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Retrospective Study of the Impact of Sensitization on COVID-19 Pandemic in Rivers State, Nigeria

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Abstract

This study examines the impact of sensitization on COVID-19 dynamics in Rivers State, Nigeria, utilizing a mathematical model and data from the Nigeria Centre for Disease Control (NCDC). Parameter estimation involves meticulous fitting, refining key parameters like β_c, c_m, α_1, α_2, α_3, α_4, and the reproduction number $\mathcal{R}_c$. The study employs a genetic algorithm for precise parameter estimation, ensuring the model aligns closely with observed COVID-19 data. Estimated values for β_c, c_m, α_1, α_2, α_3, α_4, and $\mathcal{R}_c$ provide a robust foundation for accurate simulations, enhancing the reliability of the model and facilitating a deeper understanding of the population dynamics of COVID-19 in human population. Uncertainty and sensitivity analyses highlight crucial parameters, emphasizing the relative infectiousness of asymptomatic individuals (η_s), face mask efficacy (ϵ_m), and sensitization effectiveness (ϵ_s, c_m). Numerical simulations reveal that a combined strategy of sensitization and face mask use can significantly curtail the disease progression. Targeting susceptible and exposed individuals in sensitization efforts proves most beneficial, aligning with sensitivity analysis results. Notably, the combination of sensitization and face mask use results in a remarkable 98% reduction in cumulative cases. Sensitization emphasizing various preventive measures, when doubled, shows a 99% reduction. These findings suggest that a comprehensive sensitization approach can profoundly impact COVID-19 control. Policymakers can leverage these insights to optimize sensitization programs, emphasizing the role of preventive measures beyond face mask use, ultimately guiding effective public health strategies in Rivers State and beyond.

Keywords: Covid-19, face mask, Reproduction number, Sensitization.
MSC2010: 35Q62.

1 Introduction

The COVID-19 outbreak, caused by the highly contagious Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2), resulted in 12,322,395 confirmed cases as of July 11, 2020 [1]. Starting in China in December 2019, the virus quickly spread worldwide, leading the World Health

Organization (WHO) to declare it a Public Health Emergency of International Concern on January 30, 2020 [2]. COVID-19 is transmitted through direct contact with contaminated surfaces and by inhaling respiratory droplets from infected individuals [3, 4]. To minimize its spread, various measures were promoted, including strict social (physical) distancing, community lockdowns, contact tracing, quarantine of suspected cases, isolation of confirmed cases, and the use of face masks in public [5–7].

In recent years, mathematical modeling has become a crucial tool for studying the dynamics of infectious diseases and developing strategies for control [8–14]. As a result, numerous mathematical models have been created and utilized to comprehend the mechanisms behind the spread and control of COVID-19 in different populations. For instance, the authors in [15] developed one of the earliest mathematical models to investigate the transmission dynamics of COVID-19. Using an SEIR model, the authors examined the effects of individual and government reactions, as well as a time-varying reporting rate, on the disease dynamics in Wuhan, China. Another study [16] formulated an epidemic model to analyze the outbreak of COVID-19 in Mexico. They assessed the theoretical impact of potential control measures like home quarantine, social distancing, cautious behavior, environmental cleaning, disinfection, government-imposed isolation of infected individuals, and other self-imposed measures. Results from their work suggested that social distancing and quarantine are effective controls, especially when using a Bayesian approach. The work in [5] employed a multi-group disease model to investigate the impact of mask usage in public places on reducing the spread of COVID-19 in the United States. Their findings indicated that the general public's use of face masks has a high potential for curbing community transmission and reducing the overall burden of the pandemic. Moreover, combining face masks with high public compliance and other non-pharmaceutical interventions was found to have a greater impact on reducing the disease's burden.

In another study, the study in [7] conducted a study focusing on evaluating the community-wide impact of non-pharmaceutical interventions (NPIs), such as quarantine, isolation, contact tracing, social distancing, and the use of face masks, on the burden of COVID-19. They used data from the United States for their assessment. In a different study, the authors in [6] developed a mathematical model to explore whether the use of an imperfect vaccine could lead to the elimination of COVID-19 in the United States. Their findings suggested that disease elimination is feasible if the vaccine coverage is high enough to achieve herd immunity. Specifically, they determined that a vaccine coverage of 90%, assuming a vaccine efficacy of 80%, is required to reach herd immunity in the US. The authors in [4] formulated a mathematical model for COVID-19 population dynamics in Lagos, Nigeria, using data specific to the region. Through numerical simulations, they demonstrated that if at least 55% of the population complies with social distancing and face mask use, the disease will eventually die out. Additionally, an increase in the case detection rate for symptomatic individuals to about 0.8 per day, coupled with 55% compliance with social distancing regulations, resulted in a reduction in the incidence (and prevalence) of COVID-19.

In other related studies, the authors in [17] investigated how the dynamics of the virus in a community were affected by inadequate adoption of preventive measures and insufficient knowledge of the virus's mode of transmission. Furthermore, the authors in [18] used mathematical models to explore the effects of non-pharmaceutical treatments on the dynamics of infection through transmission by symptomatic and asymptomatic infected patients [18]. Additionally, a mathematical model integrating age-specific COVID-19 transmission dynamics was developed by [19] to assess the effectiveness of treatment and immunization approaches in lowering the COVID-19 burden.

Nigeria reported its first case of COVID-19 on February 27, 2020, while Rivers State, situated in the southern Niger Delta region, recorded its first case on March 25, 2020 [1]. As of June 24, 2020, Nigeria had reported 22,020 cases of COVID-19, with 542 deaths, 13,865 active cases undergoing treatment, and 7,613 discharged cases [1]. Specifically, Rivers State had reported 930 confirmed cases, with 35 deaths, 441 active cases, and 454 discharged cases as of the same date [1]. Despite being one of the early epicenters of the disease in the southern part of Nigeria, Rivers State implemented preventive measures like border closure and early use of face masks in public during March and May, respectively. However, by June 24, the state had recorded about 930 confirmed

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cases. Despite these preventive measures, Rivers State experienced a surge in COVID-19 cases, prompting a critical question: why did cumulative cases increase despite early interventions? This study addresses this gap by specifically exploring the impact of sensitization on disease dynamics in Rivers State. Understanding the pivotal role of public compliance and awareness is crucial for refining and optimizing future interventions. This emphasizes the need to bridge the gap between implemented measures and public response in the ongoing fight against the pandemic.

Having sufficient knowledge on COVID-19 is crucial to encourage individuals to make decisions that can prevent and mitigate the pandemic. Understanding infection pathways and taking relevant precautions, such as regular hand washing, using hand sanitizers, wearing face masks, practicing respiratory etiquette, maintaining social distancing, and self-isolating when sick, plays a vital role in reducing widespread infection [20]. Research has shown that an individual's knowledge about an infectious disease can influence their behavior in ways that prevent infection. Therefore, it is essential to inform individuals about the potential risks of infections to encourage the adoption of the right precautionary measures [20]. It is noteworthy that in Nigeria, a significant number of people still consider the coronavirus disease as a scam, orchestrated by political leaders to embezzle public funds. This perception may contribute to the low compliance with some of the measures proposed by the government to curb the spread of the disease. Consequently, there is a pressing need to assess the impact of sensitization and awareness programs to address these misconceptions and improve public adherence to preventive measures.

Evaluating the impact of sensitization is crucial, given its central role in bridging the gap between implemented preventive measures and their reception by the public. Public compliance plays a vital role in pandemic control, and understanding how sensitization influences behavior is key. Dispelling misinformation, especially debunking the notion that COVID-19 is a scam, becomes essential for building trust and promoting adherence to health guidelines. This study illuminates the effectiveness of awareness campaigns in shaping behavior, offering valuable insights for future strategies to optimize sensitization efforts. The knowledge gained aims to inform authorities on tailoring sensitization to resonate with the community, ultimately enhancing pandemic control. By unraveling the intricate relationship between sensitization and public compliance, this research aspires to provide actionable data for authorities, empowering them to navigate and mitigate the challenges posed by the ongoing pandemic.

The widespread impact of COVID-19 has encouraged extensive research into its dynamics and control strategies. Mathematical modeling has emerged as a crucial tool for comprehending disease spread and devising effective interventions [1, 6, 9, 10, 20]. Numerous models globally have delved into factors influencing transmission and the effectiveness of control measures [2,11]. In this context, our study focuses on Rivers State, Nigeria, probing into its unique dynamics and evaluating the impact of awareness and sensitization on the spread of COVID-19.

2 Model Formulation

Our mathematical model divides the entire human population into distinct compartments, effectively capturing the dynamics of infection, sensitization, and behavioral changes. The interactions between unsensitized and sensitized individuals, coupled with the progression through various disease stages, are systematically represented by a set of differential equations (refer to model (2.1)). This framework enables us to delve into the influence of awareness campaigns on epidemic control, offering valuable insights for shaping future interventions.

The total human population at time t, denoted by $N(t)$, is divided into the mutually exclusive compartments of unsensitized susceptible individuals ($S_1(t)$), sensitized susceptible individuals ($S_2(t)$), unsensitized exposed individuals ($E_1(t)$), sensitized exposed individuals ($E_2(t)$) unsensitized asymptomatic individuals ($A_1(t)$), sensitized asymptomatic individuals ($A_2(t)$), unsensitized symptomatic individuals($I_1(t)$), sensitized symptomatic individuals ($I_2(t)$) hospitalized individuals ($H(t)$) and recovered individuals ($R(t)$) individuals. Such that,

$$N(t) = S_1(t) + S_2(t) + E_1(t) + E_2(t) + A_1(t) + A_2(t) + I_1(t) + I_2(t) + H(t) + R(t)$$

The population of unsensitized susceptible is depleted by being infected and by means of sensitization (α_1), while the population of the sensitized susceptible is depleted by being infected too though at a reduced rate if the sensitization programs result in behavioral change. Again, the population of unsensitized exposed is increased by the infection of unsensitized susceptible and diminished by sensitization (α_2) and progression (γ_1). In the same vein, the population of sensitized exposed individuals is generated by the infection of sensitized susceptibles and sensitization of unsensitized exposed. It is however decreased by disease progression (γ_2). In both cases of the exposed class, while a fraction progress to the asymptomatic class, the remaining fraction progress to the symptomatic class. Furthermore, the population of unsensitized asymptomatic individuals is increased by progression of unsensitized exposed and decreased by sensitization (α_3) and recovery. Again, the population of sensitized asymptomatic individuals is also increased by progression of sensitized exposed individuals as well as sensitization of unsensitized asymptomatic individuals and diminished by recovery. In addition, the population of unsensitized symptomatic individuals is increased by progression of unsensitized exposed and decreased by sensitization (α_4), recovery, death and hospitalization (σ_1). Again, the population of sensitized symptomatic individuals is also increased by progression of sensitized exposed individuals as well as sensitization of unsensitized symptomatic individuals and reduced by hospitalization (σ_2), death and recovery. It is important to state that we have assumed that $\sigma_2 > \sigma_1$ since the sensitized individuals are more likely to seek medical attention given that they are familiar with the symptoms of the infection. The population of hospitalized individuals is generated by the hospitalization of the sensitized and unsensitized symptomatic individuals and reduced by recovery and death. Finally, the population of the recovered class is generated by the recovery of individuals in the A_1, A_2, I_1, I_2 and H classes.Thus, the model for the transmission dynamics of COVID-19 in a population is given by the following system of deterministic non-linear differential equations in (2.1), with Table 1 describing the associated state variables and parameters in the model (2.1) while Figure **??** gives the flow diagram of model (2.1).

$$
\begin{aligned}
\frac{dS_1}{dt} &= -\lambda_c S_1 - \alpha_1 S_1, \\
\frac{dS_2}{dt} &= \alpha_1 S_1 - \epsilon_s \lambda_c S_2, \\
\frac{dE_1}{dt} &= \lambda_c S_1 - (\gamma_1 + \alpha_2) E_1, \\
\frac{dE_2}{dt} &= \alpha_2 E_1 + \epsilon_s \lambda_c S_2 - \gamma_2 E_2, \\
\frac{dA_1}{dt} &= (1-f)\gamma_1 E_1 - (\phi_{a1} + \alpha_3) A_1, \\
\frac{dA_2}{dt} &= (1-q)\gamma_2 E_2 + \alpha_3 A_1 - \phi_{a2} A_2, \\
\frac{dI_1}{dt} &= f\gamma_1 E_1 - (\sigma_1 + \phi_{s1} + \delta_1 + \alpha_4) I_1, \\
\frac{I_2}{dt} &= q\gamma_2 E_2 + \alpha_1 I_1 - (\sigma_2 + \phi_{s2} + \delta_2) I_2, \\
\frac{H}{dt} &= \sigma_1 I_1 + \sigma_2 I_2 - (\phi_h + \delta_3) H, \\
\frac{dR}{dt} &= \phi_{a1} A_1 + \phi_{a2} A_2 + \phi_{s1} I_1 + \phi_{s2} I_2 + \phi_h H,
\end{aligned}
\tag{2.1}
$$

where

$$
\lambda_c = (1 - c_m \epsilon_m)\beta_c \frac{(I_1 + \rho_s I_2 + \eta_s (A_1 + \rho_s A_2))}{N_h - H} \tag{2.2}
$$

is the force of infection.

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It is important to state that we have assumed that individuals in the S_2, E_2, A_2 and I_2 compartments do not join the S_1, E_1, A_1 and I_1 compartments respectively due to the fact that awareness programs are continuous in the face of the ongoing pandemic. So, the public is continuously reminded of the need to comply to safety measures.

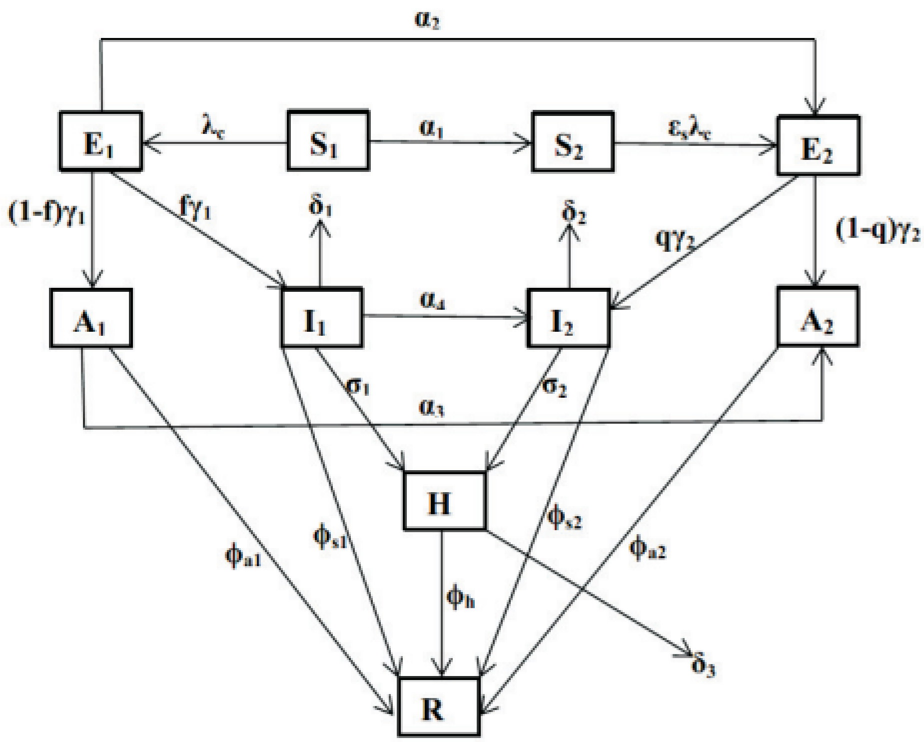

Figure 1: Schematic diagram of the model (1)

3 Analysis of the model

3.1 Basic properties of the model

The model (2.1) is biologically meaningful, if all its state variables are non-negative for all time (t) > 0 and that the region $\mathcal{C}$, defined below, is indeed bounded.

$$\mathcal{C} = \left\{(S_1, S_2, E_1, E_2, A_1, A_2, I_1, I_2, H, R) \in \mathbb{R}^{10}_+ : N \leq N(0)\right\}.$$

We claim the following:

Theorem 3.1. *Let the initial data for the model (2.1) be $S_1(0) \geq 0$, $S_2(0) \geq 0$, $E_1(0) \geq 0$, $E_2(0) \geq 0$, $A_1(0) \geq 0$, $A_2(0) \geq 0$, $I_1(0) \geq 0$, $I_2(0) \geq 0$, $H(0) \geq 0$, $R(0) \geq 0$. Then the solutions $(S_1(t), S_2(t), E_1(t), E_2(t), A_1(t), A_2(t), I_1(t), I_2(t), H(t), R(t))$ of the model (2.1) are positive for all time $t > 0$.*

Variable	Interpretation
S_1	Population of unsensitized susceptible individuals
S_2	Population of sensitized susceptible individuals
E_1	Population of unsensitized exposed individuals
E_2	Population of sensitized exposed individuals
A_1	Population of unsensitized asymptomatic individuals
A_2	Population of sensitized asymptomatic individuals
I_1	Population of unsensitized symptomatic individuals
I_2	Population of sensitized symptomatic individuals
H	Population of hospitalised individuals
R	Recovered individuals
Parameter	**Interpretation**
β_c	Effective contact rate
ϵ_m	Efficacy of face mask
c_m	Face mask usage (compliance)
ϵ_s	Efficacy of sensitization program
γ_1 (γ_2)	Progression rate for E_1 and E_2 classes respectively
σ_1 (σ_2)	Hospitalization rate for I_1 and I_2 classes respectively
$\phi_{a1}(\phi_{a2})(\phi_{s1})(\phi_{s2})(\phi_h)$	Recovery rate for $A_1(A_2)(I_1)(I_2)(H)$ classes respectively
$\delta_1(\delta_2)(\delta_3)$	Disease induced death rate for $I_1(I_2)(H)$ classes respectively
$f(q)$	fraction of unsensitized (sensitized) exposed humans who show symptoms
η_s	Modification parameter for reduced infectiousness of asymptomatic individuals
ρ_s	Modification parameter for reduced infectiousness of sensitized individuals
α_1,α_2, α_3, α_4	Sensitization rate for S_1, E_1, A_1 and I_1 respectively

Table 1: Description of variables and parameters in the model (2.1).

Proof. Let

$$t_1 = \sup\{t > 0 : S_1 > 0, S_2 > 0, E_1 > 0, E_2 > 0, A_1 > 0, A_2 > 0, \\ I_1 > 0, I_2 > 0, H > 0, R > 0 \in [0, t]\}$$

The first equation in model (2.1),

$$\frac{dS_1}{dt} = -(\lambda_c + \alpha_1)S_1 \quad \text{can be written as}$$

$$\int \frac{dS_1}{S_1} = -\int (\lambda_c + \alpha_1)dt.$$

Then,

$$S_1(t_1) = S_1(0) \exp\left[-\alpha_1 t_1 - \int_0^{t_1} \lambda_c(\tau)d\tau\right] > 0.$$

Using the same approach, we can equally show that the other state variables will remain positive for all time $t > 0$. □

Lemma 3.2. *Consider the region*

$$\mathcal{C} = \left\{(S_1, S_2, E_1, E_2, A_1, A_2, I_1, I_2, H, R) \in \mathbb{R}^{10}_+ : N \leq N(0)\right\}.$$

The closed set $\mathcal{C}$ is positively invariant and a global attractor of all positive solution of the model (2.1).

Proof. Adding the equations of the model (2.1) gives

$$\dot{N} = -\delta_1 I_1 - \delta_2 I_2 - \delta_3 H. \quad \text{With} \quad \delta_m = min(\delta_1, \delta_2, \delta_3)$$
$$\dot{N} \leq -\delta_m N_h. \quad \text{So that} \quad \int \frac{dN}{N} \leq \int -\delta_m dt. \quad \text{Thus,}$$
$$N(t) \leq N(0)e^{-\delta_m t}, \quad \text{and} \quad N(t) \to N(0) \quad \text{as} \quad t \to \infty.$$

Thus, $\mathcal{C}$ is a positively invariant set under the flow described by the model. The solutions with initial condition in $\mathcal{C}$ remain in $\mathcal{C}$ with respect the model. □

3.2 Local asymptotic stability of the disease-free equilibrium (DFE)

The model (2.1) has a disease-free equilibria (DFE), given by

$$\mathcal{D}_o = (S_1^*, S_2^*, E_1^*, E_2^*, A_1^*, A_2^*, I_1^*, I_2^*, H^*, R^*) = (S_1(0), S_2(0), 0, 0, 0, 0, 0, 0, 0, 0)$$

where $S_1(0)$ and $S_2(0)$ are the initial total sizes of the populations unsensitized and sensitized susceptible individuals, respectively (so that, $N(0) = S_1(0) + S_2(0)$).

The local stability of $\mathcal{D}_o$ can be established using the next generation operator method [21, 22]. Using closely related notations in [21], the matrices $\mathcal{F}$ and $\mathcal{V}$, for the new infection terms and the remaining transfer terms are, respectively, given by

$$\mathcal{F} = \begin{pmatrix} 0 & 0 & \frac{(1-c_m\epsilon_m)\beta_c\eta_s S_1(0)}{N(0)} & \frac{(1-c_m\epsilon_m)\beta_c\eta_s\rho_s S_1(0)}{N(0)} & \frac{(1-c_m\epsilon_m)\beta_c S_1(0)}{N(0)} & \frac{(1-c_m\epsilon_m)\beta_c\rho_s S_1(0)}{N(0)} \\ 0 & 0 & \frac{(1-c_m\epsilon_m)\epsilon_s\beta_c\eta_s S_2(0)}{N(0)} & \frac{(1-c_m\epsilon_m)\epsilon_s\beta_c\eta_s\rho_s S_2(0)}{N(0)} & \frac{(1-c_m\epsilon_m)\epsilon_s\beta_c S_2(0)}{N(0)} & \frac{(1-c_m\epsilon_m)\epsilon_s\beta_c\rho_s S_2(0)}{N(0)} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

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$$\mathcal{V} = \begin{pmatrix} g_1 & 0 & 0 & 0 & 0 & 0 \\ -\alpha_2 & \gamma_2 & 0 & 0 & 0 & 0 \\ -(1-f)\gamma_1 & 0 & g_2 & 0 & 0 & 0 \\ 0 & -(1-q)\gamma_2 & -\alpha_3 & \phi_{a2} & 0 & 0 \\ -f\gamma_1 & 0 & 0 & 0 & g_3 & 0 \\ 0 & -q\gamma_2 & 0 & 0 & -\alpha_4 & g_4 \end{pmatrix},$$

where

$$g_1 = \gamma_1 + \alpha_2, \quad g_2 = \phi_{a1} + \alpha_3 + \mu, \quad g_3 = \sigma_1 + \phi_{s1} + \delta_1 + \alpha_4,$$
$$g_4 = \sigma_2 + \phi_{s2} + \delta_2, \quad g_5 = \phi_h + \delta_3.$$

Hence it follows from [21] that the reproduction number is given by

$$\begin{aligned}\mathcal{R}_c = {} & \frac{\beta_c(1-c_m\epsilon_m)\epsilon_s\eta_s\rho_s(1-q)S_2(0)}{\phi_{a2}N(0)} + \frac{\beta_c(1-c_m\epsilon_m)\epsilon_s\rho_s q S_2(0)}{(\sigma_2+\phi_{s2}+\delta_2)N(0)} + \\ & \frac{\beta_c(1-c_m\epsilon_m)\eta_s\rho_s\alpha_2(1-q)S_1(0)}{(\gamma_1+\alpha_2)\phi_{a2}N(0)} + \frac{\beta_c(1-c_m\epsilon_m)(1-f)\eta_s\alpha_3\rho_s\gamma_1 S_1(0)}{(\gamma_1+\alpha_2)(\phi_{a1}+\alpha_3)\phi_{a2}N(0)} + \\ & \frac{\beta_c(1-c_m\epsilon_m)\rho_s\alpha_2 q S_1(0)}{(\gamma_1+\alpha_2)(\sigma_2+\phi_{s2}+\delta_2)N(0)} + \frac{\beta_c(1-c_m\epsilon_m)\gamma_1 f S_1(0)}{(\gamma_1+\alpha_2)(\sigma_1+\phi_{s1}+\delta_1+\alpha_4)N(0)} + \\ & \frac{\beta_c(1-c_m\epsilon_m)\eta_s\gamma_1(1-f)S_1(0)}{(\gamma_1+\alpha_2)(\phi_{a1}+\alpha_3)N(0)} + \frac{\beta_c(1-c_m\epsilon_m)\rho_s\alpha_4 f\gamma_1 S_1(0)}{(\gamma_1+\alpha_2)(\sigma_2+\phi_{s2}+\delta_2)(\sigma_1+\phi_{s1}+\delta_1+\alpha_4)N(0)}\end{aligned}$$

The result below follows from Theorem 2 in [21].

Lemma 3.3. *The DFE, $\mathcal{D}_o$ of the model (2.1) is locally asymptotically stable (LAS) if $\mathcal{R}_c < 1$, and unstable if $\mathcal{R}_c > 1$.*

$\mathcal{R}_c$ is the control reproduction number for the model (2.1). It represents the average number of secondary COVID-19 infections generated by a typical infectious individual both asymptomatic and symptomatic in a completely susceptible population in the presence of control measures during the period of infectiousness of the individual [21]. Biologically speaking, by Lemma 3.3 COVID-19 can be eliminated from the population whenever $\mathcal{R}_c < 1$ if the initial sizes of the population of the model are in the region of attraction of the disease free equilibrium.

4 Model fitting and parameter estimation

The model fitting process utilized a genetic algorithm (GA) [23] as our function optimizer, implemented in MATLAB. The GA algorithm helps identify the correct basin of attraction, providing initial values for the parameters under estimation. These starting values are then employed in the lsqnonlin function within the Optimization Toolbox of MATLAB. The model fitting was performed for the epidemic period, commencing from the announcement of the first COVID-19 case in Rivers State on March 25, 2020, to June 24, 2020. Plots depicting model predictions alongside observed cumulative COVID-19 reported cases and active cases for Rivers State are presented in Figure **??**. Table 2 provides the values of the parameters utilized in the simulations. It's essential to note that these parameter values were either extracted from the literature or estimated based on available COVID-19 information.

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Parameter	Value	Range	Ref
ϵ_m	0.5	(0-1)	[6]
ϵ_s	0.3	(0-1)	Inferred from [24]
γ_1, γ_2	$\frac{1}{5.1}$/day	$(\frac{1}{14}, \frac{1}{3})$	[6]
σ_1	0.0135/day	(0.0059-0.0264)	Estimated from [4]
σ_2	0.0264/day	(0.0059-0.0264)	Estimated from [4]
ϕ_{a1}	0.1429/day	$(\frac{1}{30}, \frac{1}{3})$	[6]
ϕ_{a2}	0.1429/day	$(\frac{1}{30}, \frac{1}{3})$	[6]
ϕ_{s1}	0.1429/day	$(\frac{1}{30}, \frac{1}{3})$	[6]
ϕ_{s2}	0.1429/day	$(\frac{1}{30}, \frac{1}{3})$	[6]
ϕ_h	$\frac{1}{15}$/day	$(\frac{1}{30}, \frac{1}{3})$	[4]
δ_1	0.015/day	(0.001-0.1)	[4]
δ_2	0.011/day	(0.001-0.1)	Assumed
δ_3	0.015/day	(0.001-0.1)	[4]
f	0.5	(0-1)	[6]
q	0.5	(0-1)	estimated from [6]
η_s	0.5/day	(0-1)	[4]
ρ_s	0.3/day	(0-1)	Inferred from [25]

Table 2: Values of the parameters of model (2.1) from literature.

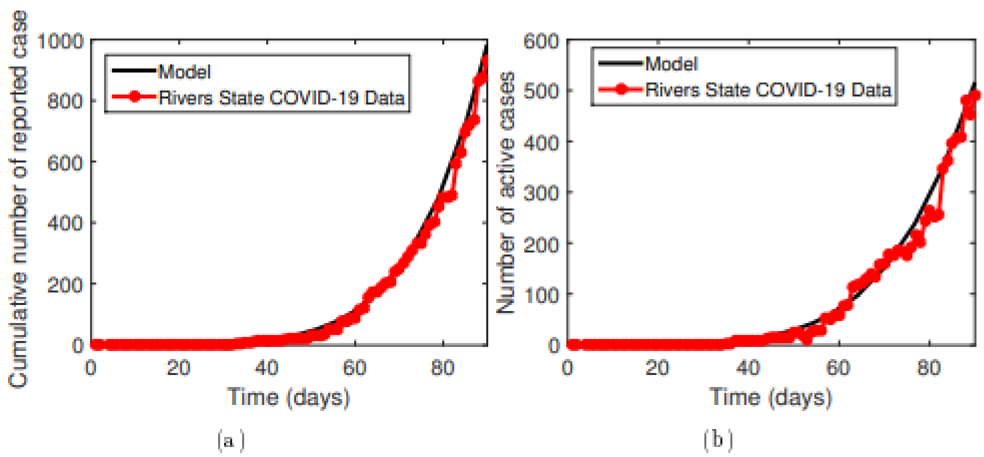

Figure 2: Fitting the cumulative number of reported cases and number of active cases.

Parameter	A	B	C
β_c	0.8474	0.8222	0.8348
c_m	0.1343	0.0543	0.0943
α_1	0.0075	0.0087	0.0081
α_2	0.0708	0.0658	0.0683
α_3	0.0997	0.0220	0.0609
α_4	0.0773	0.0773	0.0773
$\mathcal{R}_c$	2.3261	2.5815	2.4538

Table 3: Estimated parameters fitted using two different data sets.

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The parameters in model (2.1) were estimated through fitting, employing the daily cumulative number of reported cases and the number of active cases, as detailed in Table 3. The fitting process involved model (2.1), and the resulting values of β_c, c_m, α_1, α_2, α_3, α_4, and the reproduction number $\mathcal{R}_c$ are reported.

Table 3 presents the estimated parameter values obtained when fitting the model with data for the daily cumulative number of reported cases (column A) and data for the daily number of active cases (column B). Column C provides the average of the values in columns A and B.

For clarity, the notations used are as follows:
A : Cumulative Number of Reported Cases.
B : Number of Active Cases.
C : Average of A and B.

5 Simulations

5.1 Uncertainty and sensitivity analysis

Model (2.1) has twenty-four (24) parameters, and it is anticipated that uncertainties may arise from the estimation of these parameter values. In light of this, a comprehensive uncertainty and sensitivity analysis will be conducted on model (2.1) to gauge the impact of variations in each parameter, employing the Latin Hypercube Sampling technique (LHS). To quantify the influence of parameter variations on associated numerical simulations, a global sensitivity analysis will be performed using the Partial Rank Correlation Coefficients (PRCC) technique. The uncertainty and sensitivity analyses will be conducted based on the parameter values provided in Tables 2 and 3. Table 4 presents the outcomes of this analysis, using the associated reproduction number ($\mathcal{R}c$), cumulative number of reported cases, and the number of active cases as response functions. Notably, the top PRCC-ranked parameters in model (2.1) include the relative infectiousness of asymptomatic individuals (η_s), the efficacy of face masks (ϵ_m), the relative infectiousness of sensitized individuals (ρ_s), compliance with the use of face masks (c_m), effective contact rate (βc), and the fraction of unsensitized exposed individuals who become symptomatic (f).

At this point, it is crucial to emphasize that the results presented above underscore the pivotal role of sensitization in the dynamics of model (2.1), with the relative infectiousness of sensitized individuals emerging as a primary driver of the model's dynamics. Additionally, while the relative susceptibility of sensitized individuals may not be significant when considering the overall reproduction number ($\mathcal{R}_c$), it becomes noteworthy when the cumulative number of reported cases and active cases are employed as response functions. This underscores the potential of sensitization programs in effectively mitigating the spread of the infection in the population. In essence, a high-impact sensitization program has the potential to substantially reduce the susceptibility and infectiousness of sensitized individuals, thereby alleviating the burden of the disease in the population. Moreover, the analysis reveals that the parameter governing the relative infectiousness of sensitized individuals holds greater significance than the parameter associated with their relative susceptibility. Furthermore, insights from Table 4 highlight that sensitizing susceptible individuals exerts a more pronounced impact on the dynamics of model (2.1) than sensitizing any other group. As a result, it is recommended that sensitization efforts be primarily directed towards susceptible individuals, optimizing the effectiveness of interventions aimed at controlling the spread of the disease.

5.2 Numerical simulations

The population-level impact of sensitizing the public on the burden of the pandemic is numerically assessed using the parameter values tabulated in Tables 2 and 3. This evaluation involves

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Table 4: PRCC values for the parameters of the model (2.1) using $\mathcal{R}_c$, cumulative number of reported cases and number of active cases as response functions.

Parameter	$\mathcal{R}_c$	**Cumulative No. of reported cases**	**No. of active cases**
η_s	**0.7214**	**0.6376**	**0.6394**
ϵ_m	**-0.7029**	**-0.6473**	**-0.6396**
ρ_s	**0.6598**	**0.6887**	**0.6905**
c_m	**-0.6523**	**-0.6406**	**-0.6348**
β_c	**0.6261**	**0.5979**	**0.6026**
f	**0.4994**	**0.5205**	**0.5125**
ϵ_s	0.0342	**0.5013**	**0.4881**
q	0.1043	0.39457	0.4209
γ_1	0.0964	0.1174	0.1154
γ_2	-0.0316	-0.0299	0.0359
ϕ_{a1}	-0.2287	-0.0934	-0.0892
ϕ_{a2}	-0.0848	-0.0228	-0.0985
ϕ_{s1}	-0.2666	-0.1825	-0.2036
ϕ_{s2}	-0.1998	-0.1916	-0.1921
ϕ_h	-0.0029	0.0447	-0.1015
δ_1	-0.0374	-0.0434	-0.0393
δ_2	-0.0159	0.0095	-0.0013
δ_3	0.0084	0.0270	-0.0045
σ_1	-0.0231	0.0446	0.0459
σ_2	-0.0314	0.0862	0.0057
α_1	-0.0576	-0.3068	-0.3630
α_2	-0.0795	-0.1065	-0.0279
α_3	-0.0267	-0.0528	-0.0497
α_4	-0.0551	-0.0899	-0.0325

varying the values of face mask compliance (c_m), the relative susceptibility of sensitized individuals (ϵ_s), and the relative infectiousness of sensitized individuals (ρ_s). A contour plot of the control reproduction number ($\mathcal{R}_c$) of the model, as a function of face mask efficacy (ϵ_m) and compliance to face mask use (c_m), is depicted in Figure **??**. If the modification parameters for the reduced susceptibility of sensitized susceptibles (ϵ_s) and for the infectiousness of sensitized infectious individuals (ρ_s) are both set to 0.9, then a face mask efficacy of over 85% and a usage of over 90% will be needed to control the disease (Figure **??**). If the effectiveness of the sensitization program is high ($\epsilon_s = 0.1$ and $\rho_s = 0.1$), then a face mask efficacy and usage of about 70% and 75%, respectively, will be needed for effective disease control of the pandemic in Rivers State, Nigeria, considering the parameter values in Tables 2 and 3 (Figure **??**).

Figure **??** illustrates the cumulative number of reported cases (**??**) and daily active cases (**??**) when all parameters of the model (2.1) are at baseline values. We observe that 8,183 cases will be reported, with a peak occurring after 140 days if all measures remain at baseline. Assessing the impact of targeted sensitization, Figure **??** demonstrates that focusing the sensitization program on susceptible and exposed individuals will be more beneficial in the fight against the spread of the infection in the population, while the least beneficial approach is targeting susceptible and asymptomatic individuals. This result aligns with the sensitivity analysis in Table 4. Figures **??** are utilized to evaluate the impact of combining face mask use with sensitization that encourages the practice of other preventive measures.

The combined implementation of both measures shows promising potential for effective disease control and/or elimination in Rivers State, Nigeria, resulting in approximately a 98% decrease in the cumulative number of reported cases. Moreover, when sensitized symptomatic individuals are 50% less likely to transmit the disease, and sensitized susceptible are also 50% less likely to get infected without face mask use, a 68% reduction in the cumulative number of reported cases is achieved. However, in scenarios where only 50% of the population complies with face mask use in public places and the sensitized symptomatic individuals are 100% as likely to transmit the disease, and sensitized susceptible are also 100% as likely to get infected (indicating an ineffective sensitization program), an 18% reduction in the burden of the infection is observed. Thus, it is essential to note that Figure **??** may suggest that the strategy of sensitization, which encourages the practice of other preventive measures, holds better prospects for disease control than relying solely on face mask use. These results are solely based on the parameter estimates derived from the NCDC data used for model fitting.

We also evaluated the impact of increasing the sensitization rates α_1, α_2, α_3, and α_4. Figure **??** illustrates that doubling the sensitization rates leads to a remarkable 99% reduction in the cumulative number of reported cases, with all other parameters kept at baseline. This suggests that sensitization programs that encourage the practice of preventive measures, such as hand-washing, home quarantine, social distancing, cautious behavior, environmental cleaning and disinfection, and government-imposed isolation of infected individuals, have a significant impact on the dynamics of the model (2.1) and, consequently, on the control of COVID-19 in the population under study.

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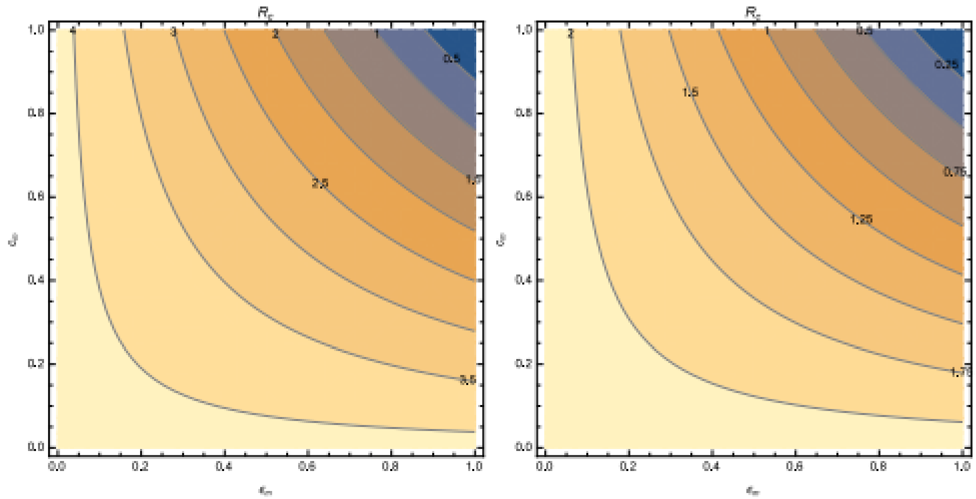
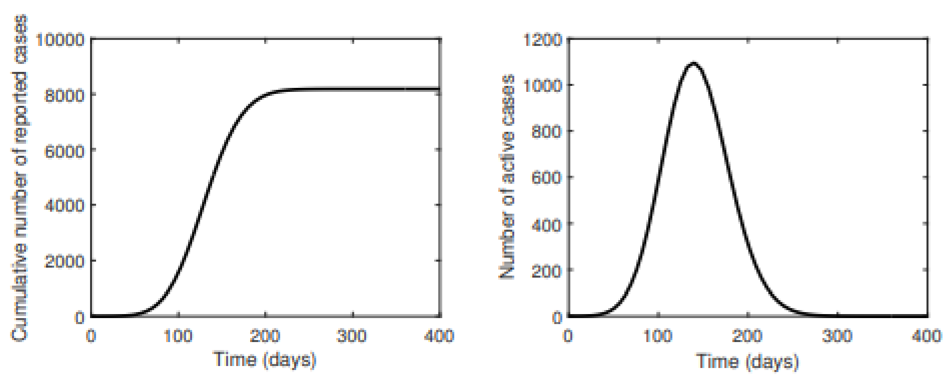


(a) $\epsilon_s = 0.9$, $\rho_s = 0.9$ (b) $\epsilon_s = 0.1$, $\rho_s = 0.1$

Figure 3: Contour plot of $\mathcal{R}_c$ showing the effect of the sensitization program on the reproduction number $\mathcal{R}_c$. Parameter values used are as given in Tables 2 and 3.



(a) (a) (b) (b)

Figure 4: Simulations of the model (1) showing (a) the cumulative number of reported cases and (b) daily active cases with all parameter values at baseline. Parameter values used are as given in Tables 2 and 3.

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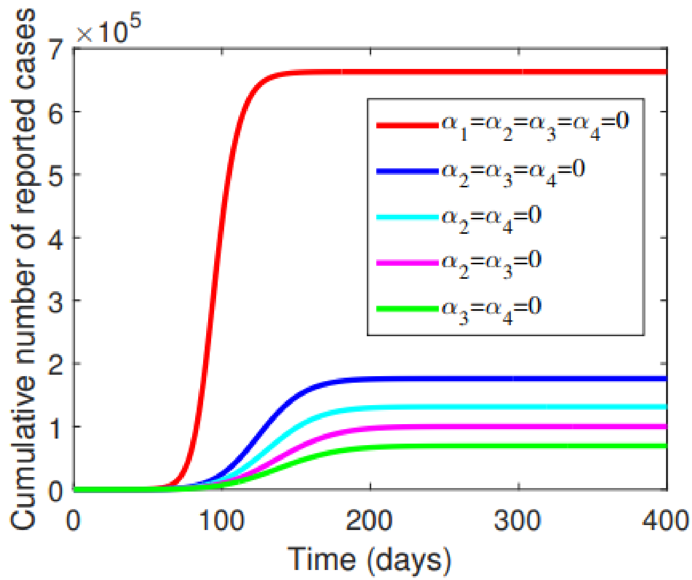

Figure 5: Simulations of the model (1) showing the impact of targetted sensitization on the cumulative number of reported cases. Parameter values used are as given in Tables 2 and 3.

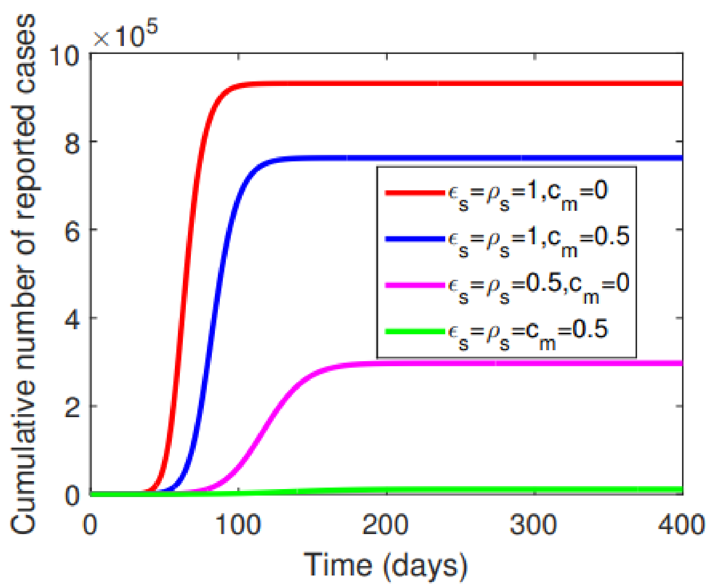

Figure 6: Simulations of the model (1) showing the impact of targetted sensitization on the cumulative number of reported cases. Parameter values used are as given in Tables 2 and 3.

6 Discussion and Conclusions

This study aims to evaluate the influence of sensitization on the dynamics of COVID-19 in Rivers State, Nigeria. We formulated a mathematical model and calibrated it using published data on the transmission dynamics of COVID-19 specific to Rivers State. Our findings offer insights into the potential outcomes of the pandemic, with a particular focus on the impact of public sensitization efforts.

The developed model underwent thorough analysis, revealing that the disease-free equilibrium of the epidemic model attains local asymptotic stability when the control reproduction number ($\mathcal{R}_c$) is less than one. Additionally, a numerical analysis was conducted to gauge the effectiveness of public sensitization regarding transmission pathways, signs and symptoms, and preventive measures. These measures include regular hand washing, the use of hand sanitizers, wearing face masks, practicing respiratory etiquette, social distancing, and self-isolation when sick. Given that an individual's knowledge about an infectious disease can shape their behavior in ways that prevent infection, our study specifically assesses the impact of the sensitization program.

The assessment focuses on the reduced likelihood of sensitized susceptible contracting the infection and the diminished potential for sensitized infectious individuals to transmit the disease, shedding light on the tangible benefits of effective public sensitization.

The uncertainty and sensitivity analysis, utilizing the associated reproduction number ($\mathcal{R}c$), the cumulative number of reported cases, and the number of active cases as response functions, highlights the top Partial Rank Correlation Coefficient (PRCC)-ranked parameters within the model (2.1). These parameters include the relative infectiousness of asymptomatic individuals (η_s), the efficacy of face masks (ϵ_m), the relative infectiousness of sensitized individuals (ρ_s), compliance with the use of face masks (c_m), the effective contact rate (βc), and the fraction of unsensitized exposed individuals who become symptomatic (f).

Numerical simulations demonstrate that, under a worst-case scenario characterized by zero compliance with face mask usage and no behavioral change among the sensitized population, Rivers State could potentially witness 931,600 cumulative COVID-19 cases. Introducing a 50% compliance rate to face mask usage without sensitization reduces this figure to 762,600, representing an 18% reduction. Furthermore, when behavioral changes resulting from sensitization are incorporated, with parameters $\epsilon_s = 0.5$ and $\rho_s = 0.5$, and in the absence of compliance with face mask usage, the cumulative cases plummet to 297,200, marking a substantial 68% reduction. These findings underscore the significance of behavioral change induced by sensitization programs, as knowledge alone about COVID-19 does not necessarily translate into behavioral modifications conducive to adherence to recommended preventive measures. It's noteworthy that, despite extensive efforts to sensitize the public about COVID-19 in Nigeria, a notable skepticism persists, with many individuals still questioning the reality of the disease.

The findings of this study carry vital implications for public health policies and ongoing sensitization efforts. The notable decrease in cumulative cases with heightened sensitization underscores the importance of targeted awareness campaigns. Authorities in Rivers State and beyond can utilize these insights to enhance and amplify sensitization strategies. Emphasizing not only the transmission pathways and symptoms but also fostering behavioral changes is crucial, as evidenced by the substantial reduction in cases when compliance and sensitization work in unison.

Efforts to counter skepticism and misinformation are crucial. Tailoring messages to address prevalent misconceptions, such as the belief that COVID-19 is a scam, can significantly enhance the effectiveness of sensitization programs. Public health interventions should prioritize the promotion of face mask usage, given its substantial impact on reducing transmission. The study advocates for a multifaceted approach, combining increased sensitization rates with behavioral changes among susceptible and infectious individuals. Our results align with findings from [17], which reported a higher rate of asymptomatic cases (infected individuals showing no symptoms) compared to symptomatic cases in new incidences. Thus, the continuous need for community sensitization is evident to maintain adherence to protective behaviors, especially as asymptomatic individuals may be less vigilant without sufficient awareness.

In summary, the study advocates combining strong awareness efforts with careful following of preventive measures, especially using face masks, is crucial for effectively controlling diseases. Policymakers and health agencies can leverage these findings to design targeted, culturally sensitive interventions, fostering behavioral changes and addressing skepticism. These efforts contribute to the development of more resilient and responsive public health strategies in Rivers State and similar contexts worldwide.

Conflicts of Interest

The authors have disclosed no conflicts of interest.

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