EFFECT OF FRANK-KAMENETSKII PARAMETER ON THERMAL EXPLOSION IN FLAMMABLE GAS CONTAINING FUEL DROPLETS

R.O. AYENI[1], O. OTOLORIN[2], K.S. ADEGBIE[3] AND T.O. AYODELE[1]

1. Department of P/A Mathematics, LAUTECH, Ogbomoso Nigeria
2. Olabisi Onabanjo University, Ago-Iwoye Nigeria
3. Federal University of Technology, Akure, Nigeria

Abstract

We investigate the effect of Frank-Kamenetskii parameter d on the thermal explosion in a flammable gas with fuel droplets. It is shown that the induction period decreases as d increases. The droplets increase the volume of the gas as they evaporate. The combustion medium is quasi-steady after explosion and the medium could no longer be regarded as homogeneous.

1. Introduction

The procedure for thermal explosion in gases, which contain fuel droplets, has been of much interest. After Semenov [5] developed the basic theory of phenomenon of thermal explosion, models that are more complicated have been suggested in [3] and [6]. The analysis of these models was carried out mainly using computers [4]. Because there is the need to understand the relative contributions of relative processes, some scientists developed asymptotic techniques to complement CFD packages. In particular, asymptotic technique was used in [4] and [1]. Usually an asymptotic technique is based on the assumption that a parameter or a ratio of some parameters is large or small. In combustion problems, the ratio of the product of the universal gas constant and the initial temperature and the activation energy is usually small. This assumption was made in the analysis of [1] and [2]. The asymptotic analysis in [4] is based on the assumption that the ratio specified above is small and further that the ratio between volumetric heat capacities of the gaseous and liquid phases is small or large. The assumption in [1], [2] and [4] is that the ratio of the product of the universal gas constant and the initial temperature and the activation energy is so small that only term of order unity is retained. Because this ratio $\varepsilon = R_u T / E$ is usually between 0 and 0.25, we consider in this paper higher orders in $\in$. The objective of the paper is to investigate physical systems where higher orders in ε are important.

2. Mathematical formulation

As in [4], we assume that the combustion gas mixture contains evaporating spherical droplets of fuel and that the medium is spatially homogeneous. We also ignore the variations in pressure in the enclosure and their influence on the combustion process. The appropriate equations [4] are

$$\rho c \frac{dT}{d\tau} = QA\mu Y \exp\left(\frac{-E}{R_u T}\right) - 4\pi R^2 nq \qquad (1)$$

$$\frac{dY}{d\tau} = -AY \exp\left(\frac{-E}{R_u T}\right) + \frac{4\pi R^2}{\mu L} nq \qquad (2)$$

+ *corresponding author* (email:)

$$\frac{dR}{d\tau} = -\frac{1}{\rho L}q \tag{3}$$

$$q = 280\sigma(T^4 - T_0^4) \tag{4}$$

where T is the temperature (K), Y is the molar concentration of the gas and fuel droplets ($kmol/m^3$), R is the radius of the droplets (m), A is pre-exponential factor (s^{-1}), E is the activation energy ($J/kmol$), R_u is the universal gas constant ($J/kmol/K$), c is the specific heat capacity ($J/kg/K$), ρ is the density (kg/m^3), q is the radiative heat flux (W/m^2), μ is the molar mass ($kg/kmol$), Q is the specific (per unit mass) combustion energy (J/kg), L is the latent heat of evaporation (J/kg), n is the number of droplets per unit volume ($1/m^3$), is the Stefan-Boltzman constant ($Wm^{-2}K^{-4}$) and is the time (s). The initial conditions are:

$$T(0) = T_0,\ Y(0) = Y_0,\ R(0) = R_0 \tag{5}$$

3. Non-dimensionalisation Analysis

Let,

$$\theta = \frac{(T-T_0)E}{RT_0^2},\ y = \frac{Y}{Y_0},\ r = \frac{R}{R_0},\ t = \frac{\tau}{\tau_0},\ \varepsilon = \frac{RT_0}{E} \tag{6}$$

Using the dimensionless variables in equation (6), the system of equations (1)-(5) become

$$\frac{d\theta}{dt} = dy\exp\left(\frac{\theta}{1+\varepsilon\theta}\right) - ar^2\left(4\theta + 6\epsilon\theta^2 + 4\epsilon^2\theta^3 + \epsilon^3\theta^4\right) \tag{7}$$

$$\frac{dy}{dt} = -fy\exp\left(\frac{\theta}{1+\varepsilon\theta}\right) + br^2\left(4\theta + 6\epsilon\theta^2 + 4\epsilon^2\theta^3 + \epsilon^3\theta^4\right) \tag{8}$$

$$\frac{dr}{dt} = -c\left(4\theta + 6\epsilon\theta^2 + 4\epsilon^2\theta^3 + \epsilon^3\theta^4\right) \tag{9}$$

$$\theta(0) = 0,\ r(0) = 1,\ y(0) = 1 \tag{10}$$

where:

$$d = \frac{\tau_0 QA\mu Y_0 A}{\rho c\varepsilon T_0}\exp\left(-\frac{E}{RT_0}\right) \tag{11}$$

$$f = \tau_0 A\exp\left(-\frac{E}{RT_0}\right) \tag{12}$$

$$a = \frac{1120\tau_0\pi\sigma R_0^2 n T_0^{\ 3}}{\rho c\varepsilon} \tag{13}$$

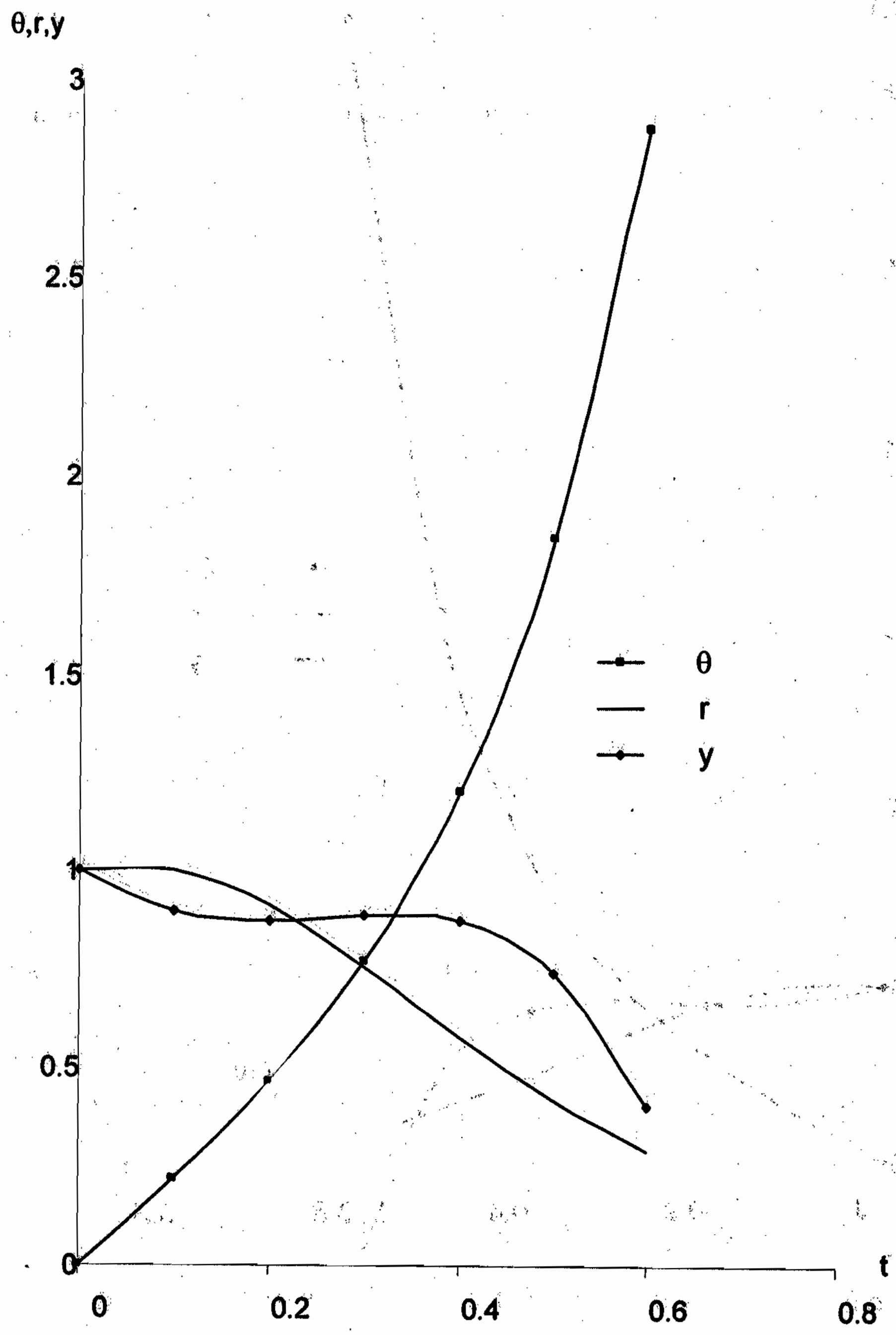

Figure 1: Graph of θ against t when a=0, b=1, c=1, f=1, ε=0.001, d=2.20059

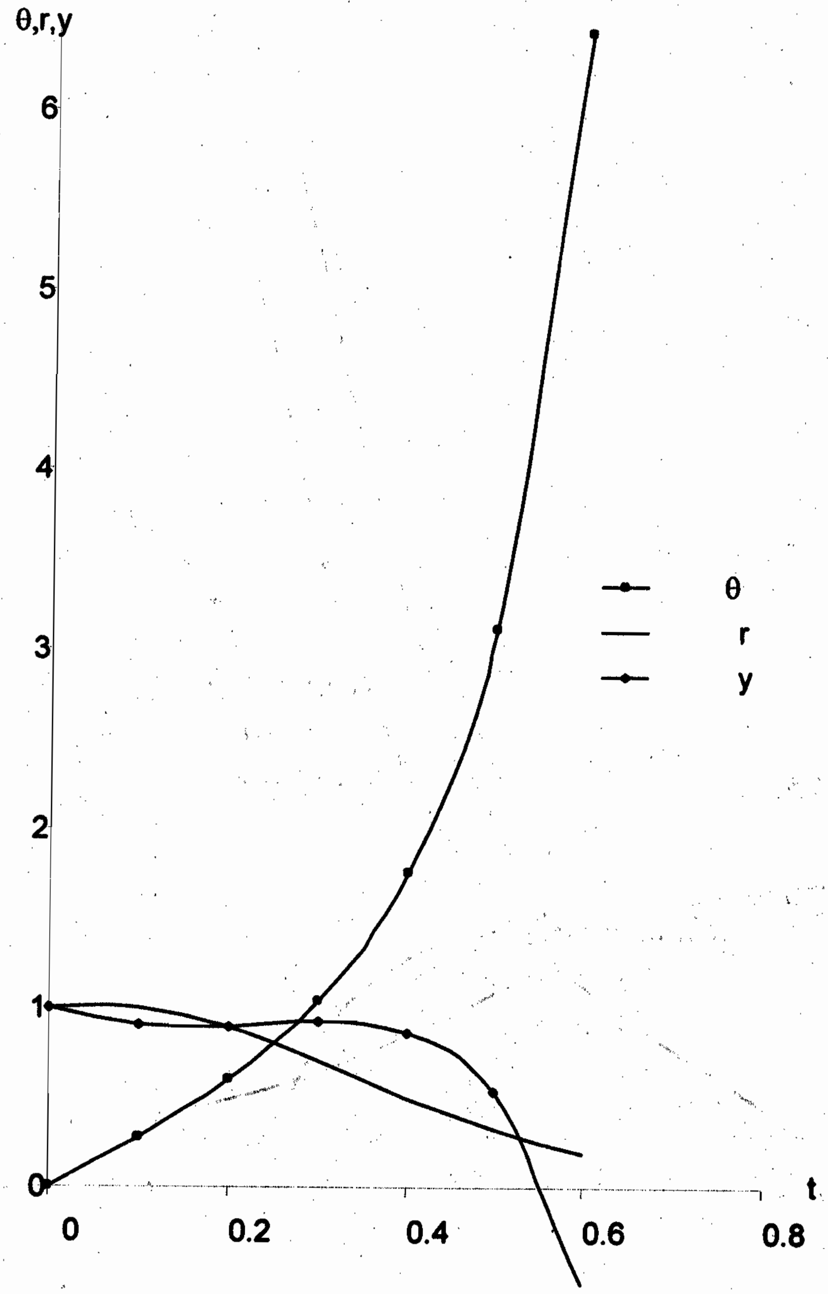

Figure 2: Graph of θ against t when a=0, b=1, c=1, f=1, ε=0.001, d=2.7398

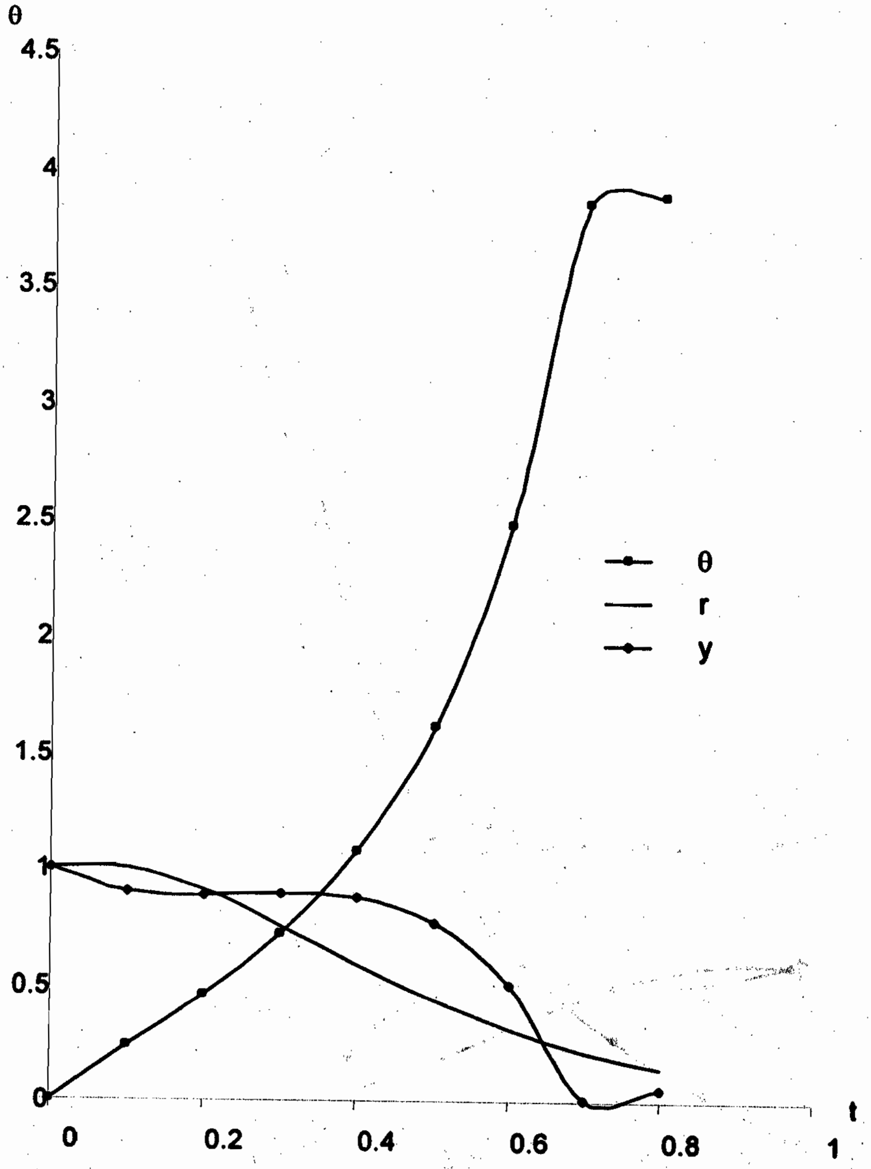

Figure 3: Graph of θ against t when a=0.5, b=1, c=1, f=1, ε=0.001, d=2.38503

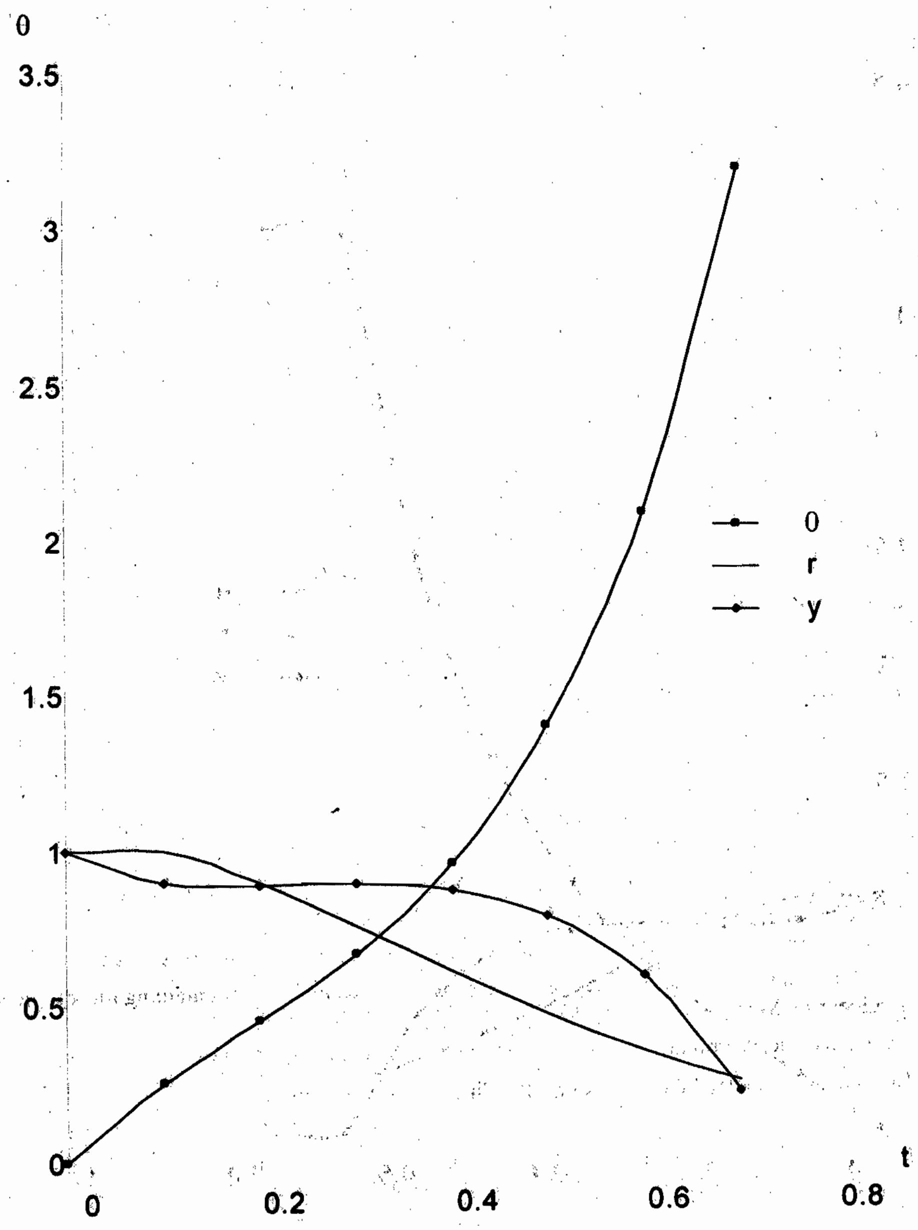

Figure 4: Graph of θ against t when $a=1$, $b=1$, $c=1$, $f=1$, $\varepsilon=0.001$, $\delta=2.56129$

$$b = \frac{1120\tau_0 \pi \sigma R_0^2 n T_0^{\ 4}}{\mu L Y_0} \tag{14}$$

$$c = \frac{280\tau_0 \sigma R T_0^{\ 5}}{\rho E L} \tag{15}$$

$$\tau_0 = 1\,\text{sec.} \tag{16}$$

4. Numerics and Discussion

Problem (7)-(10) is solved numerically for various values of *a, b, c, d* and *f* using finite difference method. Figure 1-2 show that induction time decreases as the Frank-Kamenetskii parameter d increases when there is no radiation. The figures also show that the droplets increase the volume of the gas as they evaporate. Figure 3 is quite interesting because it shows that problem has a quasi-steady solution after explosion, in this case, the medium is no longer homogeneous and the diffusion term has to be taken into consideration. The medium could not be regarded as well stirred.

Figure 4 shows even the radiation is high (higher than in figure 3) a higher Frank-Kamenetskii parameter ensures that explosion occurs and the medium may still be considered as a homogeneous.

In all these figures, not all the droplets evaporate before ignition.

Acknowledgements

The authors would like to thank the referee for comments that improve the original paper.

References

Ayeni, R.O., Ogunsola, A.W., Olajuwon, B.I. and Olanrewaju, P.O., 2005. A mathematical model of thermal explosion in flammable gas containing fuel droplets, to appear.

Ayeni, R.O., Okedoye, A.M., Popoola, A.O. and Ayodele, T.O., 2005. Effect of radiation on the critical Frank-Kamenetskii parameter of thermal ignition in a combustible gas containing fuel droplets, to appear.

Backmaster, J.D. and Ludford, G.S.S., 1982. Theory of laminar flames, Cambridge University Press, Cambridge

Goldfarb, I., Sazhin, S. and Zinoviev, A., 2004. Delayed thermal explosion in flammable gas containing fuel droplets: Asymptotic Analysis, J. Engineering Math. 50, pp. 399-414.

Semenov, N.N., 1928. Zur Theorides Verbrennungsprozesses, Z. Phys. Chem. 48, pp. 571-581.

Williams, F.A., 1985. Combustion Theory, Addison-Wesley, Reading.