STATISTICAL EVALUATION OF MATHEMATICAL METHODS IN SOLVING LINEAR THEORY PROBLEMS: DESIGN OF WATER DISTRIBUTION SYSTEMS

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ABSTRACT

Water distribution is an important factor in the development of a community. As a follow up on authors previous studies on water supply, access to safe water, water, sanitation and health (WASH) and network analysis, this paper presents evaluation of various methods (statistical and numerical) in use to solve Linear Theory equations in pipe network analysis. Three practical pipe network analyses were conducted using Guassian and Gaus-Jacobian eliminations, Microsoft excel solver, least squared and numerical methods. The flow obtained using the methods were evaluated using model of selection criterion MSC, coefficient of determination CD, reliability RD and errors. The study revealed that flow in pipe network analysis varied with the method. The flow was a function of the method used, length and diameter of the pipe; and withdraws from the node. The values of model of selection criterion and coefficient of determination varied from 1.2195 to 24.5549 and 0.7912 to 0.9751 respectively. Statistical evaluation revealed that least squared, elimination, Microsoft excel solver and numerical methods had MSC of 1.7519, 2.8709, 24.5549 and 1.2195, respectively. It was concluded that Microsoft excel solver, elimination and least squared methods were better methods for solving linear theory equations in pipe network analysis than numerical method based on CD,MSC, reliability and errors, while Microsoft excel solver was the overall best method based on MSC (24.5549), reliability (99.99 %), CD (0.9751) and errors (0.0000).

Keywords: Water Distribution, Linear Theory Equations, Eliminations, Least Squared, Numerical

INTRODUCTION

Water has been described as an important need of living things. In the design and production of potable water, distribution of the treated water has been highlighted as an essential ingredient. It is well known that treated water production and distribution are the two major ingredients that influence the quality and quantity of the water supply to the community. Water production is a continuous process which can be altered easily at any time, but distribution system (pressure at any point, pipe sizes, and flow) cannot be altered easily as the production process (Oke, 2010). The problems of water supply can be linked to production, storage, demand and distribution. The problem of inadequate water supply is a common trend in the third world or developing countries such as Nigeria, Kenya and Togo etc. It is well known that in everyday life clean water and environment are essential for people to live sustainable and healthy lives. The problem of access to adequate quality and quantity water has lead to utilization of unclean water and generation of unclean environment, which supports the spread of communicable diseases such as cholera, dysentery and others (Ojo *et al.,* 2006). Figure 1 presents global and regional access to safe water. Figure 2 shows global and regional sanitation practice. Water distribution system is one of the most important essential links in the facilities for modern society. Predetermined amount of drinking water can be delivered to the public throughout the distribution system.

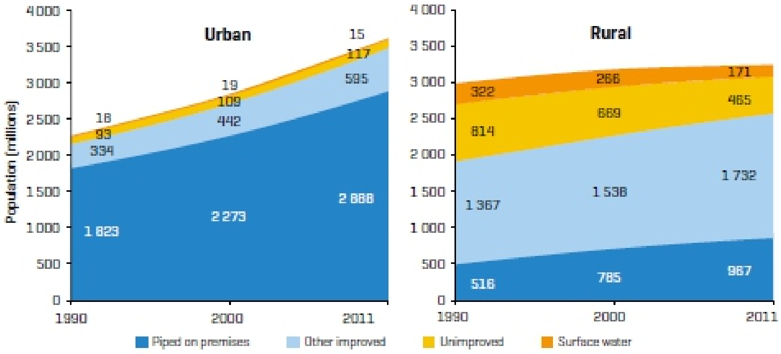

Figure 1a: Global Drinking water Coverage Trends in Urban and Rural Areas, 1990 – 2011 (JMP, 2013; Oke *et al.,* 2014)

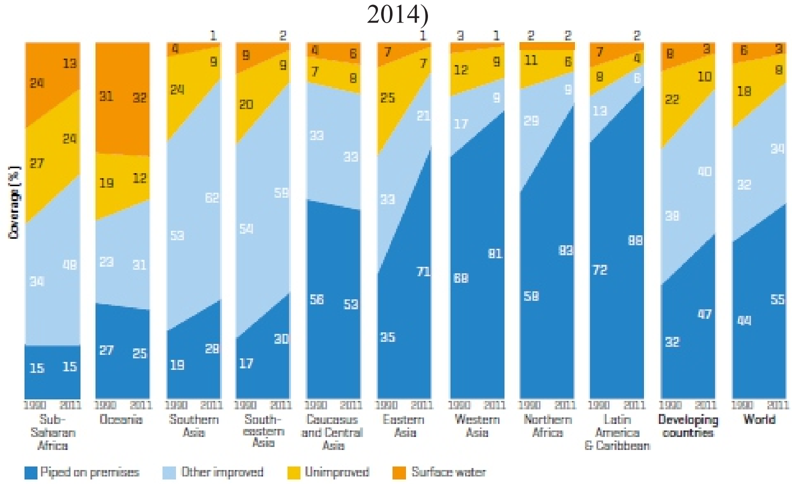

Figure 1b: Drinking water Coverage Trends by Developing Regions and the World, 1990 – 2011 (JMP, 2013; Oke *et al.,* 2014)

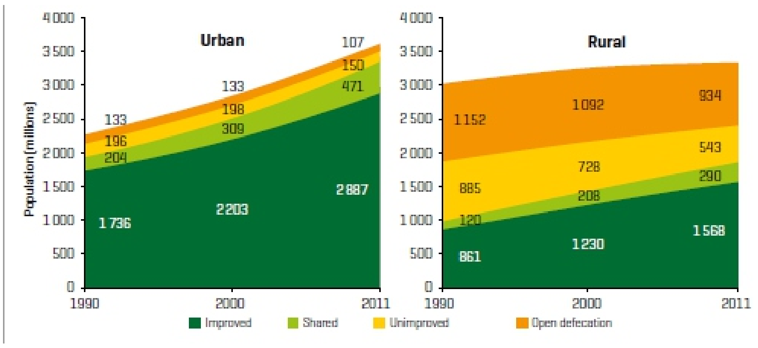

Figure 2a: Global Sanitation Coverage Trends in urban and Rural Areas, 1990 – 2011 (JMP, 2013; Oke *et al.,* 2014)

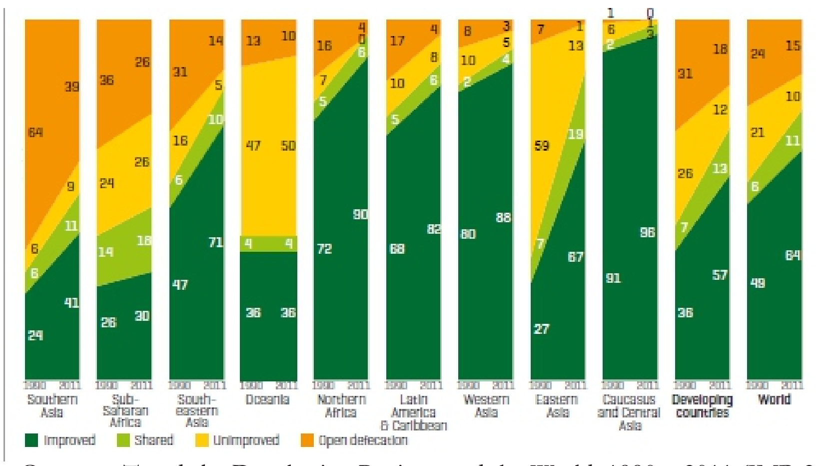

Figure 2b: Sanitation Coverage Trends by Developing Regions and the World, 1990 – 2011 (JMP, 2013; Oke *et al.,* 2014)

However, less amount of water or no water may be delivered to some places for many reasons, which can be leaking of water due to aging of pipe, leaking of water due to wrong pipe size, leaking of water due to water main breakage, leaking of water due to pipe deterioration, and so on. Furthermore, various construction environments around the place of pipe installation may cause the state of system failure. It is now necessary to figure out what kinds of problem that cause system failure in distribution system and how to fix those problems. Many studies have been conducted on the pipe breakage, leaking of water in pipes, pipe selections techniques and piping system. Researchers have presented statistical models which can predict the probability of pipe failures in a distribution system. Most of the models failed to predict the pipe failure for real city and general information and documents on pipes as well as pipe selection are very much limited (Chaudhy, 1979; Mailhot *et al.,* 2000; Watson *et al.,* 2004; Hyuk and Lee, 2008).

Though, many reasons can cause the pipe breakage or leaking of water in pipes, the most important causes include the transient effect in pipe network and wrong pipe sizing or wrong pipe size selection (Hyuk and Lee, 2008). Reliability analysis regarding transient flow has been conducted to evaluate the probability of system failure in water distribution system (Ang and Tang, 1984). Out of all the factors that contribute to inadequate water supply, water distribution techniques have not received any appropriate solutions because, until now attention on water supply has been focused on production with little or no attention to its distribution techniques. The neglect of water distribution systems can be attributed to many reasons such as its rigidity, cost of implication, its complexicity, political and economical reasons. Although, literature has reported optimization of pipe distribution network (Prasad *et al.,* 2003), detailed methods of solving pipe network analysis especially methods of solving linear theory equations are rare in literature. Literature such as Wood and Charles (1972); Steel and McGhee (1979); Isaacs and Mills (1980); Dake (1982); Featherstone and Nalluri (1982); Viessman and Hammer (1993); Ogedengbe (1985); Nielsen (1989); Ellis and Simpson (1996); Chin (2000); Wood and Charles (1972); Oke (2010) and Nelson *et al.* (2013) present techniques for pipe network analysis as Linear theory, Hardy Cross, equivalent pipe, circle theory and Newton Raphson techniques. Adeniran and Oyelowo (2013), and Dini and Tabesh (2014) highlighted different software that can be used for pipe network analysis. With known importance of adequate water supply on sanitation and health of people, documentation on mathematical methods in solving these pipe network analysis techniques is necessary. The main objective of this study is to present mathematical solution in solving Linear Theory equations in pipe network analysis and evaluate accuracy of these mathematical methods statistically. Literature such as Bowman (1962); Loveday (1980),Krasnov *et al.* (1990); Stroud (1990) stated that equations can be solved by using statistical and mathematical methods. Some of the methods are Gaussian elimination, Gauss-Jordian elimination, Matrix, least squared, Microsoft excel solver and iteration (numerical) methods. Previous studies on Microsoft excel Solver or similar package include Barati (2013) and Bhattacharjya (2010) used solver for groundwater flow; Gay and Middleton (1971) developed solutions for pipe network, Jewell (2001) and Huddleston *et al.* (2004) used excel sheet for pipe network analysis; Canakci (2007) used solver for pile foundation design while Tay *et al.* (2014) used solver for solving non-linear equation.

MATERIALS AND METHOD

Pipe network of communities were adapted from literature Oke (2007) and Jeppson (1979). These pipe networks are as presented in Figures 3, 4 and 5. Figure 3 presents a community with an overhead tank as source of water supply (reservoir) and 3 pipes network. Figure 4 shows a community with an overhead tank as source of water supply (reservoir) and 5 pipes network. Figure 5 presents a community with two overhead tanks as sources of water supply (reservoirs) and 7 pipes network. Table 1 presents basic information on the examples. Linear theory techniques were used to developed continuity equations from these pipe networks.

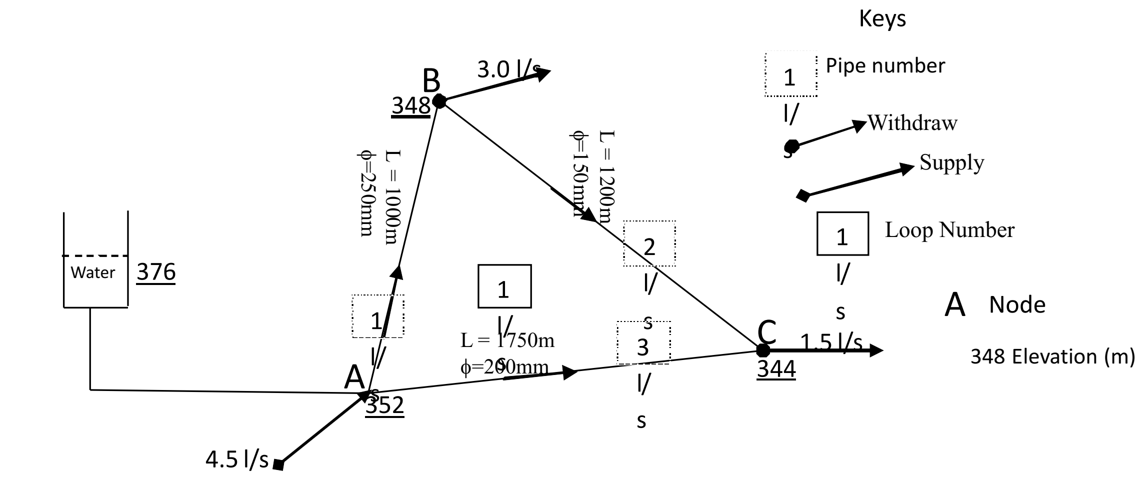

Figure 3: A Network with 3 Pipes and a Loop (Adapted from Jeppson, 1979)

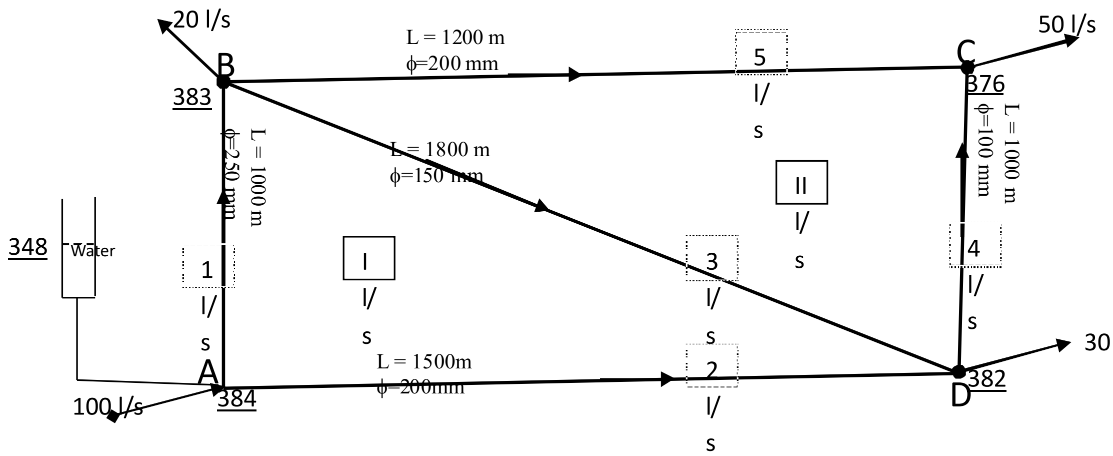

Figure 4: A Network with 5 Pipes and 2 Loops (Adapted from Oke, 2007)

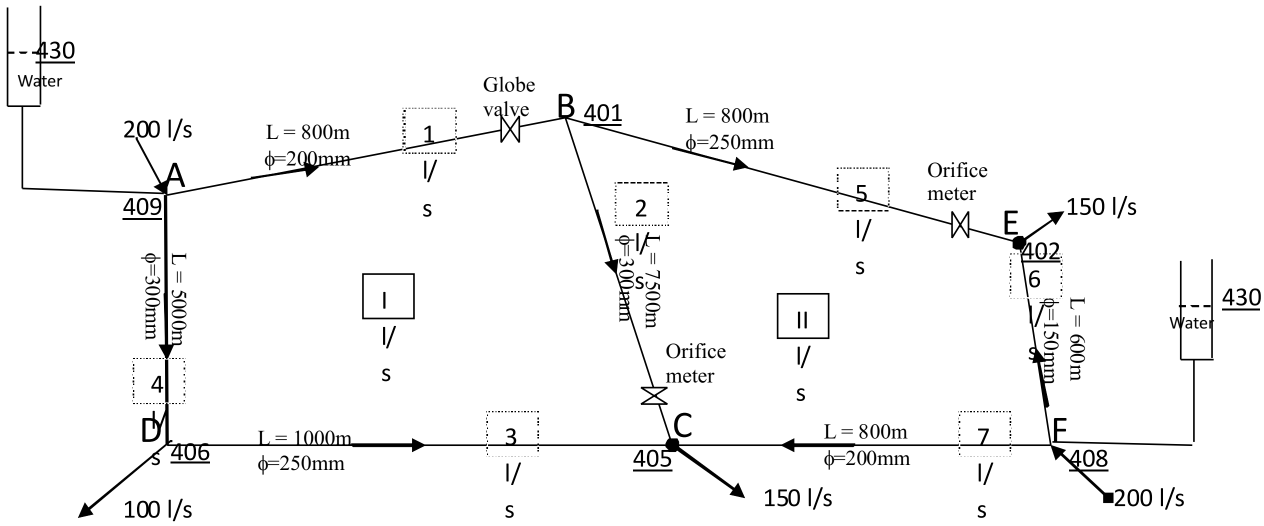

Figure 5: A Network with 7 Pipes and 2 Loops (Adapted from Jeppson, 1979)

Table 1: Basic Information on the Pipes in the Networks

Description	Pipe No	Pipe Diameter (D, mm)	Pipe Length (L, m)	Darcy Friction factor (f)	$K = \frac{8fL}{\pi^2 g D^5}$	K' = KQ	Elevation / Head of Water (H,m)	Change in Head (ΔH,m)
Example 1	1	250	1000	0.02	1690.791	3.551	352	24
	2	150	1200	0.02	26092.45	26.092	348	28
	3	200	1750	0.02	9029.796	10.836	344	32
Example 2	1	250	1000	0.02	1690.791	3.551	384	21
	2	200	1500	0.02	7739.825	16.254	383	22
	3	150	1800	0.02	39138.67	39.139	382	23
	4	100	1000	0.02	165116.3	198.140	376	29
	5	200	1200	0.02	6191.86	7.430	376	29
Example 3	1	200	800	0.02	4127.907	8.669	409	21
	2	300	750	0.02	509.6181	1.070	401	29
	3	250	1000	0.02	1690.791	1.691	405	25
	4	300	500	0.02	339.7454	0.408	406	24
	5	250	800	0.02	1352.632	1.623	402	28
	6	150	600	0.02	13046.22	15.655	408	22
	7	200	800	0.02	4127.907	4.953	405	25

Linear theory method was selected based on its advantages over Newton-Raphson and Hardy Cross methods (Jeppson, 1974; 1979), which include: it does not require an initialization and it always converges in a relatively little iterations. The detail equations are as presented in equations 1 to 23. More on Microsoft excel solver can be found in literature such as Barati (2013); Bhattacharjya (2010); Gay and Middleton (1971); Jewell (2001); Briti *et al.* (2013); Tay *et al.* (2014) and Oke *et al.* (2014a and b). The developed equations were solved using mathematical methods such as Gaussian elimination, Gauss-Jordian elimination, Matrix, least squared, Microsoft excel solver and iteration (numerical) techniques. Literature such as Bowman (1962); Loveday (1980), Krasnov *et al.* (1990); Stroud (1990) discussed more on these statistical and mathematical methods of solving Linear Theory methods. The flows obtained were evaluated statistically using model selection criterion, coefficient of determination, total error, reliability and correlation coefficient. The continuity and headloss equations used are as follows:

i Sum of flow at any node is equal to zero;

$$\sum_{i=1}^{n} Q_i = 0 \qquad (1)$$

where, Q_i is the flow at any node.

ii Sum of headloss in a closed loop is equal to zero;

$$\sum_{i=1}^{n} h_{li} = 0 \qquad (2)$$

where, h_{li} is the sum headloss in a closed loop.

Linear Theory transforms the nonlinear headloss equations into linear equations by approximating the headloss(H_l) in each pipe as:

$$H_l = \left(K_i Q_i^{n-1}\right) Q_i = K' Q_i \quad (2a)$$

Where; K' is the product of K (K $= \dfrac{8fL}{\pi^2 g D^5}$) and assumed flow (Q).

Equations (flows into the node are positive, flow out of the node or withdraw are negative) for Example 1 are as follows:

At Node A: $-Q_1 - Q_3 + 4.5 = 0$ (3)

At Node B: $Q_1 - Q_2 - 3.0 = 0$ (4)

At Node C: $Q_2 + Q_3 - 1.5 = 0$ (5)

where, Q_1; Q_2 and Q_3 are flows in pipes 1, 2 and 3 respectively.

Headloss equation (headloss in clockwise direction is positive and headloss in anticlockwise direction is negative):

$$K_1' Q_1 + K_2' Q_2 - K_3' Q_3 = 0 \quad (6)$$

where, $H_1 - H_2 = K_1 Q_1^2 = \Delta H_{12}$

$$Q_1 = \sqrt{\frac{\Delta H_{12}}{K_1}} \text{ and } K_1' = K_1 Q_1$$

The 3 x 3 matrix of these equations is as follows:

$$\begin{pmatrix} \sqrt{\frac{\Delta h_1}{K_1}} & 0 & \sqrt{\frac{\Delta h_3}{K_3}} \\ \sqrt{\frac{\Delta h_1}{K_1}} & -\sqrt{\frac{\Delta h_2}{K_2}} & 0 \\ K_1' & K_2' & -K_3' \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 4.5 \\ 3.0 \\ 0 \end{pmatrix} \quad (7)$$

Equations for Example 2 are as follows:

At Node A: $-Q_1 - Q_2 + 100 = 0$ (8)

At Node B: $Q_1 - Q_3 - Q_5 - 20 = 0$ (9)

At Node C: $Q_4 + Q_5 - 50 = 0$ (10)

At Node D: $Q_2 + Q_3 - Q_4 - 30 = 0$ (11)

Where, Q_4 and Q_5 are flows in pipes 4 and 5 respectively.

Headloss equations

Loop I $K_1' Q_1 + K_3' Q_3 - K_2' Q_2 = 0$ (12)

Loop II $K_5' Q_5 - K_4' Q_4 - K_3' Q_3 = 0$ (13)

The 5 x 5 matrix of these equations is as follows:

$$\begin{pmatrix} \sqrt{\frac{\Delta h_1}{K_1}} & \sqrt{\frac{\Delta h_2}{K_2}} & 0 & 0 & 0 \\ \sqrt{\frac{\Delta h_1}{K_1}} & 0 & -\sqrt{\frac{\Delta h_3}{K_3}} & 0 & -\sqrt{\frac{\Delta h_5}{K_5}} \\ 0 & 0 & 0 & \sqrt{\frac{\Delta h_4}{K_4}} & \sqrt{\frac{\Delta h_5}{K_5}} \\ K_1' & -K_2' & K_3' & 0 & 0 \\ 0 & 0 & -K_3' & -K_4' & K_5' \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{pmatrix} = \begin{pmatrix} 100 \\ 20 \\ 50 \\ 0 \\ 0 \end{pmatrix} \quad (14)$$

Equations for Example 3 are as follows:

At Node A: $-Q_1 - Q_4 + 200 = 0$ (15)

At Node B: $Q_1 - Q_2 - Q_5 = 0$ (16)

At Node C: $Q_2 + Q_3 + Q_7 - 150 = 0$ (17)

At Node D: $Q_4 - Q_3 - 100 = 0$ (18)

At Node E: $Q_5 + Q_6 - 150 = 0$ (19)

At Node F: $-Q_6 - Q_7 + 200 = 0$ (20)

Where, Q_6 and Q_7 are flows in pipes 6 and 7 respectively. Headloss equations

Loop I $K_1' Q_1 + K_2' Q_2 - K_3' Q_3 - K_4' Q_4 = 0$ (21)

Loop II $K_5' Q_5 + K_7' Q_7 - K_6' Q_6 - K_2' Q_2 = 0$ (22)

The 7 x 7 matrix of these equations is as follows:

$$\begin{pmatrix} \sqrt{\frac{\Delta h_1}{K_1}} & 0 & 0 & \sqrt{\frac{\Delta h_4}{K_4}} & 0 & 0 & 0 \\ \sqrt{\frac{\Delta h_1}{K_1}} & -\sqrt{\frac{\Delta h_2}{K_2}} & 0 & 0 & -\sqrt{\frac{\Delta h_5}{K_5}} & 0 & 0 \\ 0 & \sqrt{\frac{\Delta h_2}{K_2}} & \sqrt{\frac{\Delta h_3}{K_3}} & 0 & 0 & 0 & \sqrt{\frac{\Delta h_7}{K_7}} \\ 0 & 0 & -\sqrt{\frac{\Delta h_3}{K_3}} & \sqrt{\frac{\Delta h_4}{K_4}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{\frac{\Delta h_6}{K_6}} & \sqrt{\frac{\Delta h_7}{K_7}} \\ K_1' & K_2' & -K_3' & -K_2' & 0 & 0 & 0 \\ 0 & K_2' & 0 & 0 & -K_5' & -K_6' & K_7' \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \end{pmatrix} = \begin{pmatrix} 200 \\ 0 \\ 150 \\ 100 \\ 200 \\ 0 \\ 0 \end{pmatrix} \quad (23)$$

Total error (Err^2) can be computed using equation (24) as follows (Babatola *et al.,*2008; Oke, 2007):

$$Err^2 = \sum_{i=1}^{n} \left(Y_{obsi} - Y_{cali}\right)^2 \quad (24)$$

where, Y_{obsi} is the observed flow and Y_{cali} is the calculated flow

Coefficient of Determination (CD) can be expressed as follows:

$$CD = \frac{\sum_{i=1}^{n} \left(Y_{obsi} - \overline{Y}_{cali}\right)^2 - \sum_{i=1}^{n} \left(Y_{obsi} - Y_{cali}\right)^2}{\sum_{i=1}^{n} \left(Y_{obsi} - \overline{Y}_{cali}\right)^2} \quad (25)$$

where, $\overline{Y}_{obsi}$ is the average of observed flow and $\overline{Y}_{cali}$ is the average of calculated flow MSC can be computed using equation (26) as follows:

$$MSC = \ln \frac{\sum_{i=1}^{n} \left(Y_{obsi} - \overline{Y}_{obsi}\right)^2}{\sum_{i=1}^{n} \left(Y_{obsi} - Y_{cali}\right)^2} - \frac{2p}{n} \quad (26)$$

where, p is the number of parameters and n is the number of data points.

Reliability (RD) of any method is the accuracy and the validity of the method. The statistical approach developed to address reliability of any method is the testing of the hypothesis that there is no difference between the method and other methods. Sartory (2005) and Oke (2007) describe relative difference between methods statistically as follows:

$$RD = 100 - 100\left(\frac{\sum_{i=1}^{n}(Q_{obs} - Q_{cal})}{\sum_{i=1}^{n} Q_{obs}}\right) \qquad (27)$$

Where, Q_{obs} is the expected flows and Q_{cal} is the obtained flow.

RESULTS AND DISCUSSION

Results from this study are presented as follows: equations and solutions of the problems and statistical evaluation of these mathematical methods.

Equations and Solutions of the Problems

Equations 28 to 30 present the equations to the problems. The equations are square matrices. They are in matrix forms of the number of pipes (in rows and in columns). Equation 28 is the equation to problem number 1 (example 1). Equation 29 is the equation to problem number 2. Equation 30 is the equation to problem number 3. Table 2 presents solutions to the problems. The values of multipliers (q_i) for each of the method were as presented in the table. Detailed solutions by the Microsoft excel solver were as presented at the Appendices (Appendices A, B and C). From the Table the values of q_i varied with the method as well as the problem. Also, from the Table the actual flow obtained using these methods were as indicated. The values of the flows also varied with the method. These indicated that flows in pipe network analysis are functions of the network, pipe's properties and the method used. This is in agreement with literature such as Dillingham (1967); De NeufVille and James (1969); Jeppson (1979); Steel and McGhee (1979); Featherstone and Nalluri (1982); Chin (2000); Oke (2007), which stated that flow in pipes in any network depends on the method and the network in place. The network can be any of the followings, network with:

a) overhead tank as source of water (gravity supply);
b) pump from clear well as sources of water supply (direct line);
c) supply from booster station (this involves pseudo loops in the network analysis); and
d) values and meters

$$\begin{pmatrix} 0.151632 & 0.00000 & 0.065614 \\ 0.151632 & -0.04169 & 0.0000 \\ 3.522936 & 21.74652 & -6.2715 \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 4.5 \\ 3.0 \\ 0 \end{pmatrix} \qquad (28)$$

$$\begin{pmatrix} 0.145176 & 0.069379 & 0 & 0 & 0 \\ 0.145176 & 0 & -0.03085 & 0 & -0.0870999 \\ 0 & 0 & 0 & 0.016867 & 0.0870999 \\ 3.757798 & -0.30581 & 24.7427 & 0 & 0 \\ 0 & 0 & -24.7427 & -6.52396 & 0.646024465 \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{pmatrix} = \begin{pmatrix} 100 \\ 20 \\ 50 \\ 0 \\ 0 \end{pmatrix} \qquad (29)$$

$$\begin{pmatrix} 0.106675 & 0 & 0 & 0.338266 & 0 & 0 & 0 \\ 0.106675 & -0.28189 & 0 & 0 & -0.183113 & 0 & 0 \\ 0 & 0.28189 & 0.15476 & 0 & 0 & 0 & 0.092913 \\ 0 & 0 & -0.15476 & 0.338266 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0522635 & 0.092913 \\ 21.74826 & 6.03076 & -10.9849 & -2.41123 & 0 & 0 & 0 \\ 0 & -6.03076 & 0 & 0 & -7.798475 & -28.62419 & 12.07583 \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \end{pmatrix} = \begin{pmatrix} 200 \\ 0 \\ 150 \\ 100 \\ 200 \\ 0 \\ 0 \end{pmatrix} \qquad (30)$$

Statistical Evaluation of the Methods

Table 3 shows the summary of the statistical evaluation. Table 4 presents the detailed computation of the statistical evaluation. Figure 6 presents relationship between the expected and observed flows as well as the squared coefficient of correlation. Six different statistical expressions were used to evaluate the performance of the flow estimations or to compare the method of estimating the flow in the pipe. Table 3 shows the values of total error, CD, RD and MSC for each of the methods. From the Table the values of MSC are 2.8709, 1.7519, 24.5549 and 1.2195 for elimination, least squared, Microsoft excel solver and numerical methods respectively. It is well known that the higher the value of MSC the better the accuracy of the method and the higher the dependability of the method. This indicated that out of these four methods Microsoft excel solver was the most dependable and more accurate than the other three methods. The values of CD were 0.9484, 0.9485, 0.9751 and 0.7912 for elimination, least squared, Microsoft excel solver and numerical methods respectively.

Table 2: Solutions to the Problems and Various Flows of the Methods

		Factor	Multiplier (q)			Actual Q (m^3/s)			
Example	Pipe Number	A_a	Elimination (q_e)	Least Squared (q_{el})	Iteration (q_{eel})	Elimination (A_a q_e)	Least Squared (A_a q_{el})	Iteration (A_a q_{eel})	Solver
1	1	0.15163	0.01504	0.01636	0.01959	0.00228	0.00248	0.00297	0.003138
1	2	-0.0417	-0.00240	-0.00719	0.00072	0.00010	0.00030	-0.00003	0.000138
1	3	0.06561	0.03384	0.01631	0.02362	0.00222	0.00107	0.00155	0.001362
2	1	0.14513	0.50196	0.33956	0.46124	0.07285	0.04928	0.06694	0.07693
2	2	0.06938	0.39132	0.41410	0.47651	0.02715	0.02873	0.03306	0.02307
2	3	-0.03085	-0.05997	-0.04700	-0.15397	0.00185	0.00145	0.00475	0.00824
2	4	0.01687	0.27860	0.21932	0.46295	0.00470	0.00370	0.00781	0.00131
2	5	0.08710	0.52009	0.40861	0.48439	0.04530	0.03559	0.04219	0.04869
3	1	0.0929	0.38062	0.29957	0.38375	0.03536	0.02783	0.03565	0.12124
3	2	0.0523	1.29388	1.01644	1.02983	0.06767	0.05316	0.05386	0.07420
3	3	-0.1837	0.35188	0.27670	-0.35030	-0.06464	-0.05083	0.06435	-0.02124
3	4	0.3383	0.48667	0.38262	0.48581	0.16464	0.12944	0.16435	0.07876
3	5	-0.1548	0.66557	0.52326	0.52300	-0.10303	-0.08100	-0.08096	0.04704
3	6	-0.2819	-0.16662	-0.13097	-0.14516	0.04697	0.03692	0.04092	0.10296
3	7	0.1067	1.43421	1.12690	1.49091	0.15303	0.12024	0.15908	0.09704

Table 3: Summary of the Statistical Evaluation of the Methods

Statistical Evaluation	Elimination	Least Squared	Numerical (Iteration)	Solver
Model of Selection Criterion (MSC)	2.8709	1.7519	1.2195	24.5549
Coefficient of Determination (CD)	0.9484	0.9485	0.7592	0.9751
Total Error	0.00511	0.00967	0.04209	0.0000
Correlation Coefficient (R)*	0.9739	0.9739	0.8713	0.9875
Root Error✧	0.07145	0.09835	0.20517	0.0000
Reliability (%)	91.09	73.43	58.89	99.99

* Obtained from Figure 6

✧Obtained from total error (Root Error $=\sqrt{\text{Total Error}}$

Table 4: Statistical Evaluation of the Methods

*Expected Values	Obtained Value				Errors				MSC				Reliability			
	Elimin.	Least Squared	Iteration	Solver	Elimin.	Least Squared	Iteration	Solver	Elimin	Least Squared	Iteration	Solver	Elimin	Least Squared	Iteration	Solver
0.0045	0.0045	0.0035	0.0045	0.0045	0.0000	0.0000	0.0000	0.0000	0.0031	0.0019	0.0051	0.0027	0.000	0.001	0.000	0.000
0.0030	0.0022	0.0022	0.0030	0.0030	0.0000	0.0000	0.0000	0.0000	0.0034	0.0020	0.0053	0.0028	0.001	0.001	0.000	0.000
0.0015	0.0023	0.0014	0.0015	0.0015	0.0000	0.0000	0.0000	0.0000	0.0034	0.0021	0.0055	0.0030	0.001	0.000	0.000	0.000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0036	0.0022	0.0058	0.0031	0.000	0.000	0.000	0.000
0.1000	0.1000	0.0780	0.1000	0.1000	0.0000	0.0005	0.0000	0.0000	0.0016	0.0009	0.0006	0.0019	0.000	0.022	0.000	0.000
0.0200	0.0257	0.0122	0.0200	0.0200	0.0000	0.0001	0.0000	0.0000	0.0012	0.0012	0.0031	0.0013	0.006	0.008	0.000	0.000
0.0500	0.0500	0.0393	0.0500	0.0500	0.0000	0.0001	0.0000	0.0000	0.0001	0.0001	0.0007	0.0000	0.000	0.011	0.000	0.000
0.0300	0.0243	0.0265	0.0300	0.0300	0.0000	0.0000	0.0000	0.0000	0.0013	0.0004	0.0021	0.0007	0.006	0.004	0.000	0.000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0036	0.0022	0.0058	0.0031	0.000	0.000	0.000	0.000
0.0000	0.0000	0.0000	0.0678	0.0000	0.0000	0.0000	0.0046	0.0000	0.0036	0.0022	0.0001	0.0031	0.000	0.000	0.068	0.000
0.2000	0.2000	0.1573	0.2000	0.2000	0.0000	0.0018	0.0000	0.0000	0.0195	0.0121	0.0154	0.0207	0.000	0.043	0.000	0.000
0.0000	0.0707	0.0557	0.0628	0.0000	0.0050	0.0031	0.0039	0.0000	0.0001	0.0001	0.0002	0.0031	0.071	0.056	0.063	0.000
0.1500	0.1561	0.1226	0.2773	0.1500	0.0000	0.0008	0.0162	0.0000	0.0092	0.0057	0.0405	0.0088	0.006	0.027	0.127	0.000
0.1000	0.1000	0.0786	0.2287	0.1000	0.0000	0.0005	0.0166	0.0000	0.0016	0.0010	0.0233	0.0019	0.000	0.021	0.129	0.000
0.2000	0.2000	0.1572	0.2000	0.2000	0.0000	0.0018	0.0000	0.0025	0.0195	0.0121	0.0154	0.0088	0.000	0.043	0.000	0.000
0.1500	0.1500	0.1179	0.1219	0.1500	0.0000	0.0010	0.0008	0.0025	0.0080	0.0050	0.0021	0.0207	0.000	0.032	0.028	0.000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0036	0.0022	0.0058	0.0031	0.000	0.000	0.000	0.000
0.0000	0.0001	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0036	0.0022	0.0058	0.0031	0.000	0.000	0.000	0.000

Where; Elimin = Elimination

*expected values were obtained based on continuity equations (sum of flows at a node is equal to zero) and headloss in a loop (sum of headloss in a closed loop is equal to zero).

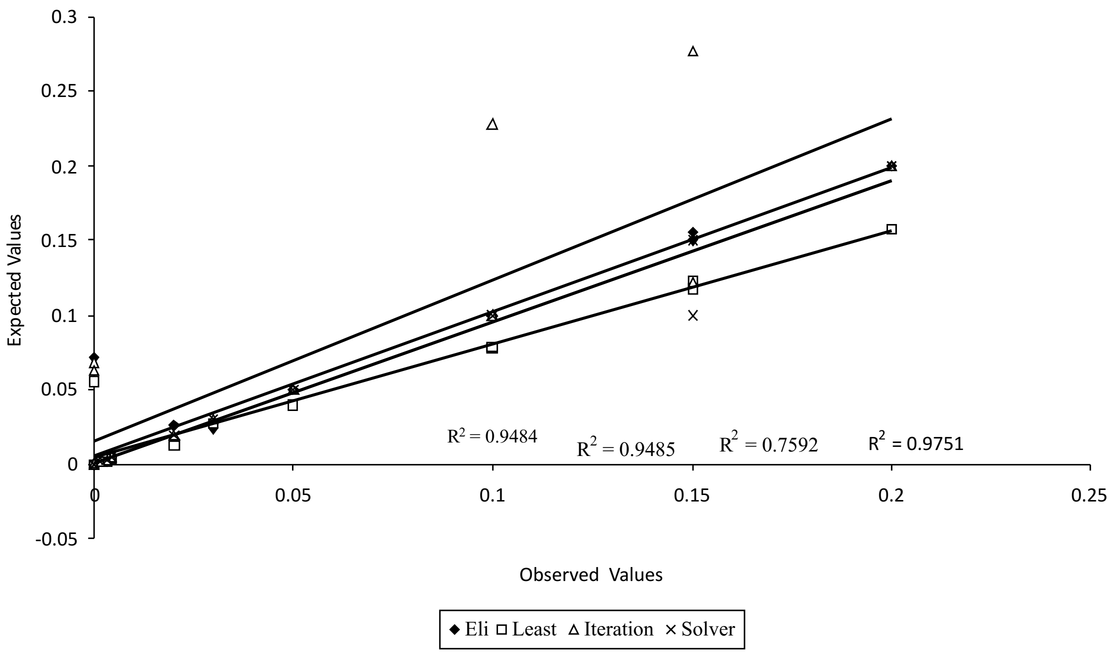

Figure 6: Statistical Evaluation of the Methods

The results show that Microsoft excel solver, least squared and elimination methods had the highest values of MSC and CD. Also, form the Table the values of correlation coefficient were 0.9739, 0.9739, 0.9875 and 0.8895 for elimination, least squared, Microsoft excel solver and numerical methods respectively. Like CD, the order of the accuracy is numerical less than (<) elimination < least squared < Microsoft excel solver. The reliabilities were in order of numerical (58.89 %), least squared (73.43 %), elimination (91.09 %) and Microsoft excel solver (99.99 %) which indicates that the order of the accuracy is numerical less than (<) least squared < elimination < Microsoft excel solver. In term of error, the values of total error and root error were 0.0051 and 0.0715; 0.0097 and 0.0984; 0.0000 and 0.0000; and 0.0421 and 0.2052 for elimination, least squared, Microsoft excel solver and numerical methods respectively. These imply that Microsoft excel solver, elimination and least squared methods are better than numerical method in solving linear theory equations (pipe network analysis) based on high values of CD, RD and MSC coupled with low values of errors (total and root errors).

CONCLUSIONS

The study was on evaluation of mathematical methods in solving linear theory equations. Three practical networks were used. Flows from these methods were evaluated statistically. It can be concluded based on the study that:

a) flow in pipes is a function of mathematical method used in the pipe network analysis;
b) Microsoft excel solver, Least squared and Gaussian elimination methods are the best based on the value of MSC, CD and RD;
c) The order of accuracy is numerical method less than (<) elimination method < least squared method < Microsoft excel solver method based on MSC, CD, RD and errors;
d) There is the need to conduct economics evaluation of the methods based on the headloss across the pipes in the loops to ascertain the reliability of the methods.

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Appendices; Detailed Solution of the Problems by Microsoft Excel Solver

Appendix A (Example 1)

Pipe No	D (mm)	L(m)	f	K	Q'	K'
1	250	1000	0.02	1690.791	0.0021	3.551
2	150	1200	0.02	26092.45	0.001	26.092
3	200	1750	0.02	9029.796	0.0012	10.836

Target	
Sum of Headloss in a loop	2.40016E-14*
Variables (Flows)	
Q1	0.003138385
Q2	0.000138385
Q3	0.001361615
Constraint	
At A	0.0045
At B	0.003
At C	0.0015

* E-14 = 10^{-14}

Appendix B (Example 2)

Pipe No	D (mm)	L(m)	f	K	Q'	K'
1	250	1000	0.02	1690.791	0.0021	3.551
2	200	1500	0.02	7739.825	0.0021	16.254
3	150	1800	0.02	39138.67	0.001	39.139
4	100	1000	0.02	165116.3	0.0012	198.140
5	200	1200	0.02	6191.86	0.0012	7.430

Target	1.65368E-13*
Sum of Headloss in a loop	
Variables (Flows)	
Q1	0.076925972
Q2	0.023074028
Q3	0.008237511
Q4	0.001311539
Q5	0.048688461
Constraint	
At A	0.1000
At B	0.0200
At C	0.0500
at D	0.0300

Appendix C (Example 3)

Pipe No	D (mm)	L(m)	f	K	Q'	K'
1	200	800	0.02	4127.907	0.0021	8.669
2	300	750	0.02	509.6181	0.0021	1.070
3	250	1000	0.02	1690.791	0.001	1.691
4	300	500	0.02	339.7454	0.0012	0.408
5	250	800	0.02	1352.632	0.0012	1.623
6	150	600	0.02	13046.22	0.0012	15.655
7	200	800	0.02	4127.907	0.0012	4.953

Target	-5.9591E-14▽
Sum of Headloss in a loop	
Variables (Flows)	
Q1	0.121243334
Q2	0.074202105
Q3	-0.02124333
Q4	0.078756666
Q5	0.047041229
Q6	0.102958771
Q7	0.097041229
Constraint	
At A	0.2000
At B	0.0000
At C	0.1500
At D	0.1000
At E	0.1500
At F	0.2000

* E-13 = 10^{-13}
▽ E-14 = 10^{-14}