ANALYSIS OF ELASTIC SCATTERING CROSS SECTIONS OF \(^{16}\text{O}\) ON \(^{27}\text{Al}\) AND \(^{154}\text{Sm}\) USING THE SEMI-MICROSCOPIC DOUBLE FOLDING MODEL

Olorunfunmi, S. D. and Olatinwo, A. S.

Department of Physics and Engineering Physics, Obafemi Awolowo University, Ile-Ife, 220005, Osun State, Nigeria.

Corresponding Author's E-mail(s): sundayolorunfunmi@gmail.com; solorunfunmi@oauife.edu.ng

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The elastic scattering data for \(^{16}\text{O} + ^{27}\text{Al}\) at a laboratory energy (\(E_{\text{lab}}\)) of 134 MeV and \(^{16}\text{O} + ^{154}\text{Sm}\) at \(E_{\text{lab}} = 85\) and 134 MeV were analyzed using the optical model-based double-folding model. The real component of the optical model potential was generated from the microscopic double-folding (DF) model, while the imaginary part was considered using both microscopic and Woods-Saxon phenomenological forms. Two density-independent effective nucleon-nucleon (NN) interactions were considered in the DF procedure: the Michigan-3-Yukawa (M3Y) and Knyazkov and Hefter (KH). The folded potential was constructed using Two-parameter Fermi (2pF) density distribution for the target nuclei and three different forms of projectile density: Two-parameter Fermi (2pF), Slater determinants consisting of harmonic oscillator single-particle wave functions (SDHO), and Dirac-Hartree-Bogoliubov (DHB) density distributions. The SDHO density exhibited slightly better agreement with the data than 2pF and DHB. The results obtained using the KH interaction were highly consistent with those achieved with the M3Y interaction. In general, the DF model-based calculations compared reasonably well with the experimental data.

Keywords: Elastic scattering, Optical model, Double folding model, Density-independent potential.

INTRODUCTION

Nuclear reactions occur in various forms, such as elastic scattering, inelastic scattering, nucleon-transfer reaction, knock-out reaction, and others. These reactions can be described using different nuclear models. Elastic scattering is often investigated using the optical model (OM) approach, which involves a complex potential with real and imaginary parts. The real part represents the reflection of the projectile waves by the target nucleus, while the imaginary part describes their absorption.

When analyzing the elastic scattering data, the real and imaginary potentials can be represented in different forms. One form is the phenomenological approach, where both the real and imaginary potentials assume the Woods-Saxon (WS) shape. However, a major drawback of the phenomenological WS form is that it contains a large number of free parameters (at least six parameters), which leads to ambiguities. These parameter values are adjusted to provide an accurate fit to the elastic scattering data.

Another approach, with fewer free parameters and ambiguities, is to construct the real part of the OM potential using the double folding (DF) model and let the imaginary part assume a phenomenological WS form. The third approach is a complete microscopic form, where both the real and imaginary potentials are constructed from the DF model. In this last approach, the number of free parameters is very small, and consequently, the ambiguities are greatly reduced.

In the present study, we have employed the last two approaches. Specifically, in the DF model, the optical potential is obtained by folding a suitable effective nucleon-nucleon (NN) interaction over the density distributions of the two interacting nuclei (Satchler and Love, 1979). Some of the commonly used effective NN interactions are Michigan-3-Yukawa (M3Y) (Bertsch et al., 1977), Knyazkov and Hefter (KH) (Knyazkov and Hefter, 1981), Jeukenne, Lejeune and Mahaux (JLM) (1977), etc. Detailed review of the DF model is presented in Brandan et al., (1997).

In recent years, there has been a renewed interest in multinucleon transfer reactions between heavy ions, particularly at energies around and above the Coulomb barrier (Corradi et al., 2009; Zhang et al., 2018; Sekizawa, 2019; Adamian et al., 2020; Roy et al., 2018; and Roy et al., 2022). The study of
The DF model offers a more comprehensive and reliable approach to analyze nuclear reactions compared to the phenomenological optical models.

The aim of this study is to reanalyze the recently measured $^{16}\text{O} + ^{27}\text{Al} (E_{\text{lab}} = 134 \text{ MeV})$ (Roy et al., 2018) and $^{16}\text{O} + ^{154}\text{Sm} (E_{\text{lab}} = 85 \text{ and } 134 \text{ MeV})$ (Roy et al., 2022) elastic scattering data using the OM based DF model. Specifically, we will calculate the real component of the OM potential using DF model with two different forms of effective nuclear interactions: M3Y and KH. Additionally, we will consider both the microscopic and WS forms of the imaginary potential.

Furthermore, we aim to test the ability of three different projectile density distributions to describe the reactions considered.

**MATERIALS AND METHODS**

**Double folding model**

In the double folding model, the real part of the OM potential is usually expressed as

$$V_n(r) = \int dr_1 \int dr_2 \rho_p(r_1)\rho_t(r_2)u_{\text{NN}}(|r_{12}|).$$  \hspace{0.5cm} (1)

where $r_{12} = |r - (r_1 - r_2)|$, $u_{\text{NN}} = \text{effective NN interaction}$, $\rho_p = \text{density distribution of the projectile nucleus}$, and $\rho_t = \text{density distribution of the target nucleus}$.

**Effective NN interaction**

Two forms of effective NN interactions are considered in this study. The first is the popular density-independent M3Y interaction derived by Bertsch et al., and parametrized by Satchler and Love as,

$$u^{\text{M3Y}}_{\text{NN}} = \exp(-4r^2) - 2134\exp(-2.5r^2) - 276\left[1 - 0.005\frac{E_{\text{lab}}}{A_p}\right]\delta(r) \text{ MeV},$$  \hspace{0.5cm} (2)

where $E_{\text{lab}} = \text{laboratory energy in MeV}$, and $A_p = \text{mass number of the projectile}$.

The direct part of the interaction potential is represented by the first and second terms in equation (2), while the third term is the exchange...
part estimated using a zero-range pseudopotential.

The second effective NN interaction is a Gaussian form of the effective NN interaction derived by Knyazkov and Hefter (1981) parametrized as follows:

\[ u_{\text{effective}}^{\text{NN}}(r) = v_1 \exp(-r^2/a_1^2) + v_2 \exp(-r^2/a_2^2) - 276 \left[ 1 - 0.005 \frac{E_{\text{lab}}}{A_p} \right] \delta(r), \]

where \( v_1 = -601.99 \text{ MeV}, v_2 = 2256.4 \text{ MeV}, a_1 = 0.8 \text{ fm} \) and \( a_2 = 0.5 \text{ fm} \), and are taken from Knyazkov and Hefter, 1981.

**Projectile density**

In the present work, three different forms of density distributions are considered for the \(^{16}\text{O}\) projectile nucleus. One of them is the Two-parameter Fermi (2pF) density distribution, which is expressed as follows (Seif and Mansour, 2015)

\[ \rho_{\text{2pF}}(r) = \rho_{\text{om}}(r) \left[ 1 + \exp \left( \frac{r - R_{\text{om}}(r)}{a_{\text{om}}(r)} \right) \right]^{-1}, \]

where \( R_{\text{om}}(r) = \text{neutron (proton) half-density radius, and } a_{\text{om}}(r) = \text{neutron (proton) surface thickness parameter.} \)

These parameters are parameterized as follows (Seif and Mansour, 2015):

\[ R_n = 0.953N^{1/3} + 0.0152Z + 0.774p_n = 0.446 + 0.0072 \left( \frac{N}{Z} \right), \]

\[ R_p = 1.322Z^{1/3} + 0.007W + 0.022p_p = 0.449 + 0.0071 \left( \frac{Z}{N} \right), \]

where \( N \) and \( Z \) are the neutron and proton numbers of the projectile nucleus. The second density distribution considered for \(^{16}\text{O}\) projectile nucleus is the one obtained by Ahmad et al., (2017) using the Slater determinants consisting of the harmonic oscillator single-particle wave functions. This is denoted as SDHO and parametrized as:

\[ \rho_{\text{SDHO}}(r) = \rho_0 \left[ 1 + \exp \left( \frac{r - c}{a} \right) \right]^{-1}, \]

where \( c = 2.845 \text{ fm}, Z = 0.569 \text{ fm for } ^{27}\text{Al} \) (Jager et al., 1974), and \( c = 5.9387 \text{ fm}, Z = 0.522 \text{ fm for } ^{154}\text{Sm} \) (De Vries et al., 1987). The central density \( \rho_0 \) can be obtained from the normalization condition:

\[ 4\pi \int \rho(r)r^2 \, dr = A, \]

where \( A \) is the mass number of the nucleus.
The Coulomb potential is expressed as,

\[ V_C(r) = \frac{1}{4\pi\varepsilon_0} \frac{Z_p Z_T e^2}{r} \quad r \geq R_c \]

\[ = \frac{1}{4\pi\varepsilon_0} \frac{Z_p Z_T e^2}{2R_c} \left(3 - \frac{r^2}{R_c^2}\right) \quad r < R_c \]

with \( R_c = r_c \left(\frac{A_p^{1/3} + A_T^{1/3}}{2}\right) \).

The real component \( V(r) \) of the OM potential is obtained using the double folding model as expressed in equation (1), with a renormalization factor \( N_r \). The imaginary part of the potential is considered to be of two forms. The first one is the phenomenological WS form given by

\[ W(r) = \frac{W_i}{1 + \frac{r}{\sigma}} \]
where \( W, r, \) and \( a \) are the depth of the imaginary potential, the reduced radius, and the diffuseness parameter, respectively. In this case, the total OM potential (equation (12)) becomes

\[
U(r) = V_c(r) - \sum N_i V_F(r) - iW(r).
\]

(16)

For this case, we denote the total potentials as 2pF(R), SDHO(R) and DHB(R) using the density distributions 2pF, SDHO, and DHB, respectively, as \(^{16}\text{O}\) density. The second one is the case where the imaginary phenomenological WS form is replaced with microscopic real folded potential. Then the total OM potential is expressed as

\[
U(r) = V_c(r) - (N_0 + iN_1)V_F(r).
\]

(17)

Similarly, for the second case, we denote the total potentials as 2pF(R+I), SDHO(R+I) and DHB(R+I) using the density distributions 2pF, SDHO, and DHB, respectively, as \(^{16}\text{O}\) density.

To evaluate the level of agreement between the calculated results and experimental data, the imaginary reduced radius \( r \) and diffuseness \( a \) were held constant at 1.2518 fm and 0.601 fm, respectively, and a search on the depth of the imaginary potential \( (W_i) \) and the renormalization constants \((N_0, \) and \( N_1) \) was carried out using the reduced chi-square:

\[
\frac{\chi^2}{N} = \sum \frac{[\sigma_{\text{th}}(\theta_k) - \sigma_{\text{ex}}(\theta_k)]^2}{\Delta \sigma_{\text{ex}}(\theta_k)}. \tag{18}
\]

Here, \( \sigma_{\text{th}}(\theta_k) \) = theoretical cross section,

\( \sigma_{\text{ex}}(\theta_k) \) = experimental cross sections,

\( \Delta \sigma_{\text{ex}}(\theta_k) \) = experimental error, and

\( N = \) number of data points.

In the study of elastic scattering cross sections, the volume integral is an important quantity, as it provides information about the nuclear density distribution and the interaction between interacting nuclei. In this study, we calculated the real \((J_R)\) and imaginary \((J_I)\) volume integrals using the formulae (Satchler and Love, 1979):

\[
J_R(E) = -\frac{4\pi}{A_p A_T} \int V(r,E) r^2 dr, \tag{19}
\]

and

\[
J_I(E) = -\frac{4\pi}{A_p A_T} \int W(r,E) r^2 dr. \tag{20}
\]

Finally, the root mean square radius of the DF potential is obtained from the expression (Varner et al., 1991):

\[
\langle r^2 \rangle^{1/2} = \sqrt{\frac{\int V_F(r)r^2 dr}{\int V_F(r)r^2 dr}}. \tag{21}
\]

The rms radius of the nucleus is an important quantity in nuclear physics that provides information about the size and structure of the nucleus.

RESULTS AND DISCUSSION

The DF model is used to calculate the real part of the optical potentials for the \(^{16}\text{O} + ^{27}\text{Al} \) reaction at 134 MeV and the \(^{16}\text{O} + ^{154}\text{Sm} \) reaction at 85 and 134 MeV, with M3Y and KH effective interactions and three different forms of projectile densities (2pF, SDHO and DHB). Figures 2 and 3 show the typical calculated DF potentials for \(^{16}\text{O} + ^{27}\text{Al} \) and \(^{16}\text{O} + ^{154}\text{Sm} \) at 134 MeV, respectively, using the aforementioned interactions and densities. It can be observed that the M3Y and KH interactions produce DF potentials with similar depths and shapes. Furthermore, the DF potential obtained from the DHB density has a slightly deeper depth than that obtained from the 2pF and SDHO distributions. This is because the DHB gives the highest density value at the center of the nucleus compared to the other two densities as shown in Figure 1(a).
Table 1 presents the root mean square (rms) radii, denoted as \( r_{\text{rms}} \) and \( R_{\text{rms}} \), calculated for the DF potentials using M3Y and KH interactions with the 2pF, SDHO, and DHB densities. As shown in the table, the rms radius of the DF potential obtained from DHB is smaller than that for the 2pF and SDHO densities. The difference is about one percent, hence, it is of no consequence. Additionally, we observe that the rms radii increase as the mass number increases for all densities. It is worth noting that the rms radii obtained with the M3Y interaction display a behavior and value that are apparently similar to those obtained with the KH interaction.

![Figure 2](image1.png)

**Figure 2**: Real folded potentials for \( ^{16}\text{O}+^{27}\text{Al} \) at 134 MeV using 2pF, SDHO and DHB density distributions with (a) M3Y and (b) KH effective interactions.

![Figure 3](image2.png)

**Figure 3**: Same as Figure 2 but for \( ^{16}\text{O}+^{154}\text{Sm} \) at 134 MeV.
To compute the elastic scattering cross sections using M3Y and KH, two methods were utilized. Method one employs a microscopic folded potential for both the real and imaginary components of the nuclear potential. In method two, a folded potential is used for the real part while the imaginary part is modeled using the Woods-Saxon form. The calculated results based on the 2pF, SDHO, and DHB density distributions with folded real and imaginary Woods-Saxon potentials are labeled as 2pF(R), SDHO(R), and DHB(R), respectively. The calculated results based on the 2pF, SDHO, and DHB density distributions with folded real and imaginary potentials are labeled as 2pF(R+I), SDHO(R+I), and DHB(R+I), respectively.

The elastic scattering cross sections of the $^{16}\text{O} + ^{27}\text{Al}$ and $^{16}\text{O} + ^{154}\text{Sm}$ reactions at 134 MeV and 85 MeV, respectively, were computed using the potentials 2pF(R), SDHO(R), DHB(R), 2pF(R+I), SDHO(R+I), and DHB(R+I) with both M3Y and KH interactions. The resulting elastic scattering cross sections normalized to Rutherford cross sections were compared to experimental data and presented in Figures 4 to 6. Tables 2 and 3 show the values of $N_a$, $N_d$, $J_a$, $J_d$, $W$, $\sigma_I$, and $X^2/N$, that produced good agreement with the data.

**Table 1:** Root mean square radii $<r^2>^{1/2}$ of the calculated DF potentials with M3Y and KH effective interactions, and three densities (2pF, SDHO and DHB) for $^{16}\text{O} + ^{27}\text{Al}$ and $^{16}\text{O} + ^{154}\text{Sm}$ reactions.

<table>
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<th>Energy (MeV)</th>
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<th>$&lt; r_{\text{M3Y}}^2 &gt;^{1/2}$ (fm)</th>
<th>$&lt; r_{\text{KH}}^2 &gt;^{1/2}$ (fm)</th>
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<td>DHB</td>
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**Figure 4:** Elastic scattering cross sections for $^{16}\text{O} + ^{27}\text{Al}$ at 134 MeV calculated using 2pF, SDHO and DHB projectile densities with (a) M3Y and (b) KH interactions. The experimental data are taken from Roy et al., 2018.
### Table 2: Real renormalization constants \( (N_r) \), depth of the imaginary potential, real and volume integrals \( (J_R \) and \( J_I) \), total reaction cross sections \( (\sigma) \) and \( \chi^2/N \) values for \(^{16}\text{O}+^{27}\text{Al}\) and \(^{16}\text{O}+^{154}\text{Sm}\) elastic scattering using the M3Y and KH potentials. The imaginary radius \( r_I \) and diffuseness \( a \) are fixed at 1.2518 fm and 0.601 fm, respectively.

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### Table 3: Real and imaginary renormalization constants \( (N_r \) and \( N_i) \), real and imaginary volume integrals \( (J_R \) and \( J_I) \), total reaction cross sections \( (\sigma) \) and \( \chi^2/N \) values for \(^{16}\text{O}+^{27}\text{Al}\) and \(^{16}\text{O}+^{154}\text{Sm}\) elastic scattering using the M3Y potential.

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Figure 5: Same as Figure 4 but for $^{16}\text{O} + ^{154}\text{Sm}$ at 85 MeV. The experimental data are taken from Roy et al., 2022.

Figure 6: Same as Figure 4 but for $^{16}\text{O} + ^{154}\text{Sm}$ at 134 MeV. The experimental data are taken from Roy et al., 2022.
Figure 4 illustrates the results obtained for the $^{16}$O + $^{27}$Al system at 134 MeV with M3Y and KH interactions. The calculated results show a reasonable agreement with the experimental data. However, Tables 2 and 3 indicate that, for the three densities (2pF, SDHO, and DHO), the values of the renormalization constants ($N_R$ and $N_i$) required for the DF potentials to reproduce the experimental data successfully are significantly different from unity. The interactions with very low $N_R$ and $N_i$ are too deep or strong. This might be attributed to the nature of the experimental data. Additionally, the experimental data in Figure 4 covers only a limited range of angles, $\theta_c.m. = 10^\circ$ - 40$^\circ$. Consequently, it is advisable to measure data at larger angles to study the realistic effect of the considered potentials on the elastic scattering results.

Moreover, the values of $\sigma_R$ obtained for the $^{16}$O + $^{27}$Al system in this study are approximately 15% higher on average than those obtained using phenomenological potentials in Ref. (Roy, 2018).

We present the results of our analysis of two data sets for the $^{16}$O + $^{154}$Sm system at energies of 85 and 134 MeV in Figures 5 and 6, respectively. Figures 5(a) and 6(a) show the results obtained with the M3Y interaction, while Figures 5(b) and 6(b) show those obtained with the KH interaction. It can be observed that the elastic scattering cross sections obtained with all the densities and their corresponding DF potentials are in good agreement with the data. Additionally, Tables 2 and 3 reveal that all densities yield comparable values of $\chi^2/N$ at both energies. Although, the DHB density slightly performs best. Moreover, we note that the values $N_R$ of increase from the range 0.7 to 1.0 at 85 MeV to approximately 1.0 to 1.3 at 134 MeV. Furthermore, the values of $\sigma_R$ increase by around 100% as the incident energy increases from 85 MeV to 134 MeV, which is expected. Finally, the values of $\sigma_R$ obtained in this study using the DF model are in close agreement with those obtained using phenomenological potentials ($\sigma_R = 1182$ and 2357 mb) (Roy et al., 2022) with a difference of less than 5%.

After analyzing the M3Y and KH effective interactions, we found that both interactions require nearly the same renormalization constant to describe the experimental data, as shown in Tables 2 and 3. Moreover, the values of $\sigma_R$ for M3Y are slightly higher than those for the KH interaction. We also observed that both interactions provide similar fits to the data, which is evident from the values of $\chi^2/N$ presented in Tables 2 and 3. Interestingly, all the density distributions for each effective interaction provide comparable quality of fits, with only a $\sim 5\%$ variation in the reaction cross-sections and similar behavior of the potential renormalization factors. However, the DHB density distribution performs best.

CONCLUSION

DF optical model potentials were developed using M3Y and KH interactions, and three distinct projectile density forms, namely 2pF, SDHO, and DHB. These potentials were then employed to study elastic scattering data for $^{16}$O + $^{27}$Al at 134 MeV and $^{16}$O + $^{154}$Sm at 85 and 134 MeV. Two forms of the imaginary potential, namely the microscopic and the phenomenological WS forms, were examined. The results indicated that the 2pF, SDHO, and DHB densities produced good agreement with the experimental data, but the DHB density demonstrated superior performance as measured by the chi-square value.

Overall, the DF elastic scattering cross sections computed using the M3Y and KH effective interactions exhibit reasonable agreement with the experimental data across all the reactions analyzed. These results suggest that the double folding potentials, which only involve a small number of fitting parameters, offer a satisfactory representation of the data. This reinforces the advantage of the DF model-based optical potential over the phenomenological form, which typically demands a greater number of fitting parameters.

DECLARATION OF CONFLICT OF INTEREST

Not applicable.
REFERENCES


