In this work, relatively cheap and easy to measure surface meteorological data such as ambient temperature, relative humidity, vapour pressure and dew-point temperature, had been collected from the Nigerian Meteorological Agency (NIMET). Using the analytically derivable Butler’s equation, precipitable water vapour (PWV) had been estimated. For the purpose of this work, Nigeria was divided into four climatic regions and for each region empirical models of the form PWV (T_d) and PWV (RH) were found; where T_d was the surface dew-point temperature and RH the surface relative humidity. The empirical models found were validated against the NCEP-DOE Reanalysis II data and found to be useful in estimating precipitable water vapour.

**Keywords:** Precipitable Water Vapour, Reanalysis II, Relative Humidity, Dew-point Temperature

**ABSTRACT**

Precipitable water vapour is by employing radiosonde (Willoughby et al., 2008) by which a vertical profile of the precipitable water vapour could be made, but radiosonde observations are not regularly made due to the expensiveness of the method. Other more expensive and technically more advanced methods of retrieval such as sun-photometer, water vapour radiometer (WVR), satellite retrieving method, Global Positioning System (GPS) receiver methods have been employed and their results found to be in good agreement with the radiosonde method (e.g., England et al., 1992; Coster et al., 1996; Linfield et al., 1997; Boccolari et al., 2002; Bai and Feng., 2003; Jade et al., 2005; Park et al., 2012).

The Butler’s equation $PWV = \frac{m_w e_o H}{\rho_i k T_o}$ could be obtained for a column of condensed liquid water with cross-sectional area A, and height h, from the atmospheric column extending from the earth’s surface to the top of the atmosphere, where $m_w$ the molecular weight of water $(=18 \text{amu} \times 1.66 \times 10^{-27} \text{kg})$, $\rho_i$ is the density of liquid water $(=1000 \text{kgm}^{-3})$, H is the scale height (assumed to be 1.5 km), $k$ is the Boltzmann’s constant $(1.38 \times 10^{-23} \text{J/K})$, $e_o$ is the surface water vapour pressure (in $\mu$bar) and $T_o$ is the surface temperature (in Kelvin). Using the values provided, the PWV (in mm) will be given as
\[ PWV \approx \frac{1}{3} \frac{e_o}{T_o} \]

The form of Equation 2, as shown in Equation 3 was used by Reitan (1963) to estimate PWV for the continental U.S.A. Leckner (1978) also employed the variant for the estimation of PWV in Canada as shown in Equation 4

\[ PWV = \frac{483e_o}{T} \]

\[ PWV = \frac{493e_o}{T} \]

where \( e_o \) is the surface water vapour pressure (in millibar), and \( T \) is the surface ambient temperature (in Kelvin), and PWV is measured in millimetre.

Reitan (1963), studying monthly means of PWV and station dew-point temperature \( T_d \) over the continental U.S.A, developed a linear relation between natural logarithm of the PWV (cm) and \( T_d \) (C) i.e.,

\[ \ln(PWV) = 0.1102 + 0.0613T_d \]

Won (1977), quoted by Okulov et al. (2002), developed a similar expression for all Canadian seasons in 1977. In Nigeria, Maduekwe and Ogunmola (1997), as well as Utah and Abimbola (2006) found a form of Equation 5 for Sokoto and Jos respectively.

It is our aim in this work to find some suitable empirical equations relating precipitable water vapour (PWV) with dew-point temperature \( T_d \) and relative humidity (RH) for the estimation of precipitable water vapour across Nigeria.

METHODOLOGY
For this work, monthly averaged surface meteorological data were obtained from the Nigerian Meteorological Agency (NIMET) for the stations at Sokoto (13.07° N, 5.23° E), Katsina (12.25° N, 7.50° E), Kano (12.00° N, 8.52° E), Maiduguri (11.83° N, 13.15° E), Ilorin (8.50° N, 4.55° E), Abuja (9.07° N, 7.48° E), Makurdi (7.73° N, 8.54° E), Yola (9.23° N, 12.46° E), Ibadan (7.40° N, 3.92° E), Osogbo (7.77° N, 4.57° E), Enugu (6.50° N, 7.50° E), Ikorodu (5.25° N, 3.57° E), Port Harcourt (4.75° N, 7.00° E) and Calabar (4.95° N, 8.33° E). For the purpose of this study, Nigeria was categorized into four climatic regions; the northern, central, southern and the coastal region, as shown in Figure 1.

Microsoft Excel® software was used for data preparation while the MATLAB® software was used for plotting, curve fittings and other analysis (cftool function of MATLAB®). The NCEP-DOE AMP II Reanalysis (known as NCEP-DOE Reanalysis II), numerical weather prediction data were obtained in a Network Common Data Form (NetCDF) file format from the Earth System Research Laboratory, Physical Sciences Division of the National Oceanic and Atmospheric Administration (NOAA), United State of America (ftp://ftp.cdc.noaa.gov/Datasets/ncep.reanalysis2.derived/surface/pr_wtr.eatm.mon.mean.nc); The data obtained had 2.5° by 2.5° global grid resolution and the nearest grid to the validation station were used. The NetCDF file was analysed using the Panoply open source software (ver. 4.3.1).

The statistical tools used for the data analysis were the coefficient of determination (\( R^2 \)), summed-squared error (SS-error) (Equation 6) and the root-mean-squared error (RMS error) (Equation 7), below

\[ SS – error = \frac{1}{n} \sum_{i=1}^{n} (y_{it} - y_{im})^2 \]

where \( n \) is the number of data points; \( y_{it} \) is the raw data value and \( y_{im} \) is the value obtained from the empirical model. A value of SS-error closer to 0 indicates that the empirical model/equation has a smaller random error component.

\[ RMS – error = \left( \frac{1}{n} \sum_{i=1}^{n} (y_{it} - y_{im})^2 \right)^{\frac{1}{2}} \]

where all parameters are as defined for Equation 6: the more the value of RMS-error approaches 0, the more the empirical model/equation obtained is useful as a predictor.
RESULTS AND DISCUSSION
The plot of natural logarithm of PWV against the dew point temperature $T_d$ for the four regions of Nigeria is as shown on Figure 2. Generally the distributions on Figure 2 are linear distributions and the empirical equations for these distributions are given in Equations 8, 9, 10 and 11 for the Northern, Central, Southern and the Coastal regions respectively

$$\ln(PWV) = 0.065T_d + 2.36$$  \hspace{1cm} 8
$$\ln(PWV) = 0.055T_d + 2.57$$  \hspace{1cm} 9
$$\ln(PWV) = 0.050T_d + 2.68$$  \hspace{1cm} 10
$$\ln(PWV) = 0.053T_d + 2.64$$  \hspace{1cm} 11

The coefficient of determination ($R^2$) for Equation 8 was found to be 0.995, the SS-error was found to be 0.60 and the RMS-error was found to be 0.04; for Equation 9, $R^2 = 0.998$, SS-error = 0.077 and RMS-error = 0.015; for Equation 10, $R^2 = 0.998$, SS-error = 0.0094 and RMS-error = 0.0063; for Equation 11, $R^2 = 0.995$, SS-error = 0.0084 and RMS-error = 0.0049.

Equations 8 to 11 show good coefficients of determination and this is expected following the work of researchers such as Reitan (1963), Ojo (1970), Maduekwe and Ogunmola (1997) and Utah and Abimbola (2006).

As observed by Utah and Abimbola (2006) the slope of PWV ($T_d$) could be used as an indicator of the moisture burden of a place, that is, the more the slope value the less water vapour mixing ratio. The slopes in Equations 8 to 11 indicate a gradual decrease in water vapour mixing ratio from the coast to the northern part of the country. However, Equations 10 and 11 show that the coastal region has a lower water vapour mixing ratio than the southern region, that is, the southern region atmosphere is slightly wetter than that of the coastal region. The maritime tropical air mass (mT) blowing in from the Atlantic will condense into clouds and some precipitations on making contact with the coastal land, the clouds will then be blown into the warmer southern region where the clouds will vapourize and increase the water vapour mixing ratio in this region.
Relative humidity (RH) is a measure of atmospheric moisture and a basic meteorological parameter; hence an attempt was made to develop empirical relation between the PWV and RH (Figure 3). For all the plots, quadratic fits were found suitable as shown in Equations 12 to 15 for Northern, Central, Southern and Coastal regions respectively. For Equation 12, SS-error was found to be 9534.61, RMS-error was found to be 5.25 while the $R^2$ was found to be 0.864. For the Central region (Eq. 13) the SS-error = 5025.30; RMS-error = 3.77 and $R^2 = 0.866$. For the Southern region (Eq. 14), the SS-error = 615; RMS-error = 1.62 and $R^2 = 0.913$. For the Coastal region (Eq. 15), we have SS-error = 1291.74; RMS-error = 1.92 and $R^2 = 0.652$

\[
\text{PWV} = -0.0063(RH)^2 + 1.26(RH) - 14.47 \quad 12
\]

\[
\text{PWV} = -0.012(RH)^2 + 1.84(RH) - 25.40 \quad 13
\]

\[
\text{PWV} = -0.026(RH)^2 + 4.06(RH) - 108.50 \quad 14
\]

\[
\text{PWV} = -0.056(RH)^2 + 9.33(RH) - 334.20 \quad 15
\]

The good correlation between the PWV and relative humidity is not unexpected since relative humidity is also a measure of atmospheric moisture content. However, the dew point temperature being a more direct measure of atmospheric moisture content than the relative humidity, generally show a better correlation with the PWV than the relative humidity does with PWV.

**Figure 2:** The Plot of Natural Logarithm of Precipitable Water Vapour $\ln$PWV, against Dew-point Temperature for (A) Northern Region; (B) Central Region; (C) Southern Region; (D) Coastal Region.
The regional variation in the coefficient of determination from the coastal region to the northern region for Equations 12 to 15, as indicated on Table 1, follows the same pattern as discussed for Equations 8 to 11.

Figure 3: Plot of Precipitable Water Vapour against Surface Relative Humidity for (A) Northern Region; (B) Central Region; (C) Southern Region; (D) Coastal Region.

In order to test the viability of the derived empirical models, the results of the empirical models were compared with the observed PWV for a location within each region and also with the PWV obtained from the NCEP-DOE Reanalysis 2 numerical weather prediction model. The results are shown in Figures 4 and 5. The northern region is compared with the PWV observed at Katsina, while the central, southern and coastal regions were compared with PWV observed at Ilorin, Osogbo and Port Harcourt respectively. The statistical results of these comparisons are shown in Tables 1 and 2. The results, as shown in Table 1, is expected as the observed PWV and calculated PWV were all obtained using the same surface meteorological data, whereas, NCEP-DOE Reanalysis II is a model derived PWV estimation for the entire atmospheric column and also due to its global grid resolution, correlation with the surface data derived PWV will be limited.

Statistical results of comparison of the derived empirical models with the NCEP-DOE Reanalysis 2 is shown in Table 2. It could be observed that the only significantly good correlation [with correlation coefficient of about 0.996 for PWV(Td) and 0.793 for PWV(RH)] was found in the Northern region and the correlations became worse progressively as we move towards the coast. This result suggests that the derived empirical models from the surface data gave more accurate result for a dry atmosphere and became
less accurate as the atmosphere became much laden with moisture. This could be due to the fact that the high variability of atmospheric water vapour introduces error in the estimation of precipitable water vapour.

Table 1: The Statistical Results of Comparison of the Precipitable Water Vapour obtained from the derived Empirical Models [PWV(T) and PWV(RH)] with the Observed PWV for each Region (Katsina for North, Ilorin for Central, Osogbo for South and Port Harcourt for the Coast).

<table>
<thead>
<tr>
<th>Region</th>
<th>PWV(T)</th>
<th>PWV(RH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>0.996</td>
<td>0.793</td>
</tr>
<tr>
<td>RMS error(mm)</td>
<td>0.93</td>
<td>6.37</td>
</tr>
<tr>
<td>Central</td>
<td>0.995</td>
<td>0.885</td>
</tr>
<tr>
<td>RMS error(mm)</td>
<td>0.42</td>
<td>2.06</td>
</tr>
<tr>
<td>South</td>
<td>0.998</td>
<td>0.904</td>
</tr>
<tr>
<td>RMS error(mm)</td>
<td>0.25</td>
<td>1.56</td>
</tr>
<tr>
<td>Coast</td>
<td>0.994</td>
<td>0.830</td>
</tr>
<tr>
<td>RMS error(mm)</td>
<td>0.28</td>
<td>1.51</td>
</tr>
</tbody>
</table>

Table 2: The Statistical Results of Comparison of the Precipitable Water Vapour obtained from the derived Empirical Models [PWV(T) and PWV(RH)] with the NCEP-DOE Reanalysis 2 Numerical Weather Prediction Model.

<table>
<thead>
<tr>
<th>Region</th>
<th>PWV(T)</th>
<th>PWV(RH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>0.893</td>
<td>0.628</td>
</tr>
<tr>
<td>RMS error(mm)</td>
<td>4.35</td>
<td>8.26</td>
</tr>
<tr>
<td>Central</td>
<td>0.518</td>
<td>0.257</td>
</tr>
<tr>
<td>RMS error(mm)</td>
<td>8.01</td>
<td>10.37</td>
</tr>
<tr>
<td>South</td>
<td>0.313</td>
<td>0.160</td>
</tr>
<tr>
<td>RMS error(mm)</td>
<td>6.28</td>
<td>7.48</td>
</tr>
<tr>
<td>Coast</td>
<td>0.236</td>
<td>0.215</td>
</tr>
<tr>
<td>RMS error(mm)</td>
<td>5.29</td>
<td>6.78</td>
</tr>
</tbody>
</table>
Figure 4: Time Series Comparison Plot of the derived Model Calculated PWV with the Observed and Reanalysis 2 PWV: (A) Northern Region with observed PWV at Katsina, (B) Central Region with Observed PWV at Ilorin.
CONCLUSION

Using monthly averaged surface meteorological data obtained from the Nigerian Meteorological Agency (NIMET) for stations across Nigeria, for years 2002 to 2011, empirical models for the estimation of precipitable water vapour were obtained for the Northern, Central, Southern and Coastal regions of Nigeria. It was found that the precipitable water vapour estimated from the dew point [i.e., PWV(Td)] generally performs better with good coefficient of determination (R²) and root mean square (RMS) error than the precipitable water vapour estimated from the relative humidity [i.e., PWV(RH)]. It was also shown that these empirical models are better predictors of precipitable water vapour in a drier atmosphere.

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