frac{d^2\overline{U}_1}{dy^2}+f_1\frac{d\overline{U}_1}{dy}+r_1^2\overline{U}_1=\frac{2\delta(y-H)}{C_{66}}=4\pi\sigma_1(y), \tag{24}$$

where

$$f_1 = 2i\xi \frac{M_{56}}{M_{66}},\ r_1^2 = \frac{\rho\omega^2}{M_{66}} - \xi^2 \frac{M_{55}}{M_{66}}$$

The heterogeneity of the lower inhomogeneous monoclinic half-space has been considered in the form

$$\begin{aligned} C_{66}^{(2)} &= C_{66}^{(0)} + \varepsilon(y-H)^2 \\ C_{56}^{(2)} &= C_{56}^{(2)} + \varepsilon(y-H)^2 \\ C_{55}^{(2)} &= C_{55}^{(2)} + \varepsilon(y-H)^2 \end{aligned} \tag{25}$$

Now, the equation of motion for the lower heterogeneous monoclinic half-space is

$$C_{66}{}^{(2)} \frac{\partial^2 u_2}{\partial y^2} + C_{55}{}^{(2)} \frac{\partial^2 u_2}{\partial z^2} + 2C_{56}{}^{(2)} \frac{\partial^2 u_2}{\partial y \partial z} + 2\varepsilon(y-H)\frac{\partial u_2}{\partial y} = \rho_0 \frac{\partial^2 u_2}{\partial t^2}, \tag{26}$$

where ρ_0 is the density of the lower half-space.

In view of substitution $u_2(y,z,t) = U_2(y,z)e^{i\omega t}$ and eq. (22), eq. (26) becomes

$$\frac{d^2\overline{U}_2}{dy^2} + f_2 \frac{d\overline{U}_2}{dy} + r_2{}^2\overline{U}_2 = 4\pi\sigma_2(y), \tag{27}$$

where

$$f_2 = 2i\xi \frac{C_{56}^{(0)}}{C_{66}^{(0)}},\ r_2{}^2 = \frac{\rho_0\omega^2}{C_{66}^{(0)}} - \xi^2 \frac{C_{55}^{(0)}}{C_{66}^{(0)}},$$

$$4\pi\sigma_2(y) = -\frac{\varepsilon}{C_{66}^{(0)}}\left\{(y-H)^2 \frac{d^2\overline{U}_2}{dy^2} + 2(y-H)\frac{d\overline{U}_2}{dy} - (y-H)^2 \xi^2 \overline{U}_2\right\}, \tag{28}$$

Now it is clear from eq. (27) that the displacement in the lower medium may be determined by assuming the lower medium to be homogeneous, isotropic having source density distribution $\sigma_2(y)$.

Substituting $\overline{U}_r(y) = \overline{U}_r{}'(y)e^{-f_r\frac{y}{2}}$ in eq. (24) and eq. (27) for $r = 1,2$ respectively, we obtain

$$\frac{d^2\overline{U}_1{}'}{dy^2} - \alpha^2\overline{U}_1{}' = 4\pi\sigma_1(y)e^{\frac{f_1 y}{2}}, \tag{29}$$

and
$$\frac{d^2\overline{U}_2{}'}{dy^2} - \beta^2\overline{U}_2{}' = 4\pi\sigma_2(y)e^{\frac{f_2 y}{2}}, \tag{30}$$

where

$$\alpha^2 = \frac{f_1{}^2}{4} - r_1{}^2,\ \beta^2 = \frac{f_2{}^2}{4} - r_2{}^2$$

The boundary conditions are as follows:

(i) Upper surface is stress free i.e. $M_{66}\left\{\frac{d\overline{U}_1}{dy} + \frac{f_1}{2}\overline{U}_1\right\} = 0$ at $y = 0$ (31)

(ii) Displacements are continuous at the common interface i.e. $\overline{U}_1 = \overline{U}_2$ at $y = H$ (32)

(iii) Stresses are continuous at the common interface i.e.

$$M_{66}\left\{\frac{d\overline{U}_1}{dy} + \frac{f_1}{2}\overline{U}_1\right\} = C_{66}{}^{(2)}\left\{\frac{d\overline{U}_2}{dy} + \frac{f_2}{2}\overline{U}_2\right\} \quad \text{at } y = H \tag{33}$$

Thus equations (29) and (30) together with prescribed boundary conditions (31) to (33) give the complete mathematical model for the problem. Now we apply Green's function technique to solve it. If $G_1(y/y_0)$ is the Green's function for the upper layer satisfying the condition $\frac{dG_1}{dy}=0$ at $y=0$ and at $y=H$, then the equation satisfied by $G_1(y/y_0)$ is

$$\frac{d^2G_1(y/y_0)_1}{dy^2}-\alpha^2G_1(y/y_0)=\delta(y-y_0) \tag{34}$$

where y_0 is a point in the upper medium and y is the field point. Multiplying the eq. (29) by $G_1(y/y_0)$ and eq. (34) by $\overline{U}_1{}'(y)$, then subtracting and integrating with respect to y from $y=0$ to $y=H$, we have

$$G_1(H/y_0)\left[\frac{d\overline{U}_1{}'}{dy}\right]_{y=H}=\frac{2}{M_{66}}e^{\frac{f_1H}{2}}G_1(H/y_0)-\overline{U}_1{}'(y_0) \tag{35}$$

Since $\frac{dG_1(y/y_0)}{dy}=0$ at $y=0$ and $y=H$.

Replacing y_0 by y and remembering that $G_1(H/y)=G_1(y/H)$, the eq. (35) gives the value of $\overline{U}_1{}'$ at any point y in the upper medium as

$$\overline{U}_1{}'(y)=\frac{2}{M_{66}}e^{\frac{f_1H}{2}}G_1(y/H)-G_1(y/H)\left[\frac{d\overline{U}_1{}'}{dy}\right]_{y=H}$$

Therefore,

$$\overline{U}_1(y)=e^{-\frac{f_1}{2}(y-H)}\left[\frac{2}{M_{66}}G_1(y/H)-G_1(y/H)\left\{\frac{d\overline{U}_1(y)}{dy}+\frac{f_1}{2}\overline{U}_1(y)\right\}_{y=H}\right] \tag{36}$$

Now, let $G_2(y/y_0)$ be the Green's function for the lower medium, as per previous discussion may be assumed to be homogeneous. We assume that $G_2(y/y_0)$ is the solution of the equation

$$\frac{d^2G_2(y/y_0)_1}{dy^2}-\beta^2G_2(y/y_0)=\delta(y-y_0) \tag{37}$$

where y_0 is the point in the lower medium, satisfying the condition $\frac{dG_2}{dy}=0$ at $y=H$ and approaches to zero as $y\to\infty$.

Multiplying eq. (30) by $G_2(y/y_0)$ and eq. (37) by $\overline{U}_2{}'(y)$, then subtracting and integrating with respect to y from $y=H$ to $y=\infty$, we have

$$-G_2(H/y_0)\left[\frac{d\overline{U}_2{}'}{dy}\right]_{y=H}=4\pi\int_H^\infty e^{\frac{f_2y}{2}}\sigma_2(y)G_2(y/y_0)\,dy-\overline{U}_2{}'(y_0) \tag{38}$$

Interchanging y by y_0 in the eq. (38), the value of $\overline{U}_2{}'(y)$ at any point y in the lower medium is

$$\overline{U}_2{}'(y)=G_2(y/H)\left[\frac{d\overline{U}_2{}'}{dy}\right]_{y=H}+4\pi\int_H^\infty e^{\frac{f_2y_0}{2}}\sigma_2(y_0)G_2(y/y_0)\,dy_0$$

Therefore,

$$\overline{U}_2(y)=e^{-\frac{f_2 y}{2}}\left[e^{\frac{f_2 H}{2}}G_2\left(\frac{y}{H}\right)\left\{\frac{d\overline{U}_2(y)}{dy}+\frac{f_2}{2}\overline{U}_2(y)\right\}_{y=H}+4\pi\int_H^{\infty}e^{\frac{f_2 y_0}{2}}\sigma_2(y_0)G_2\left(\frac{y}{y_0}\right)dy_0\right] \tag{39}$$

With the help of boundary condition (32), we have

$$\begin{aligned}\frac{2}{M_{66}}G_1(H/H)-G_1(H/H)\left\{\frac{d\overline{U}_1(y)}{dy}+\frac{f_1}{2}\overline{U}_1(y)\right\}_{y=H}&=G_2(H/H)\left\{\frac{d\overline{U}_2(y)}{dy}+\frac{f_2}{2}\overline{U}_2(y)\right\}_{y=H}\\&+4\pi e^{-\frac{f_2 H}{2}}\int_H^{\infty}e^{\frac{f_2 y_0}{2}}\sigma_2(y_0)G_2(y/y_0)dy_0\end{aligned} \tag{40}$$

Using boundary condition (33), eq. (40) can be written as

$$\left\{\frac{d\overline{U}_1(y)}{dy}+\frac{f_1}{2}\overline{U}_1(y)\right\}_{y=H}=\frac{1}{D_1}\left\{\frac{2}{M_{66}}G_2(H/H)-4\pi e^{-\frac{f_2 H}{2}}\int_H^{\infty}e^{\frac{f_2 y_0}{2}}\sigma_2(y_0)G_2(H/y_0)dy_0\right\} \tag{41}$$

where D_1 is given in appendix I.

Substituting the value of $\left\{\frac{d\overline{U}_1(y)}{dy}+\frac{f_1}{2}\overline{U}_1(y)\right\}_{y=H}$ from eq. (41) and $4\pi\sigma_2(y_0)$ from eq. (28) into eq. (36), we obtain

$$\begin{aligned}\overline{U}_1(y)=e^{-\frac{f_1}{2}(y-H)}&\left[\frac{2G_1(y/H)G_2(H/H)}{C_{66}^{(0)}G_1(H/H)+M_{66}G_2(H/H)}-\frac{\varepsilon e^{-\frac{f_2 H}{2}}G_1(y/H)}{C_{66}^{(0)}G_1(H/H)+M_{66}G_2(H/H)}\times\right.\\&\left.\int_H^{\infty}\left\{(y_0-H)^2\frac{d^2\overline{U}_2}{dy_0^2}+2(y_0-H)\frac{d\overline{U}_2}{dy_0}-\xi^2(y_0-H)^2\overline{U}_2(y_0)\right\}e^{\frac{f_2 y_0}{2}}G_2(H/y_0)dy_0\right]\end{aligned} \tag{42}$$

In view of boundary condition (33), relation (39) gives

$$\begin{aligned}\overline{U}_2(y)=e^{-\frac{f_2}{2}(y-H)}&\frac{2G_1(H/H)G_2(y/H)}{C_{66}^{(0)}G_1(H/H)+M_{66}G_2(H/H)}+\frac{\varepsilon e^{-\frac{f_2 y}{2}}G_2(y/H)M_{66}}{C_{66}^{(0)}\left\{C_{66}^{(0)}G_1(H/H)+M_{66}G_2(H/H)\right\}}\times\\&\times\int_H^{\infty}\left\{(y_0-H)^2\frac{d^2\overline{U}_2}{dy_0^2}+2(y_0-H)\frac{d\overline{U}_2}{dy_0}-\xi^2(y_0-H)^2\overline{U}_2(y_0)\right\}e^{\frac{f_2 y_0}{2}}G_2(H/y_0)dy_0-\\&-\frac{\varepsilon e^{-\frac{f_2 y}{2}}}{C_{66}^{(0)}}\int_H^{\infty}\left\{(y_0-H)^2\frac{d^2\overline{U}_2}{dy_0^2}+2(y_0-H)\frac{d\overline{U}_2}{dy_0}-\xi^2(y_0-H)^2\overline{U}_2(y_0)\right\}e^{\frac{f_2 y_0}{2}}G_2(y/y_0)dy_0\end{aligned} \tag{43}$$

$\overline{U}_2(y)$ can be obtained from the relation (43) by the method of successive approximations. The value of $\overline{U}_2(y)$ obtained from eq. (43) when substituted in eq. (42) gives the value of $\overline{U}_1(y)$. We are interested in the value of $\overline{U}_1(y)$, which will give the displacement in the upper layer, and since the higher order of ε can be neglected; we take as the first order approximation

$$\overline{U}_2(y)=\frac{2G_1(H/H)G_2(y/H)e^{-\frac{f_2}{2}(y-H)}}{C_{66}^{(0)}G_1(H/H)+M_{66}G_2(H/H)} \tag{44}$$

which gives the displacement at any point in the lower medium if it is taken as homogeneous. Putting this value of $\overline{U}_2(y)$ in eq. (42), we get

$$\overline{U}_1(y)=\frac{2G_1(y/H)G_2(H/H)e^{-\frac{f_1}{2}(y-H)}}{C_{66}^{(0)}G_1(H/H)+M_{66}G_2(H/H)}-\frac{2\varepsilon e^{-\frac{f_1}{2}(y-H)}G_1(y/H)G_1(H/H)}{\left\{C_{66}^{(0)}G_1(H/H)+M_{66}G_2(H/H)\right\}^2}\times$$
$$\int_H^\infty\left\{(y_0-H)^2\frac{d^2G_2(y_0/H)}{dy_0^{\,2}}+2(y_0-H)\frac{dG_2(y_0/H)}{dy_0}-\xi^2(y_0-H)^2G_2(y_0/H)\right\}G_2(H/y_0)\,dy_0 \tag{45}$$

The solution of eq. (45) represents the elastic displacements due to a unit impulse force in space and time. Thus the Green's function is the response of the medium to an impulsive excitation. If we know the values of $G_1(y/H)$ and $G_2(y/H)$, then the value of $\overline{U}_1(y)$ can be determined from the eq. (45). We have assumed $G_1(y/y_0)$ as the solution of eq. (34). A solution of eq. (34) may also be obtained in the following manner.

We have the equation

$$\frac{d^2\Psi}{dy^2}-\alpha^2\Psi=0. \tag{46}$$

Two independent solutions of eq. (46), vanishing at $y=-\infty$ and $y=\infty$ are

$\Psi_1(y)=e^{\alpha y}$ and $\Psi_2(y)=e^{-\alpha y}$.

Therefore the solution of the eq. (46) for an infinite medium is

$\dfrac{\Psi_1(y)\Psi_2(y_0)}{W}$ for $y<y_0$,

$\dfrac{\Psi_1(y_0)\Psi_2(y)}{W}$ for $y>y_0$,

where

$$W=\Psi_1(y)\Psi_2'(y)-\Psi_1'(y)\Psi_2(y)=-2\alpha\neq 0.$$

So, the solution of eq. (34) is

$$-\frac{e^{-\alpha|y-y_0|}}{2\alpha}.$$

Since $G_1(y/y_0)$ is to satisfy the condition

$$\frac{dG_1}{dy}=0 \text{ at } y=0 \text{ and } y=H, \tag{47}$$

Therefore, we can assume that

$$G_1(y/y_0)=C_1e^{\alpha y}+C_2e^{-\alpha y}-\frac{e^{-\alpha|y-y_0|}}{2\alpha}.$$

where C_1 and C_2 are the arbitrary constants which can be evaluated using condition (47). We finally get

$$G_1(y/y_0)=\frac{1}{2\alpha}\left[e^{-\alpha|y-y_0|}+\frac{e^{\alpha y}\left(e^{-\alpha(H+y_0)}+e^{-\alpha(H-y_0)}\right)}{e^{\alpha H}-e^{-\alpha H}}+\frac{e^{-\alpha z}\left(e^{\alpha(H-y_0)}+e^{-\alpha(H-y_0)}\right)}{e^{\alpha H}-e^{-\alpha H}}\right]. \tag{48}$$

Therefore,

$$G_1(y/H)=-\frac{1}{\alpha}\left[\frac{e^{\alpha y}+e^{-\alpha y}}{e^{\alpha H}-e^{-\alpha H}}\right], \tag{49}$$

$$G_1(H/H)=-\frac{1}{\alpha}\left[\frac{e^{\alpha H}+e^{-\alpha H}}{e^{\alpha H}-e^{-\alpha H}}\right], \tag{50}$$

Similarly, the value of $G_2(y/y_0)$ can be written as

$$G_2\left(y/y_0\right)=-\frac{1}{2\beta}\left[e^{-\beta|y-y_0|}+e^{-\beta(y+y_0-2H)}\right], \tag{51}$$

and so

$$G_2\left(H/y_0\right)=-\frac{e^{-\beta(y_0-H)}}{\beta}, \tag{52}$$

$$G_2\left(H/H\right)=-\frac{1}{\beta}. \tag{53}$$

Substituting all these values in eq. (45), we get

$$\overline{U}_1(y)=\frac{-2e^{-\frac{f_1}{2}(y-H)}\left(e^{\alpha y}+e^{-\alpha y}\right)}{C_{66}^{(0)}\beta\left(e^{\alpha H}+e^{-\alpha H}\right)+M_{66}\alpha\left(e^{\alpha H}-e^{-\alpha H}\right)}\left[1-\frac{\varepsilon\left(e^{\alpha H}+e^{-\alpha H}\right)\left(1+\frac{\xi^2}{\beta^2}\right)}{4\beta\left\{C_{66}^{(0)}\beta\left(e^{\alpha H}+e^{-\alpha H}\right)+M_{66}\alpha\left(e^{\alpha H}-e^{-\alpha H}\right)\right\}}\right] \tag{54}$$

Neglecting the higher powers of ε the eq. (54) may be approximated as

$$\overline{U}_1(y)=\frac{-2e^{-\frac{f_1}{2}(y-H)}\left(e^{\alpha y}+e^{-\alpha y}\right)}{C_{66}^{(0)}\beta\left(e^{\alpha H}+e^{-\alpha H}\right)+M_{66}\alpha\left(e^{\alpha H}-e^{-\alpha H}\right)\left[1+\frac{\varepsilon\left(e^{\alpha H}+e^{-\alpha H}\right)\left(1+\frac{\xi^2}{\beta^2}\right)}{4\beta\left\{C_{66}^{(0)}\beta\left(e^{\alpha H}+e^{-\alpha H}\right)+M_{66}\alpha\left(e^{\alpha H}-e^{-\alpha H}\right)\right\}}\right]} \tag{55}$$

Taking the inverse Fourier transform of eq. (55), the displacement in the upper medium may be obtained as

$$U_1=-2\int_{-\infty}^{\infty}\frac{e^{-\frac{f_1}{2}(y-H)}\left(e^{\alpha y}+e^{-\alpha y}\right)e^{-i\xi z}d\xi}{C_{66}^{(0)}\beta\left(e^{\alpha H}+e^{-\alpha H}\right)+M_{66}\alpha\left(e^{\alpha H}-e^{-\alpha H}\right)\left[1+\frac{\varepsilon\left(e^{\alpha H}+e^{-\alpha H}\right)\left(1+\frac{\xi^2}{\beta^2}\right)}{4\beta\left\{C_{66}^{(0)}\beta\left(e^{\alpha H}+e^{-\alpha H}\right)+M_{66}\alpha\left(e^{\alpha H}-e^{-\alpha H}\right)\right\}}\right]} \tag{56}$$

The dispersion equation of SH waves will be obtained by equating to zero the denominator of the above integral,

$$C_{66}^{(0)}\beta\left(e^{\alpha H}+e^{-\alpha H}\right)+M_{66}\alpha\left(e^{\alpha H}-e^{-\alpha H}\right)\left[1+\frac{\varepsilon\left(e^{\alpha H}+e^{-\alpha H}\right)\left(1+\frac{\xi^2}{\beta^2}\right)}{4\beta\left\{C_{66}^{(0)}\beta\left(e^{\alpha H}+e^{-\alpha H}\right)+M_{66}\alpha\left(e^{\alpha H}-e^{-\alpha H}\right)\right\}}\right]=0. \tag{57}$$

In view of the substitutions $\alpha=ik\theta_1$ and $\beta=k\theta_2$ the above eq. (57) gives the dispersion relation of shear waves in monoclinic magnetoelastic layer lying over heterogeneous monoclinic half-space

$$\tan\left(kH\theta_1\right)=\frac{C_{66}^{(0)}\theta_2}{M_{66}\theta_1}+\frac{\varepsilon}{4k^2\theta_1\theta_2M_{66}}\left[1+\frac{1}{\left\{\frac{C_{55}^{(0)}}{C_{66}^{(0)}}-\left(\frac{C_{56}^{(0)}}{C_{66}^{(0)}}\right)^2\right\}}+\frac{\left(\frac{c}{\beta_2}\right)^2}{\left\{\frac{C_{55}^{(0)}}{C_{66}^{(0)}}-\left(\frac{C_{56}^{(0)}}{C_{66}^{(0)}}\right)^2\right\}\theta_2^{\,2}}\right]. \tag{58}$$

where θ_1, θ_2 β_1 and β_2 are given in the appendix I.

3. Particular cases

Case I: When $\varepsilon = 0$ the dispersion relation (58) reduces to

$$\tan\left(kH\theta_1\right) = \frac{C_{66}^{(0)}\theta_2}{M_{66}\theta_1}$$

which is the dispersion equation of shear waves for the case of monoclinic magnetoelastic layer lying over a homogeneous monoclinic half-space due to a point source.

Case II: When $\varepsilon = 0,\ C_{56} = 0,\ C_{66} = C_{55} = \mu_1,\ C_{66}^{(0)} = C_{55}^{(0)} = \mu_2$ and $C_{56}^{(0)} = 0$ the dispersion relation (58) reduces to

$$\tan kH\left\{\frac{c^2}{\beta_3^{\,2}\left\{1+\tau_H\sin^2\phi\right\}} - \frac{\left\{1+\tau_H\cos^2\phi\right\}}{\left\{1+\tau_H\sin^2\phi\right\}}\right\}^{1/2} = \frac{\mu_2\left\{1-\dfrac{c^2}{\beta_4^{\,2}}\right\}^{1/2}}{\mu_1\left(1+\tau_H\sin^2\phi\right)\left\{\dfrac{c^2}{\beta_3^{\,2}\left\{1+\tau_H\sin^2\phi\right\}} - \dfrac{\left\{1+\tau_H\cos^2\phi\right\}}{\left\{1+\tau_H\sin^2\phi\right\}}\right\}^{1/2}}$$

where τ_H, β_3 and β_4 are given in the appendix I.

which is the dispersion equation of shear waves for the case of isotropic magnetoelastic layer lying over a homogeneous isotropic half-space due to a point source.

Case III: When $\varepsilon = 0,\ m_H = 0,\ C_{56} = 0,\ C_{66} = C_{55} = \mu_1,\ C_{66}^{(0)} = C_{55}^{(0)} = \mu_2$ and $C_{56}^{(0)} = 0$ the dispersion relation (58) reduces to

$$\tan kH\left\{\frac{c^2}{\beta_3^{\,2}} - 1\right\}^{1/2} = \frac{\mu_2\left\{1-\dfrac{c^2}{\beta_4^{\,2}}\right\}^{1/2}}{\mu_1\left\{\dfrac{c^2}{\beta_3^{\,2}} - 1\right\}^{1/2}}$$

which is the classical SH wave equation.

4. Numerical examples

For the case of a magnetoelastic monoclinic layer lying over a non-homogeneous monoclinic half space, we take the following data:

(i) For monoclinic magnetoelastic layer (Tiersten, 1969)

$C_{55} = 94\times10^9\ \text{N/m}^2$, $C_{56} = -11\times10^9\ \text{N/m}^2$,

$C_{66} = 93\times10^9\ \text{N/m}^2$, $\rho = 7,450\ \text{Kg/m}^3$.

(ii) For lower heterogeneous monoclinic half space (Tiersten, 1969)

$C_{55}^{(0)} = 57.94\times10^9\ \text{N/m}^2$, $C_{56}^{(0)} = -17.91\times10^9\ \text{N/m}^2$,

$C_{66}^{(0)} = 39.88\times10^9\ \text{N/m}^2$, $\rho_0 = 2,649\ \text{Kg/m}^3$.

Moreover the following data are used

$m_H,\ \varepsilon_0 = 0.0, 0.05, 0.10$

For the case of magnetoelastic isotropic layer lying over a non-homogeneous isotropic half- space, we select the following data:

(iii) For isotropic magnetoelastic layer (Gubbins, 1990)

$\mu_1 = 63.4\times10^9\ \text{N/m}^2$, $\rho_1 = 3364\ \text{Kg/m}^3$.

(iv) For non-homogeneous isotropic half-space (Gubbins, 1990)

$\mu_0 = 78.4\times10^9\ \text{N/m}^2$, $\rho_0 = 3535\ \text{Kg/m}^3$.

Moreover the following data are used

τ_H, $\varepsilon_1 = 0.0, 0.05, 0.10$.

The effect of magnetoelastic coupling parameter and heterogeneity on the propagation of plane SH waves in a magnetoelastic monoclinic layer lying over an heterogeneous monoclinic half spaces has been depicted by means of graphs. Fig. 2 and 4 give the variation of non-dimensional phase velocity (c/β_1) with respect to non-dimensional wave number kH for different values of magnetoelastic coupling parameters m_H and τ_H for monoclinic magnetoelastic and isotropic magnetoelastic case respectively. Fig. 3 and 5 gives the variation of non-dimensional phase velocity (c/β_1) with respect to non-dimensional wave number kH for different values of heterogeneity parameters ε_0 and ε_1 for the case of heterogeneous monoclinic half space and heterogeneous isotropic half space respectively. The small change in the non-dimensional wave number produces substantial change in non-dimensional phase velocity in all the cases. In each of these figures dotted line curves viz. 4, 5 & 6 refers to the isotopic case and solid line curves viz. 1, 2 & 3 refers to the monoclinic case. The comparative study of the graphs reveals that with the increase in heterogeneity parameters and magnetoelastic coupling parameters, the phase velocity increases for both isotropic and monoclinic cases.

The comparative study of both the cases viz. magnetoelastic monoclinic layer lying over a heterogeneous monoclinic half space and magnetoelastic isotropic layer lying over a heterogeneous isotropic half space shows that as magnetoelasticity prevails through magnetoelastic coupling parameter, the phase velocity of the SH waves due to a point source increases but the increase is found more significant in the case of monoclinic medium as compared to isotropic medium. Also, it is remarkable to notice that as heterogeneity prevails through inhomogeneity parameter the phase velocity gets increased for both the cases but the increase is more prominent in the monoclinic case.

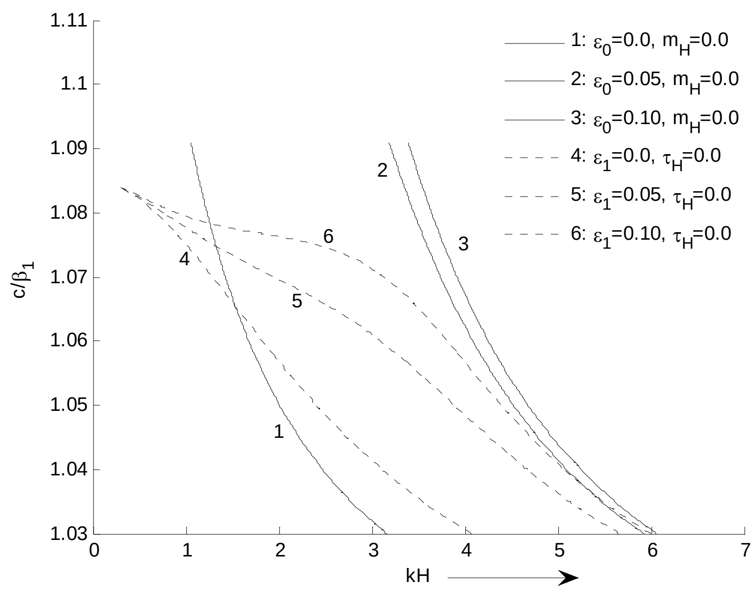

Figure 2: Dimensionless phase velocity against dimensionless wave number for $m_H = 0.0$ and $\tau_H = 0.0$.

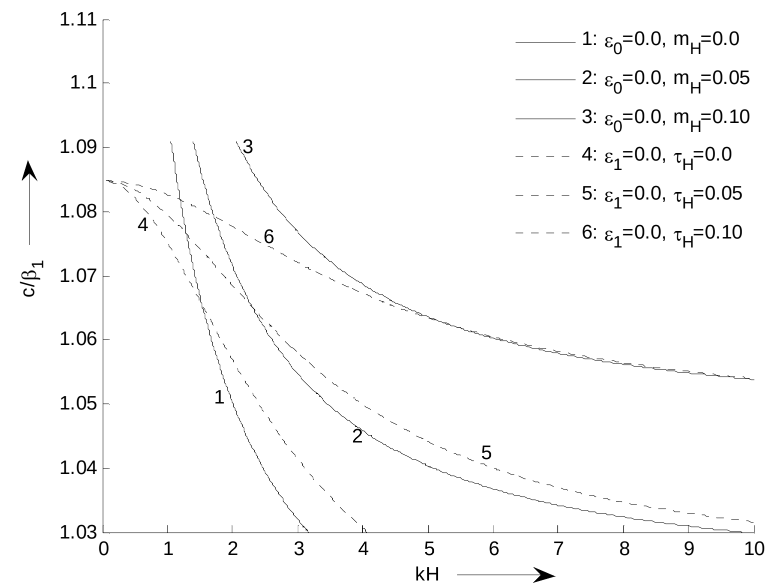

Figure 3: Dimensionless phase velocity against dimensionless wave number for $\varepsilon_0, \varepsilon_1 = 0.0$.

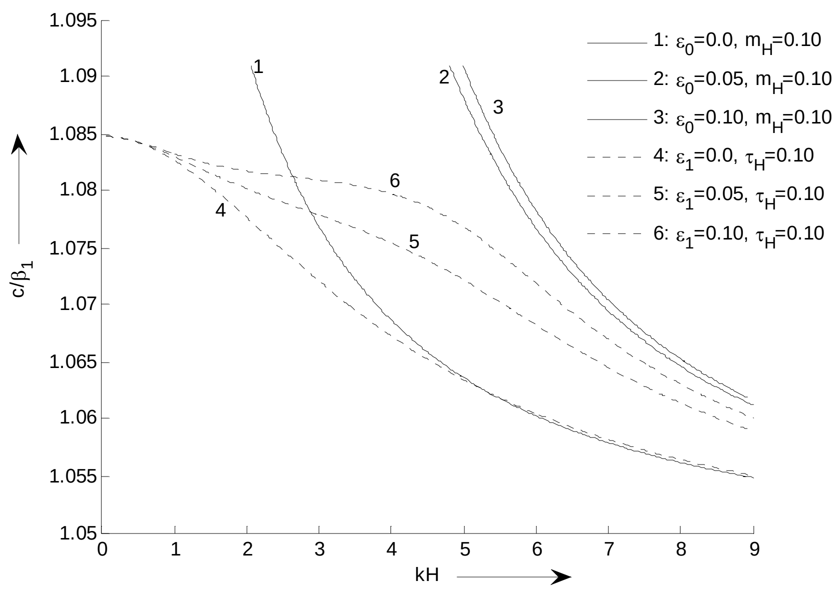

Figure 4: Dimensionless phase velocity against dimensionless wave number for $m_H = 0.10$ and $\tau_H = 0.10$.

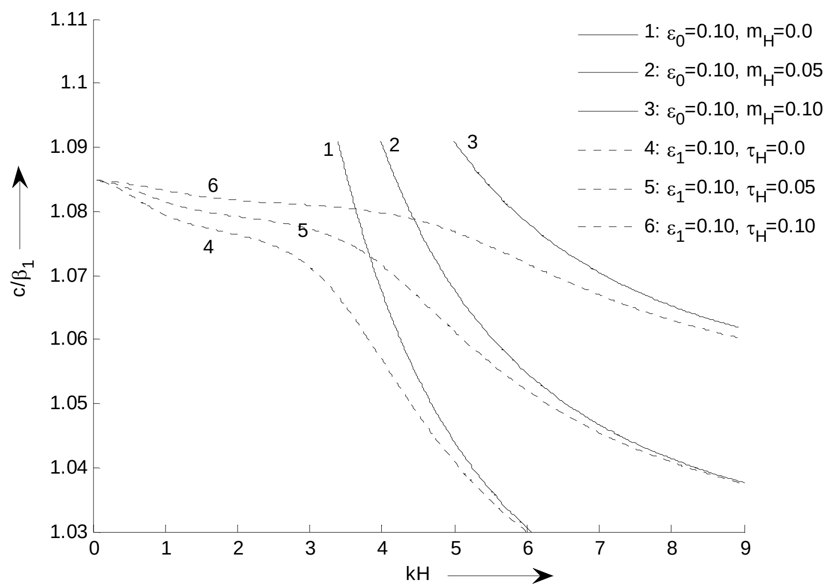

Figure 5: Dimensionless phase velocity against dimensionless wave number for $\varepsilon_0, \varepsilon_1 = 0.10$.

5. Conclusion

In this paper the dispersion relation has been obtained in the closed form. The effect of heterogeneity and magnetoelastic monoclinic parameter on the phase velocity of SH waves has been depicted and shown by means of graphs. The above study shows that as heterogeneity parameters and magnetoelastic coupling parameters increases, the phase velocity increases for both isotropic and monoclinic cases but the increase is more prominent in the monoclinic case. In the isotropic case, when heterogeneity and magnetic field are absent, the dispersion equation is matched with the classical SH wave equation. The present study has its special application to the problem of waves and vibrations where the wave signals have to travel through different layers of different material properties and containing irregularities due to continental margin, mountain roots etc. These results can also be utilized in the interpretation and analysis of data of geophysical studies. The findings will be useful in forecasting formation details at greater depth through signal processing and seismic data analysis. The present study may be effectively utilized to generate initial data prior to exploitation of the formation. This study may be useful to geophysicist and metallurgist for analysis of rock and material structures through Non-Destructive Testing (NDT).

Appendix I

$$D_1 = G_1\left(H/H\right) + \left(\frac{M_{66}}{C_{66}^{(2)}}\right)_{y=H} G_2\left(H/H\right)$$

$$\theta_1 = \left\{\left(\frac{M_{56}}{M_{66}}\right)^2 + \frac{c^2\rho}{M_{66}} - \frac{M_{55}}{M_{66}}\right\}^{1/2}, \theta_2 = \left\{\frac{C_{55}^{(0)}}{C_{66}^{(0)}} - \frac{c^2\rho_0}{C_{66}^{(0)}} - \left(\frac{C_{56}^{(0)}}{C_{66}^{(0)}}\right)^2\right\}^{1/2}$$

$$\beta_1 = \sqrt{\frac{C_{66}}{\rho}},\ \beta_2 = \sqrt{\frac{C_{66}^{(0)}}{\rho_0}},\ \beta_3 = \sqrt{\frac{\mu_1}{\rho}},\ \beta_4 = \sqrt{\frac{\mu_2}{\rho_0}}\ ,\ \tau_{\mathrm{H}} = \frac{\mu_e {H_0}^2}{\mu_1}, \varepsilon_0 = \frac{\varepsilon H^2}{C_{66}} \text{ and } \varepsilon_1 = \frac{\varepsilon H^2}{\mu_1}.$$

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