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Analysis of deformation due to inclined load in generalized thermodiffusive elastic medium

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Abstract

The present investigation deals with study of deformation in homogeneous, isotropic thermodiffusion elastic half-space as a result of inclined load. The inclined load is assumed to be a linear combination of normal load and tangential load. The integral transform technique is used to solve the problem. As an application of the approach distributed and moving forces have been taken. The transformed components of displacement, stresses, temperature distribution and concentration are inverted using numerical inversion technique. The effect of relaxation times and response of two theories of thermoelasticity i.e. Green and Lindsay (G-L) theory and coupled theory (CT) on these quantities have been depicted graphically for a particular model. Some particular cases are also deduced.

Keywords: Inclined load, Distributed sources and Moving force, Stresses, Temperature distribution and Concentration

1. Introduction

Biot (1956) developed the coupled theory of thermoelasticity to deal with a defect of the uncoupled that mechanical causes have no effect on the temperature. However, this theory shares a defect of the uncoupled theory in that it predicts infinite speeds of propagation for heat waves.The subject of generalized thermoelasticity has drawn the attention of many researchers during last few decades as these theories have been attempted mainly to overcome the shortcomings of the classical coupled dynamical theory of thermoelasticity, which predicts two phenomenon not compatible with physical observations. First, the equation of heat conduction of this theory does not contain any elastic terms. Second, the heat equation is of parabolic type, predicting infinite speeds of propagation for heat waves. Therefore, the generalized theories are characterized with finite speed of thermal disturbance. The first two generalized thermoelastic models are Lord-Shulman model (L-S) (1967) and Green-Lindsay (G-L) (1972). In L-S model, one thermal relaxation time parameter is introduced in the Fourier's law of heat conduction,whereas in the G-L model, two thermal relaxation times are introduced in the constitutive relations for stress tensor and entropy equation.

Diffusion can be defined as the random walk, of an ensemble of particles, from regions of great concentration to regions of lower concentration. There is now a great deal of interest in the study of this phenomenon, due to its many applications in geophysics and industrial applications. In integrated circuit fabrication, diffusion is used to introduce "dopants" in controlled amounts into the semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors, form integrated resistors, form the source/drain regions in MOS transistors and dope poly-silicon gates in MOS transistors. Nowacki (1974(a), 1974(b), 1974(c),1976) developed the theory of coupled themoelastic diffusion. This implies infinite speeds of propagation of theromelastic waves. Olesiak and Pyryev (1995) discussed a coupled quasi-stationary problem of thermodiffusion for an elastic cylinder. They studied the influence of cross effects, the thermal excitation results in an additional to mass concentration and the mass concentration generates the additional field of temperature.

Sherief *et. al* (2004) developed the theory of generalized thermoelastic diffusion with one relaxation time, which allows the finite speed of propagation of waves. Sherief and Shaleh (2005) investigated a half space problem in the theory of generalized thermoelastic diffusion with one relaxation time. Singh (2005) investigated the reflection of P and SV waves at the free surface of generalized thermoelastic diffusion. Aouadi (2006, 2007(a), 2007(b), 2008) investigated the different types of problems in thermoelastic diffusion. Sharma *et.al* (2008(a), 2008(b)) and Kumar and Kansal (2009) study various types of problem in thermoelastic diffusion. Kumar *et.al* (2005(a),2005(b),2007) investigated different problems in micropolar elastic medium due to inclined load. Recently Sharma *et.al* (2009) and Sherief and El-Maghraby (2009) discussed different source problems in generalized thermoelastic diffusion. The deformation at any point of the medium is useful to analyze the deformation field around mining tremors and drilling into the crust of the earth. It can also contribute to the theoretical consideration of the seismic and volcanic sources since it can account for the deformation field in the entire volume surrounding the source region.

The purpose of the present paper is to determine the components of stress, temperature distribution and concentration in generalized thermodiffusive elastic medium due to inclined load as result of distributed sources and moving force by applying the integral transform technique. The results of the present problem may be applied to a wide class of geographical problems involving temperature, shape and concentration. Physical applications are found in the mechanical engineering, geophysical and indusrial activities. The present model is very useful for understanding the nature of interaction between mechanical, diffusive and thermal fields since most of the structural elements of engineering industries are often subject to mechanical, diffusive and thermal stresses at an elevated temperature and concentration. This study is also of geophysical interest, particularly in the investigation concerned with earthquake and other phenomenon in seismology and engineering.

2. Basic Equations

Following Green and Lindsay (1972) and Sherief *et al.*(2004), the governing equations in a homogeneous, isotropic generalized thermodiffusive elastic solid in the absence of body forces and heat sources are:

The constitutive relations:

$$t_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e_{kk} - \beta_1(T + \tau_1\dot{T}) - \beta_2(C + \tau^1\dot{C})], \tag{1}$$

$$P = -\beta_2 e_{kk} + b(C + \tau^1\dot{C}) - a(T + \tau_1\dot{T}), \tag{2}$$

The equations of motion:

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} - \beta_1(T + \tau_1\dot{T})_{,i} - \beta_2(C + \tau^1\dot{C})_{,i} = \rho\ddot{u}_i, \tag{3}$$

The equation of heat conduction:

$$\rho C_E(\dot{T} + \tau_0\ddot{T}) + \beta_1 T_0\dot{e} + aT_0(\dot{C} + \tau^0\ddot{C}) = KT_{,ii}, \tag{4}$$

Equation of mass diffusion:

$$D\beta_2 e_{,ii} + Da(T + \tau_1\dot{T})_{,ii} + \dot{C} - Db(C + \tau^1\dot{C})_{,ii} = 0, \tag{5}$$

where

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})(i, j = 1,2,3), \quad \beta_1 = (3\lambda + 2\mu)\alpha_t, \quad \beta_2 = (3\lambda + 2\mu)\alpha_c,$$

λ, μ -Lame's constants, α_t, α_c - coefficient of linear thermal and linear diffusion expansion. $T = T - T_0$, T- absolute temperature and T_0 - temperature of the medium in the natural state assumed to be such that $\left|T/T_0\right| < 1$. t_{ij} - component of the stress tensor, u_i - component of the displacement vector, ρ - density, e_{ij} - components of the strain tensor, $e = e_{kk}$, P- chemical potential, C- Concentration, C_E -Specific heat, K- Coefficient of thermal conductivity , τ_0, τ_1 and τ^0, τ^1 -Thermal and Diffusion relaxation times, D, a, b -constants, δ_{ij} - Kronecker delta.

$\tau_1 \ge \tau_0 \ge 0, \quad \tau^1 \ge \tau^0 \ge 0$ For G-L theory, $\qquad \tau_1 = \tau_0 = \tau^1 = \tau^0 = 0$ For CT theory

3. Formulation and Solution of the problem

We consider a homogeneous, isotropic generalized thermodiffusive elastic medium in the undeformed state at temperature T_0. The rectangular cartesian co-ordinate system (x_1, x_2, x_3) having origin on the surface x_3=0 with x_3 - axis pointing normally into the medium is introduced. Suppose that an inclined line load, per unit length, is acting on the x_2 -axis and its inclination with the

x_3-axis is δ (Fig.1) as shown in appendix I.

For two dimensional problem, we take

$$\vec{u} = (u_1, 0, u_3), \tag{6}$$

The initial and regularity conditions are given by

$$u_1(x,x_3,0) = 0 = \dot{u}_1(x,x_3,0), \quad u_3(x,x_3,0) = 0 = \dot{u}_3(x,x_3,0), \quad T(x,x_3,0) = 0 = \dot{T}(x,x_3,0),$$
$$C(x,x_3,0) = 0 = \dot{C}(x,x_3,0), \ P(x,x_3,0) = 0 = \dot{P}(x,x_3,0), \quad \text{for } x_3 \geq 0, \ -\infty < x < \infty,$$
$$u_1(x,x_3,t) = u_3(x,x_3,t) = T(x,x_3,t) = C(x,x_3,t) = P(x,x_3,t) = 0, \quad \text{for } t > 0 \text{ when } x_3 \to \infty.$$

To facilitate the solution, following dimensionless quantities are introduced:

$$x_1' = \frac{\omega_1^*}{c_1}x, \quad x_3' = \frac{\omega_1^*}{c_1}x_3, \quad u_1' = \frac{\omega_1^*}{c_1}u_1, \quad u_3' = \frac{\omega_1^*}{c_1}u_3, \quad t_{31}' = \frac{t_{31}}{\beta_1 T_0},$$
$$t_{33}' = \frac{t_{33}}{\beta_1 T_0}, \quad t' = \omega_1^* t, \quad C' = \frac{\beta_2 C}{\rho c_1^2}, \quad T' = \frac{\beta_1}{\rho c_1^2}T, \quad P' = \frac{P}{\beta_2},$$
$$\gamma' = \omega_1 \gamma, \quad \tau_1^{'} = \omega_1^* \tau_1, \quad \tau_0^{'} = \omega_1^* \tau_0, \quad \tau^{1'} = \omega_1^* \tau^1, \quad \tau^{0'} = \omega_1^* \tau^0, \tag{7}$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho} \ and \quad \omega_1^* = \frac{\rho C_E c_1^2}{K}.$$

The displacement components, $u_1(x_1,x_3,t)$ and $u_3(x_1,x_3,t)$, may be written in terms of the potential functions $\phi(x_1,x_3,t)$ and $\psi(x_1,x_3,t)$ in dimensionless form as

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}, \quad u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1}, \tag{8}$$

we define Laplace and Fourier transforms

$$\hat{f}(x_1,x_3,s) = \int_o^\infty e^{-st} f(x_1,x_3,t)dt,$$

and

$$\tilde{f}(\xi,x_3,s) = \int_{-\infty}^\infty e^{i\xi x} \hat{f}(x_1,x_3,s)dx_1, \tag{9}$$

Applying the Laplace and Fourier transform defined by (9) on equations (3)-(5), after using (6)-(8)(suppressing the primes for convenience) and eliminating $\tilde{\phi}$, $\tilde{T}$, $\tilde{C}$ and $\tilde{\psi}$ from the resulting expressions, we obtain

$$(\frac{d^6}{dx_3^6} + R\frac{d^4}{dx_3^4} + Q\frac{d^2}{dx_3^2} + S)(\tilde{\phi}, \tilde{T}, \tilde{C}) = 0, \tag{10}$$

$$(\frac{d^2}{dz^2} - \lambda_4^2)\tilde{\psi} = 0, \tag{11}$$

Solving equations (10) and (11) and making use of the radiation conditions that $\tilde{\phi}, \tilde{T}, \tilde{C}, \tilde{\psi} \to 0$ as $x_3 \to \infty$, we obtain

$$(\tilde{\phi}, \tilde{T}, \tilde{C}) = \Sigma(1, d_i, e_i)A_i e^{-\lambda_i x_3}, \qquad \tilde{\psi} = A_4 e^{-\lambda_4 x_3} \quad i = 1,2,3, \tag{12}$$

where d_i, e_i are given in appendix-III and $\lambda_i\ (i=1-3)$ and λ_4 are the roots of the following indical equation

$$\left(\frac{d^6}{dx_3^6}+R\frac{d^4}{dx_3^4}+Q\frac{d^2}{dx_3^2}+S\right)=0,$$

$$\left(\frac{d^2}{dz^2}-\lambda_4^2\right)=0,$$

where R,Q,S are given in appendix II.

4. Boundary Conditions

Consider a normal line load F_1 per unit length, acting in the positive x_3-axis on the plane boundary x_3=0 along the x_2-axis and a tangential line load F_2 per unit length, acting at the origin in the positive x_1- axis, then boundary condition are

$$(i)t_{33}=-F_1\psi_1(x_1)H(t) \tag{13}$$

$$(ii)t_{31}=-F_2\psi_2(x_1)H(t) \tag{14}$$

$$(iii)\frac{\partial T}{\partial x_3}=0 \tag{15}$$

$$(iv)\frac{\partial C}{\partial x_3}=0 \tag{16}$$

where $H(t)=1$ for $t\geq 0$, H(t)=0 for $t<0$

F_1 and F_2 are the magnitude of forces, $\psi_1(x)$ and $\psi_2(x)$ are specify the vertical and horizontal load distributions respectively as shown in appendix I(Fig.b). $H(t)$ is the Heavyside unit step function. Making use of equations (7) and (8) in the boundary conditions (13)-(16) and applying the Laplace and Fourier transforms defined by (9).Then substituting values of $\tilde{\phi},\tilde{T},\tilde{\psi},\tilde{C}$ from the equation (12) in the resulting equations, we obtain the expressions of displacements, stresses, temperature distribution and concentration as

$$\tilde{u}_1=-\iota\xi(A_{11}e^{-\lambda_1x_3}+A_{12}e^{-\lambda_2x_3}+A_{13}e^{-\lambda_3x_3})+\lambda_4A_{14}e^{-\lambda_4x_3} \tag{17}$$

$$\tilde{u}_3=-(\lambda_1A_{11}e^{-\lambda_1x_3}+\lambda_2A_{12}e^{-\lambda_2x_3}+\lambda_3A_{13}e^{-\lambda_3x_3}+\iota\xi A_{14}e^{-\lambda_4x_3}) \tag{18}$$

$$\tilde{t}_{33}=S_1A_{11}e^{-\lambda_1x_3}+S_2A_{12}e^{-\lambda_2x_3}+S_3A_{13}e^{-\lambda_3x_3}+S_4A_{14}e^{-\lambda_4x_3} \tag{19}$$

$$\tilde{t}_{31}=R_1A_{11}e^{-\lambda_1x_3}+R_2A_{12}e^{-\lambda_2x_3}+R_3A_{13}e^{-\lambda_3x_3}+R_4A_{14}e^{-\lambda_4x_3} \tag{20}$$

$$\tilde{T}=d_1A_{11}e^{-\lambda_1x_3}+d_2A_{12}e^{-\lambda_2x_3}+d_3A_{13}e^{-\lambda_3x_3} \tag{21}$$

$$\tilde{C}=e_1A_{11}e^{-\lambda_1x_3}+e_2A_{12}e^{-\lambda_2x_3}+e_3A_{13}e^{-\lambda_3x_3} \tag{22}$$

where respective values of $A_{11}, A_{12}, A_{13}, A_{14}$ are given in appendix IV

Case (i) Uniformly distributed force:

The solution due to uniformly distributed force (Figure UDF) applied on the half-space is obtained by setting

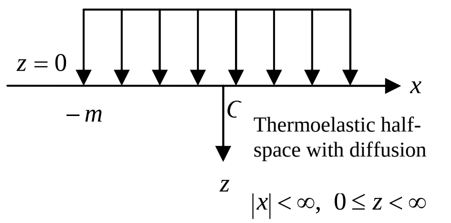

Figure (UDF): Uniformly distributed force

$$[\psi_1(x_1), \psi_2(x_1)] = H(x_1 + d) - H(x_1 - d) \qquad (23)$$

Applying Laplace and Fourier transforms defined by (9) on equation (23), for the case of uniform strip load of non dimensional width $2d$, applied on the half-space is obtained by setting

$$[\tilde{\psi}_1(\xi), \tilde{\psi}_2(\xi)] = 2\frac{(\sin \xi d)}{\xi} \qquad (24)$$

Case (ii) Linearly distributed force:

The solution due to linearly distributed force (Figure LDF) over a strip of non dimensional width 2d, applied on the half-space is obtained by setting

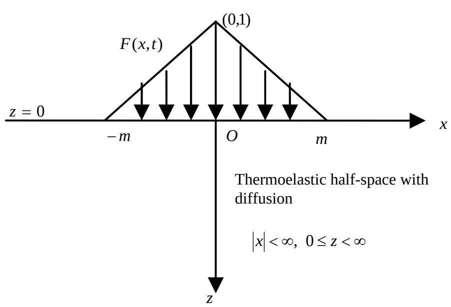

Figure LDF: Linearly distributed force

$$[\psi_1(x), \psi_2(x)] = \begin{cases} 1 - \dfrac{|x|}{d} & if\ |x| \le d \\ 0 & if\ |x| > d \end{cases} \qquad (25)$$

in equations (13) and (14). Applying Laplace and Fourier transforms defined by (9) on equation (25), we obtain

$$[\tilde{\psi}_1(\xi), \tilde{\psi}_2(\xi)] = \frac{2(1 - \cos(\xi d))}{\xi^2 d}, \xi \ne 0, \qquad (26)$$

Case (iii) Moving force:

The solution due to an impulsive force, moving along the x_1-axis with uniform speed V at $x_3 = 0$ is obtained by setting

$$[\psi_1(x_1), \psi_2(x_1)]H(t) = \psi(x_1, t) = \delta(x_1 - Vt) \qquad (27)$$

in equations (13)-(14). Applying Laplace and Fourier transform's defined by equation (9) on the equation (27) yield,

$$[\tilde{\psi}_1(\xi), \tilde{\psi}_2(\xi)]/s = \psi(\xi, s) = \frac{1}{s - \imath\xi V} \qquad (28)$$

This type of situtations occurs in many branches of engineering e.g. bridges and rail/ road tracks caused by moving vehical. Substitute the values of $\tilde{\psi}_1(\xi), \tilde{\psi}_2(\xi)$ from (24), (26) and (28) in equations (17)-(22), we obtain the corresponding expressions for Uniformly distributed force, Linearly distributed force and Moving force respectively.

5. Applications

For an inclined line load F_0, per unit length(Appendix-I(b)), we have

$$F_1 = F_0 \cos\delta, \quad F_2 = F_0 \sin\delta \qquad (29)$$

Using equation (29) in (17)-(22) and with the help of equations (24),(26) and (28), we obtain corresponding expressions for distributed forces and moving force respectively.

6. Special Case

1. Neglecting diffusion effect $(\beta_2 = b = a = 0)$: in equations (17)-(22), we obtain the corresponding expression for generalized thermoelastic medium and are given in Appendix V
2.By vanishing the thermal relaxation times, i.e $\tau_0 = \tau^0 = \tau_1 = \tau^1 = 0$ in equations (17)-(22), we obtain corresponding expressions in coupled thermoelastic diffusive media.

7. Inversion of the transforms

The transformed components of displacement, stress, temperature distribution and chemical potential are function of x_3, the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $\tilde{f}(\xi, x_3, s)$. To obtain the solution of the problem in physical domain, we must invert the Laplace and Fourier transform by using the method applied by Sharma *et.al* (2009)

8. Numerical result and discussion

With the view of illustrating theoretical results obtained in preceding section, we now present some numerical results. The material parameter chosen for this purpose are given by Sherief and Saleh (2005)

$$\lambda = 7.76\times10^{10}\,Kg\,m^{-1}\,s^{-2}, \quad \mu = 3.86\times10^{10}\,Kg\,m^{-1}\,s^{-2}, \quad \rho = 8.954\times10^{3}\,Kg\,cm^{-3},$$
$$K = 0.386\times10^{3}W\,m^{-1}\,K^{-1}, \quad \tau^1 = 0.04s, \quad \tau_1 = 0.03s, \quad \tau^0 = 0.02s, \quad \tau_0 = 0.01s.$$
$$\alpha_t = 1.78\times10^{-5}\,K^{-1}, \quad \alpha_c = 1.98\times10^{-4}\,m^3\,kg^{-1}, \quad a = 1.2\times10^{4}\,m^2 \quad s^{-2}\,k^{-1},$$
$$b = 0.9\times10^{6}\,m^5\,Kg^{-1}\,s^{-2}, \quad j = 0.2\times10^{-15}\,cm^2, \quad D = 0.85\times10^{-8}\,Kg\,s\,m^{-3}.$$
$$C_E = 0.3831J\,Kg^{-1}\,K^{-1}, \qquad T_0 = 0.293\times10^{3}\,K,$$

The values of normal stress (t_{33}), tangential stress (t_{31}), temperature distribution T and mass Concentration C are presented graphically for G-L theory with thermoelastic diffusion and without diffusion effects in the range $0 \le x \le 10$. The solid line, dashed line and small dashed line corresponds for thermoelastic diffusion (GLWD) and solid line, dash line and small dashed line with centre symbols corresponds for thermoelastic theory (GLD) due to various sources for $\delta = 0$ (Initial angle), $\delta = 45$ (Intermediate angle), $\delta = 90$ (Extreme angle)

Uniformly Distributed Source

Figure 1 depicts the variations of t_{33} with distance x. It is noticed that values of t_{33} for GLWD and GLD at all δ decrease in range $0 \le x \le 3$ except for GLD at extreme angle, which shows opposite behaviour i.e. value increases. With further increase in x, the values of t_{33} for GLWD increase and decrease monotonically at all δ, whereas values of t_{33} for GLD approaches towards origin.

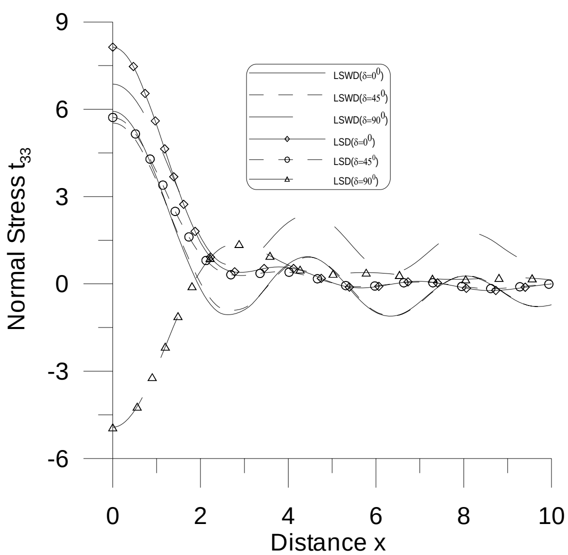

Fig1: Variation of normal stress t_{33} with (Uniformly distributed normal force)

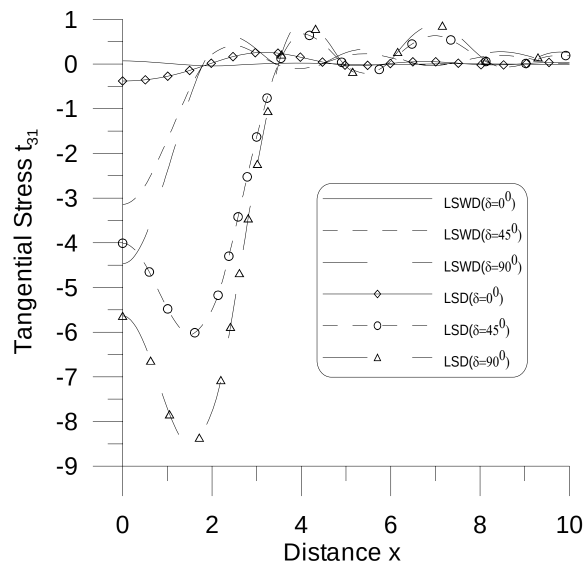

Fig 2: Variation of Tangential stress t_{31} x with distance x distance (Uniformly distributed normal force)

The variation of t_{31} with distance is shown in figure 2. It is noticed that the values of t_{31} for GLWD and GLD at initial angle shows small variations about zero value, whereas for remaining δ values of t_{31} for GLWD and GLD shows opposite behaviour in the range $0 \le x \le 4$, which reveals the impact of diffusion, then with increase in x, values of t_{31} for GLD shows oscillatory behaviour while for GLWD values converges towards origin.

Figure 3 shows the variations of T with distance x. It is noticed that trends of T for GLWD and GLD for all angles are opposite in behaviour in range $0 \le x \le 3$, except for GLD at $\delta = 90$, which shows steady state about origin in entire range. As x increases, the values of T for GLWD follows oscillatory behaviour with decreasing magnitude for all δ, while for GLD at $\delta = 0{,}45$ tends to approach towards origin.

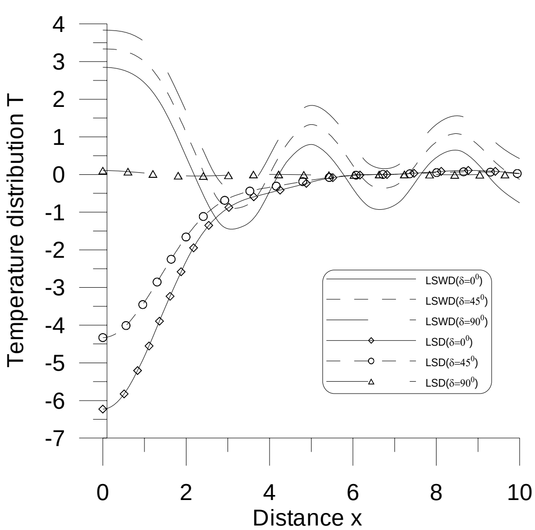

Fig 3: Variation of Temperature distribution T with distance x (Uniformly distributed normal force)

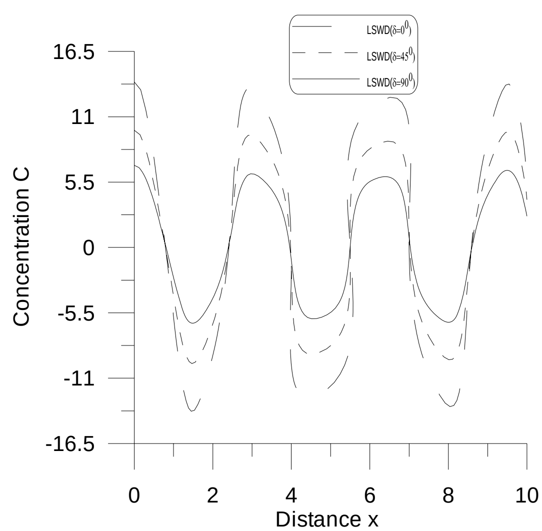

Fig 4: Variation of Concentration C with distance x (Uniformly distributed normal force)

The variations of C with distance x is depicted in figure 4. It is noticed that the values of C for GLWD shows similar behaviour in entire range i.e. their values increase and decrease alternately.

Linearly Distributed Source

Figure 5 shows the variations of t_{33} with distance x. It is noticed that trends of variations of t_{33} for GLWD and GLD are similar in nature at $\delta = 0{,}45$ and at $\delta = 90$ both GLWD and GLD shows opposite behaviour in range $0 \le x \le 3$, with increase in x, values of t_{33} for GLWD shows oscillatory behaviour for all δ, while for GLD values shows small variations near zero value.

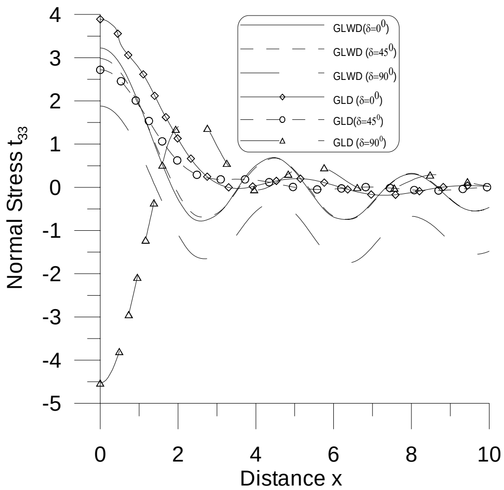

Fig 5: Variation of normal stress t_{33} with distance x
(Linearly distributed force)

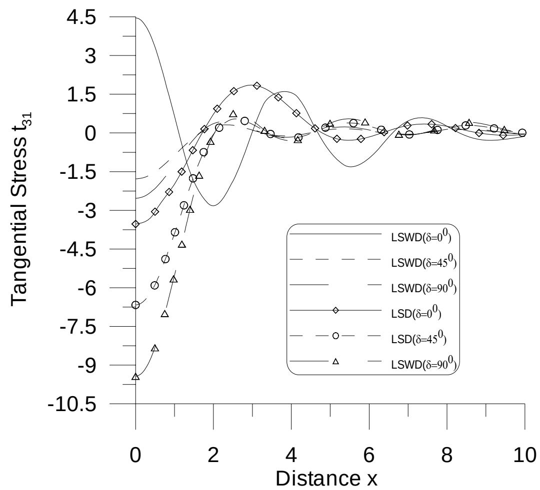

Fig 6: Variation of Tangential stress t_{31} with distance x
(Linearly distributed normal force)

Figure 6 depicts the variations of t_{31} with distance x. It is noticed that values of t_{31} for GLWD and GLD increase in the range $0 \le x \le 3$ for all δ except at $\delta = 0$ for GLWD, which shows opposite behaviour i.e. value decreases abruptly and then follows an oscillatory behaviour with decreasing magnitude.

The variations of T with distance x is shown in figure 7. It is noticed the trends of T for GLWD and GLD are opposite in behaviour near the loading surface at $\delta = 0{,}45$, while at $\delta = 90$ trends of T for GLWD and GLD are similar in nature i.e. values decrease in range $0 \le x \le 3$, magnitude being greater for GLD. As x increases, values of T for GLWD and GLD increase and decrease alternately for all δ, while values of T for GLD converge towards origin, which shows the impact of diffusion.

Figure 8 show the variations of C with x. It is noticed that trends of variations of C for GLWD at all δ are similar in nature with significant difference in their magnitude.

Moving Force

Figure 9 shows the variations of t_{33} with distance x. It is noticed that values of t_{33} for GLWD increase abruptly in initial range for all δ, then follows an oscillatory behaviour with decreasing magnitude. Also for GLD at $\delta = 0{,}45$ are similar in nature except in range $0 \le x \le 3$, where it shows opposite behaviour. Also, values of t_{33} for GLD at initial angle increase abruptly and then converges towards origin.

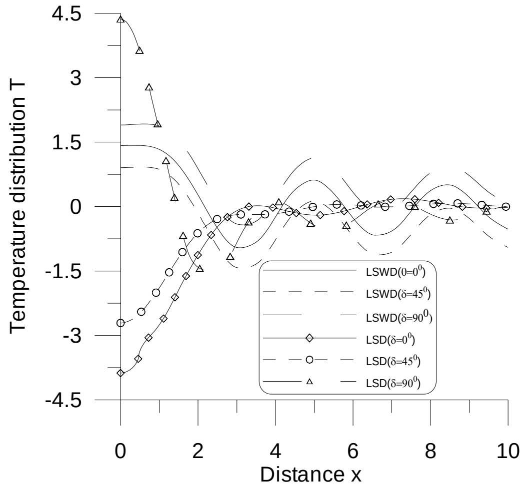

Fig 7: Variation of Temperature distribution T x (Linearly distributed normal force)

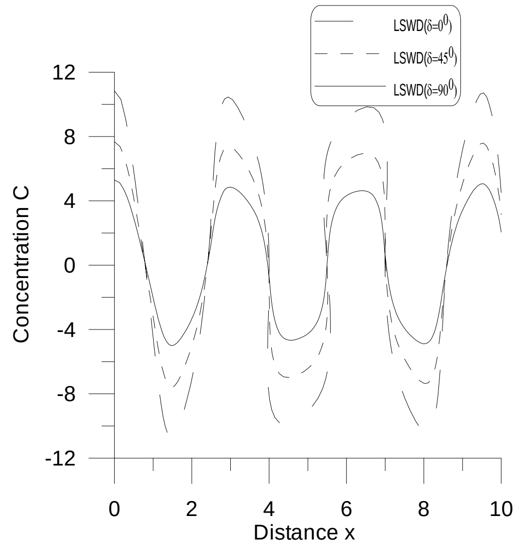

Fig 8: Variation of Concentration C with distance x (Linearly with distance distributed normal force)

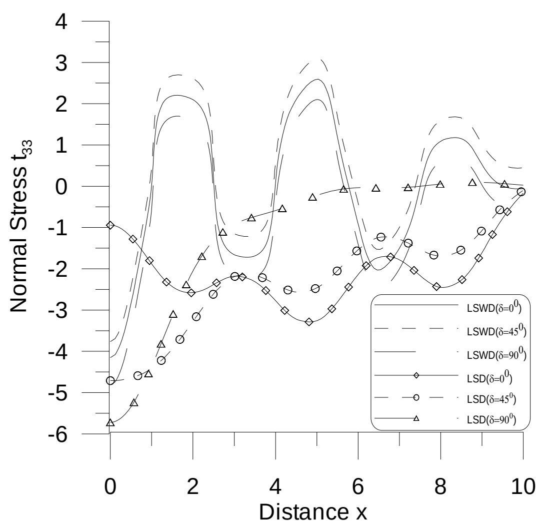

Fig 9: Variation of normal stress t_{33} with distance x (Moving force)

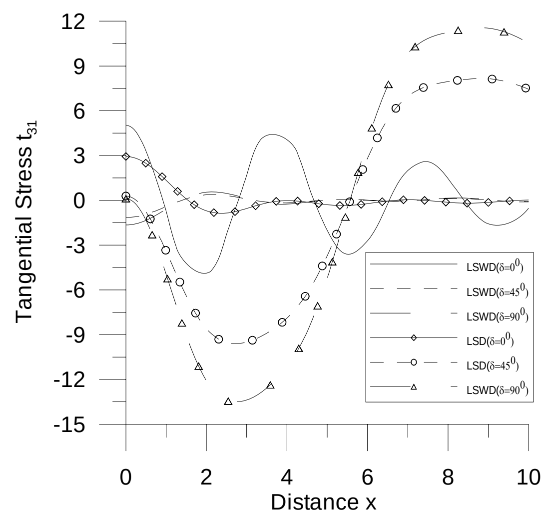

Fig 10: Variation of Tangential stress t_{31} distance x (Moving force)

It is noticed from figure 10, which is plot for t_{31} with x, that trends of variations of t_{31} for GLWD at $\delta = 45{,}90$ are opposite to that noticed for GLD at $\delta = 0$ in range $0 \le x \le 4$. In remaining range, value of t_{31} for GLD at initial angle converges toward origin, while values of t_{31} for GLWD at $\delta = 0$ increase and decrease alternately with x. Also values of t_{31} for GLD at $\delta = 45{,}90$ decrease in range $0 \le x \le 3$ and increase in remaining range.

Figure 11 depict the variations of T with x. It is noticed that trends of variations of T for GLWD at all δ are similar in nature with significant difference in their magnitude i.e. their values increase in range $2 \le x \le 4$, $6 \le x \le 8$ and decreases in remaining range. Also opposite behaviour is noticed for GLD in initial range at $\delta = 0{,}45$, with further increase in x, both shows similar pattern also, values of T for GLD at $\delta = 0$ decreases abruptly in range $0 \le x \le 4$ and in remaining range shows small variations.

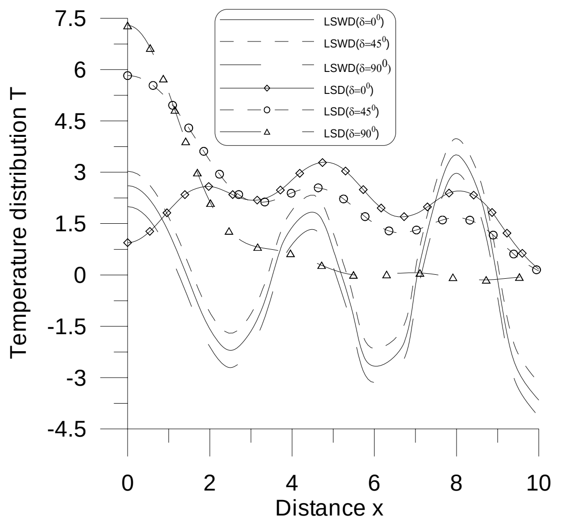

Fig 11: Variation of Temperature distribution T with distance x (Moving force)

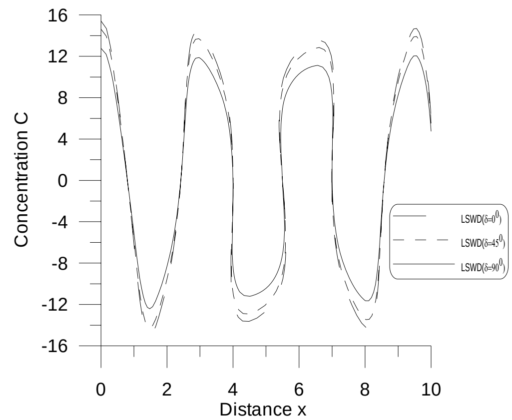

Fig 12: Variation of Concentration C with distance x (Moving force)

The variations of C with x are shown in figure 12. It is noticed that values of C for GLWD at $\delta = 0{,}45{,}90$ increase and decrease alternately with significant difference in their magnitude.

9. Conclusion

The article presents an in-depth, analysis of inclined load problem in the context of G-L theory and CT theory of thermoelasticity. It is noticed that trends of variations of stresses, temperature distribution and chemical potential on the application of distributed sources are similar in nature with significant difference in their magnitude. The present problem can provide useful contribution to the theoretical considerations of the seismic and volcanic sources, since it can can account for the deformation fields in the entire volume surrounding the source region.

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Nomenclature

λ, μ -Lame's constants

α_t, α_c - coefficient of linear thermal and linear diffusion expansion

T- absolute temperature

T_0 - temperature of the medium in the natural state

t_{ij} - component of the stress tensor

u_i - component of the displacement vector

ρ - density

e_{ij} - components of the strain tensor

P- chemical potential

C- Concentration

C_E -Specific heat

K- Coefficient of thermal conductivity

τ_0, τ_1 - Thermal relaxation times

τ^0, τ^1 -Diffusion relaxation times

D, a, b -constants

δ_{ij} - Kronecker delta.

Appendix I

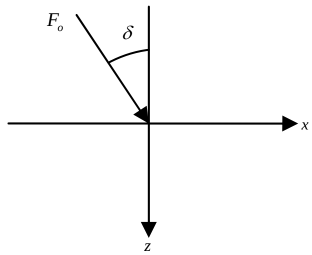

Fig. (a) Inclined load over thermoelastic half-space

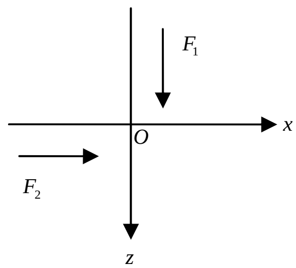

Fig. (b) Normal and tangential loadings

Appendix II

$$R = \frac{F - 3\xi^2 E}{E}, \quad Q = \frac{G - 2F\xi^2 + 3\xi^4 E}{E}, \qquad S = \frac{F\xi^4 - G\xi^2 + H - E\xi^6}{E}, \quad \lambda_4^2 = \xi^2 + \frac{s^2}{\delta},$$

$$E = f_2(1-\varepsilon_2), \quad F = \varepsilon_2(f_2 f_3 + s a_2 + s^2 f_2) + \varepsilon_1 s f_1 f_2(\varepsilon_2 + a_1) + \varepsilon_1 a_1 f_4(1+a_1) - f_2 f_3,$$

$$G = -[\varepsilon_2 a_2 s(f_3 + s^2 + \varepsilon_1 s f_1) + s^2 \varepsilon_2 f_2 f_3 + \varepsilon_1 a_1^2 s^2 f_1 f_4], \quad H = \varepsilon_2 a_2 s^3 f_3,$$

$$a_1 = \frac{a(\lambda+2\mu)}{\beta_1\beta_2}, \quad a_2 = \frac{K}{bD\rho C_E}, \quad \delta = \frac{\mu}{\lambda+2\mu}, \quad \varepsilon_1 = \frac{\beta_1^2 T_0}{\rho C_E(\lambda+2\mu)}, \quad \varepsilon_2 = \frac{b(\lambda+2\mu)}{\beta_2^2}$$

$$f_1 = (1+\tau_1 s), \quad f_2 = (1+\tau^1 s), \quad f_3 = s(1+\tau_0 s), \quad f_4 = s(1+\tau^0 s),$$

Appendix III

$$d_i = \frac{\lambda_i^4 P_1 - \lambda_i^2 P_2 + P_3}{P_{12}}, \quad e_i = \frac{\lambda_i^6 + \lambda_i^4 P_4 + P_5}{P_{12}}, \quad P_{12} = \varepsilon_2 f_2 \lambda_i^4 - \lambda_i^2 P_{10} + P_{11} \qquad \text{(i=1,2,3)}$$

$$P_1 = \varepsilon_1(a_1 f_1 + \varepsilon_2 f_2 s), \quad P_2 = 2P_1\xi^2 + P_6, \quad P_3 = P_1\xi^4 + P_6\xi^2, \quad P_4 = P_5 - 3\xi^2,$$

$$P_5 = 3\xi^4 - 2P_9\xi^2 + P_9\xi^4, \quad P_6 = \varepsilon_1\varepsilon_2 a_2 s^2, \quad P_7 = \varepsilon_2(f_2 f_3 + a_2 s) + \varepsilon_1 a_1^2 f_1 f_4$$

$$P_8 = \varepsilon_2 a_2 f_3 s, \quad P_9 = \varepsilon_1 a_1 s - f_3, \quad P_{10} = P_7 + 2\varepsilon_2 f_2 \xi^2, \quad P_{11} = \varepsilon_2 f_2 \xi^4 + P_7\xi^2 + P_8$$

Appendix IV

$$A_{11} = \frac{W_1(-R_4 F_1 \tilde{\psi}_1(\xi) + S_4 F_2 \tilde{\psi}_2(\xi))}{\Delta}, \quad A_{12} = \frac{W_2(R_4 F_1 \tilde{\psi}_1(\xi) - S_4 F_2 \tilde{\psi}_2(\xi))}{\Delta},$$

$$A_{13} = \frac{W_3(-R_4 F_1 \tilde{\psi}_1(\xi) + S_4 F_2 \tilde{\psi}_2(\xi))}{\Delta}, \quad g_1 = \frac{\lambda}{\beta_1 T_0}, \quad g_2 = \frac{\rho c_1^2}{\beta_1 T_0}, \quad g_3 = \frac{\mu}{\beta_1 T_0},$$

$$A_{14} = \frac{F_1\tilde{\psi}_1(\xi)(W_1R_1 - W_2R_2 + W_3R_3) - F_2\tilde{\psi}_2(\xi)(W_1S_1 - W_2S_2 + W_3S_3)}{\Delta}$$

$$S_i = -\xi^2 g_1 + \lambda_i^2 g_2 - a_i f_1 - b_i f_2, S_4 = \iota\xi\lambda_4(g_2 - g_1), R_i = 2\iota\xi\lambda_i g_3, R_4 = -g_3(\lambda_4^2 + \xi^2)$$

for (i=1,2,3)

$$W_1 = \frac{\lambda_2\lambda_3(d_2e_3 - d_3e_2)}{s}, \quad W_2 = \frac{\lambda_1\lambda_3(d_1e_3 - d_3e_1)}{s}, \quad W_3 = \frac{\lambda_1\lambda_2(d_1e_2 - d_2e_1)}{s}$$

and

$$\Delta = \begin{vmatrix} S_1 & S_2 & S_3 & S_4 \\ R_1 & R_2 & R_3 & R_4 \\ \lambda_1 d_1 & \lambda_2 d_2 & \lambda_3 d_3 & 0 \\ \lambda_1 e_1 & \lambda_2 e_2 & \lambda_3 e_3 & 0 \end{vmatrix}$$

Appendix V

$$\tilde{u}_1 = -\iota\xi(B_{11}e^{-\lambda_1 x_3} + B_{12}e^{-\lambda_2 x_{13}}) + \lambda_4 B_{13}e^{-\lambda_4 x_3},$$

$$\tilde{u}_3 = -(\lambda_1 B_{11}e^{-\lambda_1 x_3} + \lambda_2 B_{12}e^{-\lambda_2 x_3} + \iota\xi B_{13}e^{-\lambda_4 x_3}),$$

$$\tilde{t}_{33} = S_1 B_{11}e^{-\lambda_1 x_3} + S_2 B_{12}e^{-\lambda_2 x_3} + S_4 B_{13}e^{-\lambda_4 x_3},$$

$$\tilde{t}_{31} = R_1 B_{11}e^{-\lambda_1 x_3} + R_2 B_{12}e^{-\lambda_2 x_3} + R_4 B_{13}e^{-\lambda_4 x_3},$$

$$\tilde{T} = d_1 B_{11}e^{-\lambda_1 x_3} + d_2 B_{12}e^{-\lambda_2 x_3},$$

where

$$B_{11} = \frac{X_1[F_1R_4\tilde{\psi}_1(\xi) - F_2S_4\tilde{\psi}_2(\xi)]}{\Delta_0}, \quad B_{12} = -\frac{X_2[-F_1R_4\tilde{\psi}_1(\xi) + F_2S_4\tilde{\psi}_2(\xi)]}{\Delta_0}$$

$$B_{13} = \frac{F_1\tilde{\psi}_1(\xi)[R_{12}X_2 - R_{11}X_1] + F_2\tilde{\psi}_2(\xi)[S_{11}X_1 - S_{12}X_2]}{\Delta_0}, \quad X_1 = \frac{\lambda_2 d_2}{s},$$

$$X_2 = \frac{\lambda_1 d_1}{s}, \quad S_i = -\xi^2 g_1 + \lambda_i^2 g_2 - a_i, \quad S_4 = \iota\xi\lambda_4(g_2 - g_1), \quad R_i = 2\iota\xi\lambda_i g_3,$$

$$R_4 = -g_3(\lambda_4^2 + \xi^2)$$

for i= 1,2

$$\Delta_0 = \begin{vmatrix} S_{11} & S_{12} & S_4 \\ R_{11} & R_{12} & R_4 \\ \lambda_1 d_1 & \lambda_2 d_2 & 0 \end{vmatrix}$$

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