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Combined effects of radiation and chemical reaction on MHD flow past a moving plate with Hall current

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Abstract

Influence of radiation and chemical reaction on MHD flow past a moving plate with Hall current is studied here. Earlier, we (2016) have studied unsteady MHD flow in porous media over exponentially accelerated plate with variable wall temperature and mass transfer along with Hall current. To study further, we are changing the model by considering radiation and chemical reaction on flow, and changing geometry of the model. Now, we are taking the plate positioned vertically upward. Laplace method is used to solve the flow model. The results obtained have been analyzed with the help of graphs drawn for different parameters. The numerical values obtained for the drag, Sherwood and Nusselt numbers at boundary have been tabulated. Here too, the results are found to be in agreement with the actual flow.

Keywords: MHD flow, radiation, chemical reaction.

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1. Introduction

MHD flow problems over an impulsively started vertical plate play important role in many branches of science and technology. The effects of radiation and chemical reaction on MHD flow are also significant in many cases. Some such problems already studied are mentioned here. The Hall effect in the viscous flow of ionized gas between parallel plates under transverse magnetic field was studied by Sato (1961). Mazumder and Deka (2007) have considered MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation. Ibrahim and Makinde (2010) have investigated chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction. Reddy et al (2014) have studied MHD flow considering free convection over a porous plate. Balla and Naikoti (2015) have considered unsteady MHD flow with convective heat and mass diffusion. MHD flow over a stretching surface was analyzed by Jonnadula et al (2015). Malapati and Polarapu (2015) have worked on MHD flow with natural convection. Unsteady MHD flow in porous media was investigated by us (2016). Rajput and Kanaujia (2016) have worked on chemical reaction in MHD flow past a vertical plate with mass diffusion and constant wall temperature with Hall current. The motive of this study is to analyze the combined effects of radiation and chemical reaction on fluid flow over a moving plate in the presence of transversely applied uniform magnetic field and Hall current. The fluid model under consideration has been solved by Laplace transform method. The model contains equations of motion, diffusion equation of energy. To analyze the solution of the model, desirable sets of the values of the parameters have been considered. The numerical data obtained is discussed with the help of graphs and tables.

2. Mathematical Analysis

The physical model is shown in Figure-1



Consider an unsteady MHD flow past an impulsively started vertical plate. The fluid is electrically conducting. The x axis is along the vertical plate and z is perpendicular to it. Thus the z axis lies in the horizontal plane. The uniform magnetic field B_0 is applied perpendicular to the fluid. Initially it has been considered that the plate as well as the fluid is at the same temperature T. The species concentration in the fluid is taken as C. At time t > 0, the plate starts moving with a velocity u_0 . The wall temperature T_w and the concentration C_w in the boundary region are raised in proportion with time. So, under above assumptions, the governing equations are as follows:

Momentum Equation

$$\frac{\partial u}{\partial t} = \hat{\partial} \frac{\partial^2 u}{\partial z^2} + gS(T - T_{\infty}) + gS^*(C - C_{\infty}) - \frac{\dagger B_0^2(u + mv)}{\dots(1 + m^2)}$$
(1)

$$\frac{\partial v}{\partial t} = \hat{D} \frac{\partial^2 v}{\partial z^2} + \frac{\dagger B_0^2 (mu - v)}{\dots (1 + m^2)}$$
(2)

Concentration Equation

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial z^2} - K_c (C - C_{\infty})$$
(3)

Energy Equation

$$\dots C_{p} \frac{\partial T}{\partial t} = k \frac{\partial^{2} T}{\partial z^{2}} - \frac{\partial q_{r}}{\partial z}$$

$$\tag{4}$$

Here K_c is chemical parameter, u and v are the primary and secondary velocities along x and z directions respectively. Other symbols used have the following description:

C- species concentration in the fluid, m -the Hall current parameter, *T*-temperature of the fluid, T_w - temperature of the plate at z = 0, C_w -species concentration at the plate z = 0, B_0 - the uniform magnetic field, - electrical conductivity. * - volumetric

coefficient of concentration expansion, ν the kinematic viscosity, - the density, C_p - the specific heat at constant pressure, g-the

acceleration due to gravity, -volumetric coefficient of thermal expansion, *t*-time, *k*- thermal conductivity of the fluid, *D*-the mass diffusion coefficient.

The boundary conditions taken are as under:

$$t \le 0: u = v = 0, \ T = T_{\infty}, \ C = C_{\infty}, \ \text{for every } z.$$

$$t > 0: u = u_0, \ v = 0, \ T = T + (T_w - T_{\infty})A, \ C = C_{\infty} + (C_w - C_{\infty})A, \ \text{at } z=0.$$

$$\left.\right\}$$
(5)

$$u \to 0, v \to 0, T \to T_{\infty}, C \to C_{\infty}$$
 as $z \to \infty$

By using Rosseland approximation (Brewster (1992)), the radiative heat flux q_r is given by

$$\frac{\partial q_r}{\partial z} = -4a^* \dagger \left(T_{\infty}^4 - T^4 \right) \tag{6}$$

where a^* is absorption constant.

The temperature difference in the flow is considered sufficiently small, therefore T^4 can be approximated by Taylor series. Hence $T^4 \cong 4T_{\infty}^3 T - 3T_{\infty}^4$ (7)

Energy equation (4) is transformed under equations (6) and (7), which is as below:

$$\dots C_{p} \frac{\partial T}{\partial t} = k \frac{\partial^{2} T}{\partial z^{2}} - 16a^{*} \dagger T_{\infty}^{3} (T - T_{\infty})$$
(8)

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To transform equations (1), (2), (3) and (8) into dimensionless form, we use the following quantities:

$$\overline{z} = \frac{zu_0}{u_0}, \ \overline{u} = \frac{u}{u_0}, \ \overline{v} = \frac{v}{u_0}, \quad = \frac{(T - T_{\infty})}{(T_w - T_{\infty})}, \ S_c = \frac{1}{D}, \ \mu = -\tau, \ \overline{C} = \frac{(C - C_{\infty})}{(C_w - C_{\infty})}, \ R = \frac{16a^{*-2} \dagger T_{\infty}^3}{ku_0} \\
G_r = \frac{gS^{*}(T_w - T_{\infty})}{u_0^{3}}, \ M = \frac{B_0^2}{u_0^2}, \ G_m = \frac{g^{**}(C_w - C_{\infty})}{u_0^{3}}, \ K_0 = \frac{K_c}{u_0^2} P_r = \frac{\mu u_0}{k}, \ \overline{t} = \frac{tu_0^2}{k}.$$
(9)

The symbols in dimensionless form are as under:

 K_0 - chemical reaction, R - Radiation parameter, \overline{u} - primary velocity, \overline{v} - secondary velocity, P_r - Prandtl number, S_c - Schmidt number, \overline{t} - time, - temperature, \overline{C} - concentration, G_r - thermal Grashof number, G_m - mass Grashof number, μ - coefficient of viscosity, M- magnetic parameter.

The flow model in dimensionless form is

$$\frac{\partial \overline{u}}{\partial \overline{t}} = \frac{\partial^2 \overline{u}}{\partial \overline{z}^2} + G_r + G_m \overline{C} - \frac{M(\overline{u} + m\overline{v})}{(1 + m^2)}$$
(10)

$$\frac{\partial \overline{v}}{\partial \overline{t}} = \frac{\partial^2 \overline{u}}{\partial \overline{z}^2} + \frac{M(m\overline{u} - \overline{v})}{(1+m^2)}$$
(11)

$$\frac{\partial \overline{C}}{\partial \overline{z}} = \frac{1}{c} \frac{\partial^2 \overline{C}}{\partial \overline{z}^2} - K_0 \overline{C}$$
(12)

$$\frac{\partial t}{\partial \bar{t}} = \frac{1}{P_r} \frac{\partial^2 t}{\partial \bar{z}^2} - \frac{R_t}{P_r}$$
(13)

The boundary conditions (5) become:

$$\overline{t} \leq 0: \overline{u} = \overline{v} = = \overline{C} = 0, \text{ for every } \overline{z} .$$

$$\overline{t} > 0: \overline{u} = 1, \ \overline{v} = 0, = \overline{t}, \ \overline{C} = \overline{t}, \text{ at } \overline{z} = 0.$$

$$\overline{u} \to 0, \ \overline{v} \to 0, \quad \to 0, \ \overline{C} \to 0, \text{ as } \overline{z} \to \infty.$$

$$(14)$$

Removing bars, for simplicity, we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial z^2} + G_m C - \frac{M(u+mv)}{(1+m^2)}$$
(15)

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial z^2} + \frac{M(mu - v)}{(1 + m^2)}$$
(16)

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2} - K_0 C \tag{17}$$

$$\frac{\partial_{"}}{\partial t} = \frac{1}{P_{r}} \frac{\partial^{2}_{"}}{\partial z^{2}} - \frac{R_{"}}{P_{r}}$$
(18)

$$t \le 0: u = v = C = 0, \text{ for every } z.$$

$$t > 0: u = 1, v = 0, = t, C = t, \text{ at } z=0.$$

$$u \to 0, v \to 0, \longrightarrow 0, C \to 0, \text{ as } z \to \infty.$$

$$(19)$$

Writing the equations (15) and (16) in combined form (using q = u + i v)

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2} + G_r + G_m C - qa$$
⁽²⁰⁾

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2} - K_0 C \tag{21}$$

$$\frac{\partial_{n}}{\partial t} = \frac{1}{P_{r}} \frac{\partial^{2}_{n}}{\partial z^{2}} - \frac{R_{n}}{P_{r}}$$
(22)

The boundary conditions (19) are reduced to:

$$t \le 0: q = = C = 0, \text{ for every z.}$$

$$t > 0: q = 1, = t, C = t, \text{ at } z=0.$$

$$q \to 0, \quad \to 0, \quad C \to 0, \quad \text{as } z \to \infty.$$

$$(23)$$

The equations (20) to (22), using equation (23), are solved by the Laplace method. The solution obtained is as under:

$$\begin{split} q &= \frac{1}{2} exp(-\sqrt{a}z)A_{33} + \frac{G_r}{4(a-R)^2} ((exp(-\sqrt{a}z)(2RtA_1 - 2atA_1 + z\sqrt{a}A_2 + 2A_1(P_r+1)) - \frac{A_2z}{\sqrt{a}}) - \frac{2A_5P_rz}{\sqrt{A_{32}}A_{11}} (at - Rt + P_r-1) + \frac{2A_{28}A_6P_rz}{A_{11}} (P_r-1) - 2A_{26}A_3(P_r-1) - \frac{P_rz\sqrt{A_{32}P_r}}{A_{10}f\sqrt{R}} (\frac{1}{a} - \frac{1}{R})) + \frac{G_m}{4(a-K_0S_c)^2} ((exp(-\sqrt{a}z)(z\sqrt{a}A_2 + 2A_1(S_1 - 1)) + 2A_1(S_1 - 1)) - 2A_2(S_1 - 2A_1(S_1 - 1)) + 2A_1(S_1 - 1)) + \frac{2A_2P_r}{A_{11}} (2\sqrt{a}A_2 + 2A_1(S_1 - 1)) + \frac{G_m}{4(a-K_0S_1 - 2)^2} ((exp(-\sqrt{a}z)(z\sqrt{a}A_2 + 2A_1(S_1 - 1))) + 2A_2(S_1 - 2A_1(S_1 - 1)) + 2A_1(S_1 - 2A_1(S_1 - 2A_1(S_1 - 1))) + 2A_1(S_1 - 2$$

$$C = \frac{e^{-z\sqrt{S_cK_0}}}{4\sqrt{K_0}} \{ erfc[\frac{z\sqrt{S_c} - 2t\sqrt{K_0}}{2\sqrt{t}}](-z\sqrt{S_c} + 2t\sqrt{K_0}) + e^{2z\sqrt{S_cK_0}}erfc[\frac{z\sqrt{S_c} + 2t\sqrt{K_0}}{2\sqrt{t}}](z\sqrt{S_c} + 2t\sqrt{K_0}) \}.$$

The expressions for the symbols involved in the above solution are given in the appendix.

3. Skin Friction

The dimensionless skin-friction at the surface is

$$\left(\frac{dq}{dz}\right)_{z=0} = \ddagger_x + i\ddagger_y \cdot$$

The numerical values of τ_x and τ_y , for different parameters, are given in table-1.

4. Nusselt number

The dimensionless Nusselt number is given by

$$Nu = \left(\frac{\partial_{u}}{\partial z}\right)_{z=0} = erfc[\frac{\sqrt{Rt}}{\sqrt{tP_{r}}}](\sqrt{Rt} - \frac{\sqrt{R}}{2}t + \frac{P_{r}}{4\sqrt{R}}) - erfc[-\frac{\sqrt{Rt}}{\sqrt{tP_{r}}}](\frac{\sqrt{R}}{2}t + \frac{P_{r}}{4\sqrt{R}}) - \frac{e^{-\frac{P_{r}}{P_{r}}}\sqrt{tP_{r}}}{\sqrt{f}}.$$

5. Sherwood number

The dimensionless Sherwood number at the surface is

$$S_{h} = \left(\frac{\partial C}{\partial z}\right)_{z=0} = erfc[-\sqrt{tK_{0}}](-\frac{1}{4\sqrt{K_{0}}}\sqrt{S_{c}} - \frac{t\sqrt{S_{c}K_{0}}}{2}) + \sqrt{S_{c}}erfc[\sqrt{tK_{0}}](\frac{1}{4\sqrt{K_{0}}} + t\sqrt{K_{0}}) - \frac{e^{-tK_{0}}\sqrt{tS_{c}K_{0}}}{\sqrt{tK_{0}}}) + \frac{1}{4\sqrt{K_{0}}}\sqrt{tS_{c}K_{0}} + t\sqrt{K_{0}} + t\sqrt{K_{0}}$$

Rt

6. Results and discussion

The study is carried out to examine the effects of radiation with chemical reaction on unsteady MHD flow past a moving plate with variable wall temperature and mass diffusion in the presence of Hall current. The behavior of other parameters like magnetic parameter, Hall current and thermal buoyancy is almost similar to the earlier model studied by us (2016). The analytical results are shown in figures 2 to 7. The numerical values of skin-friction, Sherwood number and Nusselt number are presented in Table-1, 2 and 3, respectively. Chemical reaction effect on fluid flow behavior is shown by figures 2 and 3. It is seen here, when chemical reaction parameter K_0 increases, *u* and *v* decrease throughout near the surface. Figures 4 and 5 indicate that effect of radiation in the flow near the plate tends to accelerate velocities. This is due to the fact that the large values of radiation parameter tend to accelerate velocity of the fluid in the region near the surface of the plate. Further, it is noticed that the temperature and concentration of the fluid near the plate decrease when radiation and chemical reaction parameters are increased (figures 6 and 7).

Skin friction is given in table 1. The values of Skin friction x and y increase with the increase in R and decrease with K_o . Sherwood number is given in table 2. The value of S_h decreases with the increase in K_o , S_c and t. Nusselt number is given in table 3. The value of Nu decreases with increase in P_r , R and t.







Figure 6: θ vs z



Figure	7:	С	vs	Ζ
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Table 1	l : Skin-f	riction for	different	parameters
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М	т	Pr	Sc	Gm	Gr	R	K_0	t		
2	1	0.71	2.01	100	10	04	01	0.4	01215.7783	01284.2268
2	1	0.71	2.01	100	10	06	01	0.4	06052.4816	09261.1973
2	1	0.71	2.01	100	10	08	01	0.4	41596.3813	80992.4326
2	1	0.71	2.01	100	10	04	05	0.4	01214.7722	01248.1817
2	1	0.71	2.01	100	10	04	10	0.4	01213.9525	01248.1458

Table2. Sherwood number for different Parameters.

K ₀	Sc	Т	S_h
01	2.01	0.2	-0.762200
05	2.01	0.2	-0.933049
10	2.01	0.2	-1.118240

01	3.00	0.2	-0.931175
01	4.00	0.2	-1.075230
01	2.01	0.3	-0.961323
01	2.01	0.4	-1.141570

Table 3: Nusselt number for different parameters

Pr	R	t	Nu
0.71	2	0.4	-0.805273
7.00	2	0.4	-1.959260
0.71	3	0.4	-0.894014
0.71	4	0.4	-0.976083
0.71	2	0.5	-0.950956
0.71	2	0.6	-1.094940

7. Conclusion

The results obtained are in agreement with the usual flow. It is observed that the velocities near the plate surface are increased with radiation parameter, and decrease with chemical reaction. Further, due to the radiation the drag at the plate surface increases. However the drag decreases with chemical reaction. Also the values of S_h and N_u decrease with K_0 and R. It is as per expected flow behavior.

Appendix:

$$\begin{split} a &= \frac{M\left(1-im\right)}{1+m^2}, \ A = \frac{u_0^2 t}{1+m^2}, \ A_1 = (1+A_{12}+e^{2\sqrt{az}}(1-A_{13})), \ A_2 = (1+A_{12}-e^{2\sqrt{az}}(1-A_{13})), \\ A_3 &= (A_{14}-1+A_{29}(A_{15}-1)), \ A_4 = (A_{16}-1+A_{30}(A_{17}-1)), \ A_5 = (A_{18}-1+A_{33}(A_{19}-1)), \\ A_6 &= (A_{20}-1+A_{31}(A_{21}-1)), \ A_7 = (e^{2z\sqrt{K_0S_c}}(A_{23}-1)-A_{22}-1)), \ A_8 = (e^{2z\sqrt{K_0S_c}}(A_{23}-1)+A_{22}+1)), \\ A_6 &= (A_{30}(A_{25}-1)-A_{24}-1), \ A_{10} = (1-A_{18}+A_{32}(A_{19}-1)), \ A_{11} = Abs[z]Abs[P_r], \ A_{12} = erf[\frac{1}{2\sqrt{t}}(2\sqrt{at}-z)], \\ A_{13} &= erf[\frac{1}{2\sqrt{t}}(2\sqrt{at}+z)], \ A_{14} = erf[\frac{1}{2\sqrt{t}}(z-2t\sqrt{\frac{aP_r-R}{P_r-1}})], \ A_{15} = erf[\frac{1}{2\sqrt{t}}(z+2t\sqrt{\frac{aP_r-R}{P_r-1}})], \\ A_{16} &= erf[\frac{1}{2\sqrt{t}}(z-2t\sqrt{\frac{(a-K_0)S_c}{S_c-1}})], \ A_{17} = erf[\frac{1}{2\sqrt{t}}(z+2t\sqrt{\frac{(a-K_0)S_c}{S_c-1}})], \ A_{18} = erf[\frac{A_{11}}{2\sqrt{t}} - \sqrt{\frac{tR}{P_r}}], \\ A_{19} &= erf[\frac{A_{11}}{2\sqrt{t}} + \sqrt{\frac{R}{P_r}}], \ A_{20} = erf[\frac{A_{11}}{2\sqrt{t}} - \sqrt{\frac{(R-aP_r)t}{P_r-P_r^2}})], \ A_{21} = erf[\frac{A_{11}}{2\sqrt{t}} + \sqrt{\frac{(R-aP_r)t}{P_r-P_r^2}})], \\ A_{22} &= erf[\frac{1}{2\sqrt{t}}(2t\sqrt{K_0} - z\sqrt{S_c})], \ A_{23} = erf[\frac{1}{2\sqrt{t}}(2t\sqrt{K_0} + z\sqrt{S_c})], \ A_{24} = erf[\frac{1}{2\sqrt{t}}(2t\sqrt{\frac{(a-K_0)S_c}{S_c-1}} - z\sqrt{\frac{S_c}{S_c-1}})], \\ A_{25} &= erf[\frac{1}{2\sqrt{t}}(2t\sqrt{\frac{(a-K_0)S_c}{S_c-1}} + z\sqrt{S_c})], \ A_{28} &= \frac{1}{A_{31}}exp(\frac{at}{P_r-1} - \frac{Rt}{P_r-1}), \ A_{29} &= exp(2z\sqrt{\frac{-R+aP_r}{P_r-1}}), \\ A_{30} &= exp(2z\sqrt{\frac{(a-K_0)S_c}{S_c-1}}), \end{aligned}$$

$$\begin{split} A_{31} &= \exp(2Abs[z] \sqrt{\frac{P_r(aP_r - R)}{P_r - 1}}), \ A_{32} = \exp(2Abs[z] \sqrt{P_r R}), \ A_{33} = 1 + A_{34} + \exp(2\sqrt{az})A_{35}, \\ A_{34} &= erf[\frac{1}{2\sqrt{t}}(2\sqrt{at} - z)], \ A_{35} = erfc[\frac{1}{2\sqrt{t}}(2\sqrt{at} + z)], \end{split}$$

References

- Balla C. S. and Naikoti K., 2015. Radiation effects on unsteady MHD convective heat and mass transfer past a vertical plate with chemical reaction and viscous dissipation. Alexandria Engineering Journal, Vol. 54, pp 661–671.
- Brewster M.Q., 1992. Thermal Radiative Transfer and Properties. John Wiley & Sons, New York.
- Ibrahim S. Y., and Makinde O. D., 2010. Chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction. Scientific Research and Essays, Vol. 5, No. 19, pp. 2875-2882.
- Jonnadula M., Polarapu P., Reddy M. G. and Venakateswarlu M., 2015. Influence of thermal radiation and chemical reaction on MHD flow, heat and mass transfer over a stretching surface. International Conference on Computational Heat and Mass Transfer, Procedia Engineering (Elsevier), Vol. 127, pp. 1315 1322.
- Malapati V. and Polarapu P., 2015. Unsteady MHD free convection heat and mass transfer in a boundary layer flow past a vertical permeable plate with thermal radiation and chemical reaction. Procedia Engineering, 127, pp 791 799.
- Mazumder M. and Deka R. K., 2007. MHD flow past an impulsively started infinite vertical plate in presence of thermal radiation, Romanian Journal of Physics, Vol. 52, No. (5-6), pp 529-535.
- Rajput U.S. and Kumar G., 2016. Unsteady MHD flow in porous media past over exponentially accelerated inclined plate with variable wall temperature and mass transfer along with Hall current. International Journal of Engineering, Science and Technology, Vol. 8, No. 2, pp. 01-10.
- Rajput U. S. and Kanaujia N., 2016. Chemical reaction in MHD flow past a vertical plate with mass diffusion and constant wall temperature with hall current. International Journal of Engineering, Science and Technology Vol. 8, No. 4, pp. 28-38.
- Reddy N. A, Raju M.C. and Varma S.V.K., 2014. Analytical study of MHD free convective, dissipative boundary layer flow past a porous vertical surface in the presence of thermal radiation, chemical reaction and constant suction. Ain Shams Engineering Journal, Vol. 5, pp 1361–1369.
- Sato H., 1961. The Hall effect in the viscous flow of ionized gas between parallel plates under transverse magnetic field. Journal of the Physical Society of Japan, Vol. 16, No. 7, pp 1427–1433.

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