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Effect of constant heat flux at outer cylinder on stability of viscous flow in a narrow-gap annulus with radial temperature gradient

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Abstract

In this paper, the stability of the Couette flow of a viscous incompressible fluid between two concentric rotating cylinders is studied in the presence of a radial temperature gradient, when the outer cylinder is maintained at a constant heat flux. The analytical solution of the eigen-value problem is obtained by using the trigonometric series method, when the gap between the cylinders is narrow. The numerical values of the critical wave number and critical Taylor number are computed from the obtained analytical expressions for the first, second and third approximations. It is found that the difference between the numerical values of the critical Taylor number corresponding to the second and third approximations are very small as compared to the difference between first and second approximations. The critical Taylor numbers obtained by the third approximation agree very well with the earlier results computed numerically by using the shooting method. This clearly indicates that for the better result one should obtain the numerical values by taking more approximations. Also, the amplitude of the radial velocity and the cell-pattern are shown on the graphs for different values of the ratio of the angular velocities.

Keywords: Radial temperature gradient, trigonometric series method, constant heat flux, Taylor number.

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1. Introduction

The stability of flow phenomenon of a viscous incompressible fluid between two concentric rotating cylinders with the inner cylinder rotating or both rotating was first studied by (Taylor, 1923). He used a parameter called Taylor number *Ta* and found that the flow becomes unstable, when the Taylor number exceeds its critical value Ta_c. To determine Ta_c different methods were given by Taylor (1923), Chandrasekhar (1953), DiPrima (1953), Duty and Reid (1964) and Harris and Reid (1964). In all these papers, it was basically assumed that the two cylinders are at the same temperature and as a result of which radial temperature gradient does not exist. However, in many chemical, electrical and mechanical engineering applications the temperature of both cylinders cannot remain the same. Thus, due to the change in the temperature of both cylinders, there exist a temperature gradient and the stability of the fluid flow is affected by the temperature gradient.

The effects of a radial temperature gradient on the stability of the flow of a viscous fluid was theoretically as well as experimentally studied by Chandrasekhar (1954), Bjorklund and Kays (1959), Hass and Nissan (1961), Yih (1961), Beeker and Kays (1962), Ho et al. (1964), Aoki et al. (1967), Sharman et al. (1973), Walowit et al. (1964), Soundalgekar et al. (1981), Takhar et al. (1985), Singh et al. (1994) and others. Further, the effect of radial temperature gradient on the stability of Dean flow was investigated by Ali et al. (1998) under narrow gap approximation and the effect of radial temperature gradient on the circular Couette flow was analysed by Mutabazi et al. (2001). The Taylor–Dean flow through a curved duct of square cross section, in which walls of the duct except the outer wall rotate around the center of curvature and an azimuthal pressure gradient was imposed, was analyzed by Yamamoto et al. (2004). After that, Soleimani and Sadeghy (2011) investigated numerically the stability of the Bingham fluids in Taylor–Dean flow between two concentric cylinders at arbitrary gap spacing. The three dimensional linear stability analysis of the Couette flow between two axial cylinders for shear-thinning fluids with and without yield stress was performed by Alibenyahia et al. (2012).

In all of these studies, it was assumed that the two cylinders are mentioned at the different temperatures. In technological applications, there is another situation in which the outer cylinder is maintained at constant temperature while the inner cylinder can get heated from the heat generated by an external source of energy. Mathematically this situation is studied by imposing a variable or constant heat flux at the inner cylinder while the outer cylinder is maintained at a constant temperature. The stability of flow formation between two concentric rotating cylinders, when there is a constant heat flux at the inner cylinder while the outer one is maintained at a constant temperature was studied by Takhar et al. (1988a), Prasad et al. (2012), Takhar et al. (1988b) for the narrow gap case and wide gap case, respectively. Recently, Mahapatra et al. (2013) studied the effect of radial temperature gradient on the stability of Taylor-Dean flow between two arbitrarily spaced concentric cylinders.

Instead of a constant heat flux at the inner cylinder, we can apply the constant heat flux at the outer cylinder and then find its effect on the stability of flow. This was studied by Eagles et al. (1997) for wide gap case and Takhar et al. (1992) for narrow gap case and solved the eigen-value problem by the shooting method. Stability of pure pressure maintained flow in a curved channel is studied by Pandey and Prasad (2012) and same problem is also discussed by Prasad and Pandey (2012) in the presence of axial magnetic field. A linear stability analysis for the Taylor-Dean flow, between concentric horizontal cylinders with a constant azimuthal pressure gradient, keeping the cylinders at different temperatures, when the inner cylinder is rotating and outer one is stationary was studied by Deka and Paul (2013) for the narrow gap case. Here, our aim is to study the narrow-gap Taylor-stability problem in the case when there is a constant heat flux at the outer cylinder while the inner one is maintained at a constant temperature. We have solved this problem by using the trigonometric series method and the results are compared with those obtained by Takhar et al. (1992) by using a numerical method. Also the amplitude of the radial velocity and the cell-pattern are shown on graphs for different values of the ratio of the angular velocities.

2. Mathematical Analysis

Consider the flow of an incompressible viscous fluid between two concentric rotating cylinders of radii R_1 and R_2 (R_1, radius of the inner cylinder; R_2 , radius of the outer cylinder) with angular velocities Ω_1 and Ω_2 respectively. T is the temperature of the fluid and T_1 is the constant temperature of the inner cylinder respectively. Assuming stationary marginal state, the following differential equations have been obtained to govern the stability of the viscous flow in a narrow-gap annular-space (Takhar et al., 1992):

$$(D^2-a^2)^2 u = -a^2 T_a\left[g(x)v + N.\left(g(x)\right)^2\theta\right], \tag{1}$$

$$(D^2-a^2)v = u, \tag{2}$$

$$\left(D^2-a^2\right)\theta = u. \tag{3}$$

with boundary conditions

$$\begin{aligned} &u = Du = v = \theta = 0 \;\; at \;\; x = 0, \\ &u = Du = v = D\theta = 0 \;\; at \;\; x = 1. \end{aligned} \tag{4}$$

where,

$$d = R_2 - R_1, x = (r - R_1)/d, D = \frac{d}{dx},$$

$$a = \lambda d, \mu = \Omega_2/\Omega_1, \alpha = \mu - 1, g(x) = 1 + \alpha x, Pr = \nu / K,$$

$$\theta = 2AKT/(\Omega_1 \Pr qR_2), Ra = \Pr \Omega_1^2 d^4 (qR_2)\beta/(\nu^2 K), u = (\nu/2Ad^2)u,$$

$$N = -\{\Pr(\frac{qR_2}{K})\beta\Omega_2\}/(4A) = Ra/Ta, Ta = -(4A\Omega_1 d^4)/\nu^2. \tag{5}$$

Here, *Ra* and *Ta* are respectively, the Rayleigh number and the Taylor number.

As suggested by Chandrasekhar (1953, we take a sine series for v in order to satisfy the boundary conditions given by Eq. (4) as follows:

$$v = \sum_{m=1}^{\infty} A_m \sin(m\pi x). \tag{6}$$

Substituting Eq. (6) in Eq. (2) and then with the help of Eq. (2) and (3), we obtain the value of θ. Using these values of θ and v in Eq. (1), we have obtained the general solution for u as follows:

$$u = -a^2 Ta \sum_{m=1}^{\infty} \frac{A_m}{K_1^{\,2}} [(A_1^{(m)} + xA_2^{(m)})\cos h(ax) + (A_3^{(m)} + xA_4^{(m)})\sin h\left(ax\right) + (1+\alpha x)\sin(m\pi x) + \frac{4m\alpha\pi\cos\left(m\pi x\right)}{K_1}\{1+N(1+\alpha x)\} + N\{(\left(1+\alpha x\right)^2 + \frac{4\alpha^2 K_2}{K_1^{\,2}})\sin\left(m\pi x\right) + \frac{\left(-1\right)^{m+1} m\pi K_1^{\,2} x^2 K_4}{48a^5 K_*}\}]. \tag{7}$$

where, $A_1^{(m)}$, $A_2^{(m)}$, $A_3^{(m)}$, $A_4^{(m)}$ are the constants of integration. In view of the boundary conditions (4) and solving the resulting equations, we have

$$A_1^{(m)} = -\frac{4m\pi\alpha\left(1+2N\right)}{K_1}, \tag{8}$$

$$A_2^{(m)} = \frac{m\pi}{\Delta}[\alpha_m \sin K_0^2 a - a\gamma_m K_0 - a\alpha K_5 \delta_m + a^2\left(\beta_m - \lambda_m \mu_m\right)], \tag{9}$$

$$A_3^{(m)} = \frac{m\pi}{\Delta}\left[a\alpha_m - \gamma_m K_0 - \alpha K_5 \delta_m + a\beta_m - a\lambda_m \mu_m\right], \tag{10}$$

$$A_4^{(m)} = -\frac{m\pi}{\Delta}[K_6 + \mu_m a(K_7 + K_8)], \tag{11}$$

where,

$$K_* = \cosh(a), K_0 = \sinh(a), K_1 = m^2\pi^2 + a^2, K_2 = a^2 - 5m^2\pi^2, K_3 = a^2 - m^2\pi^2,$$

$$K_4 = a^2(6 + \alpha^2 x^2 + 4\alpha x)\sin h\left(ax\right) - (\alpha x + 3)\cos h(ax), K_5 = aK_* + K_0,$$

$$K_6 = \alpha_m C_1 + C_2\left(\gamma_m + \beta_m K_*\right) + a^2\beta_m K_0 + \frac{4a^2\alpha\left(-1\right)^{m+1}(1+2N\left(\alpha+1\right))K_0}{K_1}$$

$$C_1 = \left(a - K_0 K_*\right), C_2 = \left(aK_* - K_0\right), \alpha_m = 1 + N + \frac{12\left(-1\right)^{m+1} NK_3\alpha^2}{K_1^{\,2}},$$

$$\beta_m = \frac{4\alpha\left(1+2N\right)}{K_1}, \Delta = \left(K_0^{\,2}a - a^2\right),$$

$$\gamma_m = \left(-1\right)^{m+1}(\alpha+1)\left(1+\left(\alpha+1\right)N\right) + \frac{12\left(-1\right)^{m+1} NK_3\alpha^2}{K_1^{\,2}} - \beta_m K_*,$$

$$\delta_m = -\frac{4\left(-1\right)^{m+1}\left(1+2N(1+\alpha)\right)}{K_1^{\,2}} - \frac{\left(-1\right)^{m+1} NK_1^{\,2}}{48a^5}\left\{4a\left(3+\alpha\right) - 9\alpha\tan ha\right\},$$

$$\lambda_m = K_0^{\,2}a\left(\alpha^2\left(a^2 + 3aK_* + 18\right) - 6a^2 + 4\alpha a\left(a + 6K_*\right)\right),$$

$$\mu_m = \frac{\left(-1\right)^{m+1} NK_1^{\,2}}{48a^6 K_*}.$$

By inserting the mathematical expressions of u and v from Eqs. (7) and (6), respectively, in Eq. (2), we have

$$\sum_{n=1}^{\infty} A_n R\sin(n\pi x) = -a^2 Ta\sum_{m=1}^{\infty}\frac{A_m}{K_1^2}[(A_1^{(m)}+xA_2^{(m)})\cos h(ax)+(A_3^{(m)}+xA_4^{(m)})\sin h(ax)+R_1$$
$$+N(R_2+R_3)]. \tag{12}$$

Multiplying Eq. (12) by $\sin(m\pi x)$ and then integrating over the range $0 \le x \le 1$, we obtain a system of linear homogeneous equations for the constants and the requirement that these constants are to all zero leads to the following secular equation:

$$\|\frac{n\pi}{R}[A_1^{(m)}R_5 + A_2^{(m)}R_6 + A_3^{(m)}(-1)^{n+1}K_0 + A_4^{(m)}R_7] + \frac{(-1)^{m+1}m\pi K_1^2 N}{48a^5 K_*} + R_8 + R_9$$
$$+N\alpha^2 Y_{mn} + \alpha X_{mn} - \frac{R^3}{2a^2 Ta}\delta_{mn}\| = 0. \tag{13}$$

where,

$$R = n^2\pi^2 + a^2,$$
$$R_1 = (1+\alpha x)\sin(m\pi x) + \frac{4m\alpha\pi\cos(m\pi x)}{K_1},$$
$$R_2 = (1+\alpha x)^2\sin(m\pi x) + \frac{8m\pi\alpha cos(m\pi x)}{K_1}(1+\alpha x),$$
$$R_3 = \frac{4\alpha^2 K_2\sin(m\pi x)}{K_1^2} + \frac{(-1)^{m+1}m\pi K_1^2 x^2 R_4}{48a^5\cos ha},$$
$$R_4 = a^2(6+\alpha^2x^2+4\alpha x)\sin h(ax) - (\alpha x+3)\cos h(ax),$$
$$R_5 = 1+(-1)^{n+1}K_*, R_6 = (-1)^{n+1} + \frac{2(-1)^n aK_0}{R},$$
$$R_7 = (-1)^{n+1}K_0 + \frac{2a}{R}((-1)^n K_* - 1)),$$
$$R_8 = 3(2a^2+3\alpha^2)I_1 + a^2\alpha(\alpha I_2 + 4I_5) - 4a\alpha(\alpha I_3 + 3I_4),$$
$$R_9 = \frac{1}{2}\left[1+N\left\{1+\frac{4\alpha^2K_2}{K_1^2}I_1\right\}\right]\delta_{mn}, R_{10} = \mathrm{a}^2 - \mathrm{n}^2\pi^2,$$
$$R_{11} = \mathrm{n}^4\pi^4 - 5\mathrm{a}^2(\mathrm{a}^2 - 2\mathrm{n}^2\pi^2), R_{12} = \mathrm{n}^2\pi^2 - 3\mathrm{a}^2, R_{13} = (\mathrm{n}^2-\mathrm{m}^2),$$
$$R_{13} = 3(\mathrm{n}^2\pi^2 - 2\mathrm{a}^2)K_0 + \mathrm{n}^2\pi^2K_*, R_{14} = [(-1)^{n+m} - 1],$$
$$R_{15} = \left[1+(1+\alpha)(-1)^{n+m}\right], R_{16} = \left[(-1)^{n+1} + (1+\alpha)(-1)^{m+1}\right],$$
$$I_1 = \frac{\mathrm{n}\pi}{R^3}(-1)^{\mathrm{n}}[K_0\{R^2 + 2(3\mathrm{a}^2 + \mathrm{n}^2\pi^2)\} + 4\mathrm{a}K_*R],$$
$$I_2 = \frac{\mathrm{n}\pi}{R^5}(-1)^{\mathrm{n}}[K_0\{R^4 + 12R^2R_{12} + 24R_{11}\} + 8RK_*\{aR^2 + 12R_{10}\}],$$
$$I_3 = \frac{\mathrm{n}\pi}{R^4}(-1)^{\mathrm{n}}[-K_*R^3 + 6a\{(4R_{10}+R^2)K_0 + \mathrm{R}K_*(\mathrm{n}^2\pi^2 - 2\mathrm{a})\}],$$

$$I_4=\frac{n\pi}{R^4}\left(-1\right)^n[-K_*R(R^2-2R_{12})+4aR^2K_0-2n\pi R_{12}],$$

$$I_5=\frac{n\pi}{R}\left(-1\right)^n[K_0+\frac{2R_{13}}{R^2}+\frac{6aK_*}{R}(1+\frac{4a^2}{R^2})]-\frac{24R_{10}n\pi a}{R^4},$$

$$\delta_{mn}=\begin{cases}1, & if\;\; m=n\\ 0, & if\;\; m\neq n\end{cases}$$

$$X_{mn}=\begin{cases}\frac{1}{4}+\frac{N}{2}, & if\;\; m=n\\ \frac{2mn((-1)^{m+1}-1)\left(1+2N\right)}{R_{13}}\left[\frac{1}{\pi^2R_{13}}-\frac{2}{K_1}\right], & if\;\; m\neq n\end{cases}$$

$$Y_{mn}=\begin{cases}\frac{1}{6}-\frac{1}{4m^2\pi^2}-\frac{2}{K_1}, & if\;\; m=n\\ \frac{4mn\left(-1\right)^{m+1}}{R_{13}}\left[\frac{1}{\pi^2R_{13}}-\frac{2}{K_1}\right], & if\;\; m\neq n\end{cases} \tag{14}$$

On substituting the values of $A_1^{(m)}$, $A_2^{(m)}$, $A_3^{(m)}$, $A_4^{(m)}$ and I_1, I_2, I_3, I_4, I_5 from Eqs. (8) - (11), and (14) respectively and taking $N=0$ we have,

$$\|\frac{4mn\pi^2R_{14}\alpha}{RK_1}+\frac{2amn\pi^2C_1}{R^2\Delta}\{R_{15}+R_{16}+\frac{4a\alpha K_0}{K_1C_1}[K_0+a\left(-1\right)^{m+1}]R_{14}\}+\frac{1}{2}\delta_{mn}+\alpha X_{mn}$$
$$-\frac{R^3}{2a^2Ta}\delta_{mn}\|=0. \tag{15}$$

For $m=n=1$ we get,

$$Ta=\frac{2}{\left(\mu+1\right)}.\frac{\left(\pi^2+a^2\right)^3}{a^2[1-\frac{16\pi^2 a\cosh^2(a/2)}{\left(\pi^2+a^2\right)^2(K_0+a)}]}. \tag{16}$$

Eqs. (15) and (16) are the result obtained by (Chandrashekhar, 1961).

3. Result and Discussion

The numerical value of Ta_c computed from Eq. (13) corresponding to the first, second and third approximations are listed in **Table 1**. In this table Ta_1, Ta_2, Ta_3 represent the numerical values corresponding to the first, second and third approximations, while Ta_c is the values obtained by Takhar et al. (1992). This table clearly shows that the values of Ta_c obtained by the third approximation agree very well with the values obtained numerically by Takhar et al. (1992) using the shooting method. When μ is positive or negative i.e., the two cylinders are either co-rotating or counter rotating, the value of Ta_c is observed to decrease with increasing N and hence flow gets destabilized owing to increasing value of N, when the two cylinders are either co-rotating or counter rotating. In the case of co-rotating cylinders, when μ increases (due to increasing the rotational speed of outer cylinder) Ta_c decreases. But in the presence of counter rotating cylinders, when the angular speed of outer cylinder increases as compared to that of inner cylinders, the flow gets more and more stable, because the value of Ta_c is observed to increase by increasing μ.

Table 1: Value of critical Taylor and wave numbers

μ	N	a_i	Ta_1	Ta_2	Ta_3	Ta_c
0.0	1.0	2.977	2066.7246	2052.55	2052.55	2057.5
	0.75	3.005	2296.6501	2281.02	2281.02	2282.8
	0.50	3.037	2583.1528	2560.16	2560.16	2562.7
	0.25	3.077	2949.6938	2933.41	2933.41	2919.6
	0.00	3.127	3434.3928	3412.97	3412.97	3390.0
0.25	1.0	2.929	1525.9376	1526.02	1535.63	1530.9
	0.75	2.962	1718.2650	1716.74	1724.73	1720.5
	0.50	3.003	1965.0648	1961.94	1966.18	1962.8
	0.25	3.053	2292.6948	2288.85	2288.85	2283.1
	0.00	3.120	2747.4587	2738.23	2738.23	2725.3
0.50	1.0	2.887	1167.1129	1166.86	1175.92	1175.2
`	0.75	2.925	1331.6896	1332.27	1339.94	1338.2
	0.50	2.972	1549.2590	1550.63	1555.69	1552.8
	0.25	3.035	1849.6797	1847.75	1846.04	1847.3
	0.00	3.118	2289.5393	2284.67	2384.67	2275.4
0.75	1.0	2.851	918.7109	909.09	909.09	926.3
	0.75	2.892	1061.1198	1061.57	1068.26	1067.9
	0.50	2.944	1254.7781	1254.71	1260.40	1259.5
	0.25	3.016	1532.6458	1533.74	1534.45	1532.7
	0.00	3.118	1962.4639	1960.78	1946.66	1951.5
1.0	1.0	2.820	740.4821	714.29	769.23	746.7
	0.75	2.861	864.6610	833.33	833.33	870.5
	0.50	2.918	1037.9725	1036.91	1041.99	1042.5
	0.25	2.998	1295.8606	1295.34	1297.19	1296.7
	0.00	3.119	1717.1574	1718.31	1714.09	1707.8
-0.25	1.0	3.030	2923.0413	2871.09	2871.09	2866.4
	0.75	3.050	3214.9263	3151.59	3151.59	3148.6
	0.50	3.076	3570.8037	3491.62	3491.62	3491.9
	0.25	3.107	4013.7922	3913.89	3913.89	3918.2
	0.00	3.145	4579.7085	4518.75	4518.75	4461.4
-0.50	1.0	3.080	4352.3574	4168.40	4168.40	4151.0
	0.75	3.103	4794.1382	4564.13	4564.13	4554.3
	0.50	3.129	5334.11621	5063.29	5065.86	5043.2
	0.25	3.161	6009.129883	5659.31	5659.31	5647.8
	0.00	3.199	6875.7700	6406.15	6406.15	6413.8
-0.75	1.0	3.108	6834.3462	6218.91	6222.78	6231.1
	0.75	3.169	7835.2312	6988.12	6993.01	6967
	0.50	3.236	9185.72461	7968.13	7980.85	7880.5
	0.25	3.312	11075.3740	9242.14	9250.69	9032.5
	0.00	3.407	13901.3730	10882.6	10888.5	11361.8
-1.0	1.0	2.989	10638.8349	9115.77	9165.90	9507.0
	0.75	3.276	14869.6768	11348.2	11429.9	11297.3
	0.50	3.567	23751.3926	13020.8	13132.0	13380.5
	0.25	3.808	50519.9844	16460.9	16622.3	15794.5
	0.00	3.999	-1754513792.00	18331.8	18508.2	18662.9

Other interesting phenomenon is to know the behavior of the amplitude of the radial velocity and the corresponding cell-pattern. So for a set of values of a_c and Ta_c, the values of $A_2^{(m)} / A_1^{(m)}, A_3^{(m)} / A_1^{(m)}, A_4^{(m)} / A_1^{(m)}$ are determined from Eq.(4). The eigen-functions thus obtained are normalised so that the amplitude of the radial component of the velocity perturbation is unity. These eigen-functions $u(x)$ and the corresponding cell-patterns for the stream function $\Psi = \mathrm{u}(\mathrm{x})\cos(a_c \mathrm{z})$ at the onset of instability for different values of μ and N are shown in Figs. 1-3.

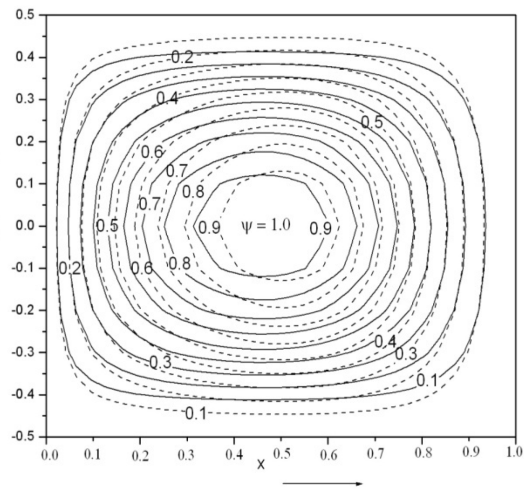

Fig.1. Comparison of the cell patterns at the onset of instability for $N = 0.75$ (shown by continuous curve) and $N = 0.25$ (shown by broken curve) at constant $\mu = 0.75.$.

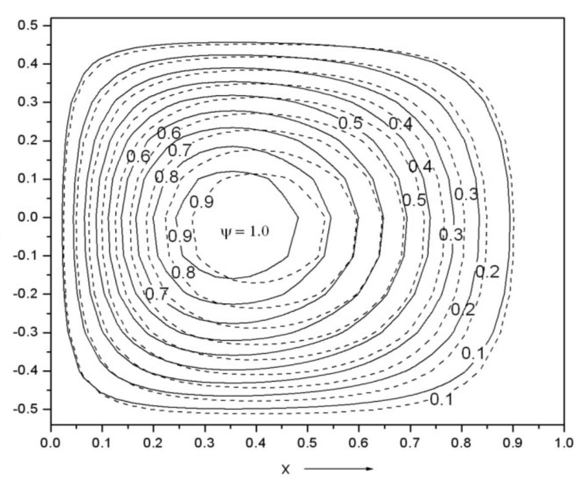

Fig.2. Comparison of the cell patterns at the onset of instability for $N = 0.50$ (shown by broken curve) and $N = 0.75$ (shown by continuous curve) at constant $\mu = -0.25$.

In **Fig. 1** and **2,** the cell patterns are shown for the cases of co-rotating cylinders and counter rotating cylinders respectively. From both figures**,** we found that the cells shift from outer cylinder toward the inner cylinder as N increases. Physically, this is true because the convection currents are flowing from the outer cylinder toward the inner one, and hence the shifting phenomenon is seen from the outer cylinder toward the inner cylinder, when N increases. This also confirms the destabilization of flow as

N increases. From **Fig.2**, we also see that as we increase the value of N the left hand edges of the cells becomes closer toward the inner most cell. Also, the left hand edge of the innermost cell is straightened and the corners are formed at the upper and lower ends of the innermost cell, and if N is further raised, the cells will start breaking through these corners, making flow unstable.

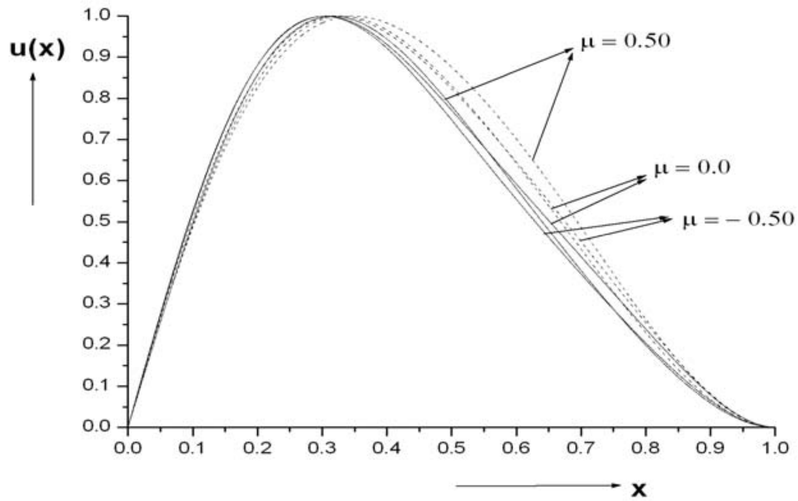

Fig.3. The radial eigen-function u(x) for $N = 0.0$ (shown by continuous curve) and for $N = 0.50$ (shown by broken curve) for different values of μ.

In Fig. 3, the radial eigen-function $u(x)$ is shown when $N = 0.0$ and $N = 0.50$ for different values of μ. We found that, the maximum value of $u(x)$ for $N = 0.0$ is found at $x = 0.3057,\ 0.3164,\ \ 0.3026$ and for $N = 0.50$ is found at $x = 0.3302,\ 0.3479,\ 0.3229$, when $\mu = 0, 0.50, -0.50$ respectively. From these we observe that for $N = 0.0$, due to co-rotation of the two cylinders, the maximum of $u(x)$ shifts toward the outer cylinder as compared to the case of $\mu = 0$, whereas in the presence of counter rotating cylinders the maximum of $u(x)$ shifts toward the inner cylinder, but in the presence of a positive radial temperature gradient, the maximum of $u(x)$ shifts more and more toward the outer cylinder as N increases.

4. Conclusions

The stability of Couette flow of a viscous incompressible fluid between two concentric rotating cylinders is studied in the presence of a radial temperature gradient, when the outer cylinder is maintained at a constant heat flux. The following conclusions have been obtained from the analysis:-

1. The flow is more stable when the two cylinders are counter rotating.
2. The fluid flow gets destabilized owing to increasing N for all value of μ. The destabilizing is greater when the angular speed of outer cylinder (μ is +ve) increases.
3. The flow gets stable, when angular speed of outer cylinder (μ is -ve) increases when N is constant.
4. When the cylinders are counter rotating, the maximum of $u(x)$ shifts toward the inner cylinder as compared to the case, when only inner cylinder is rotating, whereas in the presence of co-rotating cylinders the maximum of $u(x)$ shifts toward the outer cylinder,
5. In the presence of a positive radial temperature gradient, the maximum of $u(x)$ shifts more and more toward the outer cylinder as N increases.

Nomenclature

a	Dimensionless wave number
r	Distance from the axis
d	Difference between two radii of the cylinders

R_1, R_2	Radii of inner and outer cylinders respectively
u, v, w	Velocity components in r, θ and z directions respectively
K	Thermal conductivity
q	Constant heat flux at the outer cylinder
Ra	Rayleigh number
Ta	Taylor number
Pr	Prandtl number
N	Ratio of Rayleigh number and Taylor number (Ra/Ta)
T	Temperature of fluid
T_1	Temperature of inner cylinder

Greek symbols

β	Coefficient of thermal expansion
Ω_1, Ω_2	Angular velocity of the inner and outer cylinders respectively
μ	Ratio of angular velocities Ω_2 / Ω_1
ν	Kinematic viscosity

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