# ON THE PERFORMANCE OF AUTOREGRESSIVE MOVING AVERAGE POLYNOMIAL DISTRIBUTED LAG MODEL

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#### ABSTRACT

This study focused on the performance of Autoregressive Moving Average Polynomial Distributed Lag Model among all other distributed lag models. Four models were considered; Distributed Lag (DL) model, Polynomial Distributed Lag (PDL) model, Autoregressive Polynomial Distributed Lag (ARPDL) model and Autoregressive Moving Average Polynomial Distributed Lag (ARMAPDL) model. The parameters of these models were estimated using least squares and Newton Raphson iterative methods. To determine the order of the models, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) were used. To determine the best model, the residual variances attached to these models were studied and the model with the minimum residual variance was considered to perform better than others. Using numerical example, DL, PDL, ARPDL and ARMAPDL models were fitted. Autoregressive Moving Average Polynomial Distributed Lag Model (ARMAPDL) model performed better than the other models.

Keywords: Distributed Lag Model, Selection Criterion, Parameter Estimation, Residual Variance.

# INTRODUCTION

Economic decisions have consequences that may last a long time. When the income tax is increased, consumers have less disposable income, reducing their expenditures on goods and services, which reduce profits of suppliers, the demand for productive inputs and the profits of the input suppliers. These effects do not occur instantaneously but are spread, or distributed, over future time periods. Economic actions or decisions taken at one point in time, *t*, have effects on the economy at time *t*, but also at times *t*+1, *t*+2, and so on (Judge *et al.*, 2000).

The reasons for lag in a model could be due topsychological, technological, institutional, political, business and economic decisions (Ojo, 2013). Due to this underlining fact, Distributed Lag Model has been applied in various fields in the past few decades and a remarkable success in its application has been made which help in the diverse areas of the economy (Kocky, 1954; Almon, 1965; Zvi, 1961; Robert and Richard, 1968; Frank, 1972; Dwight, 1971; Krinsten, 1981 and Wilfried, 1991).

Econometric analysis of long-run relations has been the focus of much theoretical and empirical research in economics. In the case where the variables in the long-run relation of interest are trend stationary, the general practice has been to de-trend the series and to model the de-trended series as stationary distributed lag or autoregressive distributed lag (ARDL) model (Hashem and Yongcheol, 1995).

In autoregressive distributed lag model, the regressors may include lagged values of the dependent variable and current and lagged values of one or more explanatory variables. This model allows us to determine what the effects are of a change in a policy variable (Chen, 2010). It is imperative to see that adding an instrumental variable such as Moving Average (MA) to Autoregressive Polynomial Distributed Lag (ARPDL) model there is the likelihood of having a better model. This study sought to critically examine the Autoregressive Moving Average Polynomial Distributed Lag (ARMAPDL) model in terms of its residual variancerelative to that of the aforementioned distributed lag models

## **MATERIALS AND METHODS**

**Distributed Lag Model** 

Distributed Lag Model is given as  $Y_{t} = \alpha + \beta_{0}X_{t} + \beta_{1}X_{t-1} + \dots + \beta_{j}X_{t-j} + \varepsilon_{t} \quad (1)$  Assumptions of the model are:

- The model is linear in parameters:  $y_t = b_0 + b_1 x_{t1} + \ldots + b_1 x_{ti} + \varepsilon_t$
- There is the need to make a zero conditional mean assumption: E(ε<sub>t</sub> | X) = 0, t = 1, 2, ..., n.
- The Xs are strictly exogenous
- \*  $\varepsilon_t$  is independent with mean zero and variance of  $\sigma^2$
- ★ There is no serial correlation: Corr. ( $\varepsilon_t \varepsilon_s | X$ )=0 for *t* ≠ *s*

#### **Polynomial Distributed Lag Model**

Polynomial Distributed Lag Model is obtained from a finite distributed lag given as

$$Y_t = \beta_0 X_t + \beta_1 X_{t-1} + \dots + \beta_j X_{t-j} + \varepsilon_t, \quad (2)$$

where

 $\beta_j$  is approximated by polynomial of lower degree.

$$\beta_j = d_o + d_1 j + d_2 j^2 + \dots + d_r j^r \tag{3}$$

r is the degree of polynomial while j is the number of lag of the decay. Assuming j=3 and r=2, we have;

$$Y_t = \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \varepsilon_{t.}(4)$$

Substituting the (3) into (4) and factorizing the equation, we obtain

$$Y_t = d_o Z_1 + d_1 Z_2 + d_2 Z_3 + \varepsilon_t \tag{5}$$

where

$$Z_{1} = X_{t} + X_{t-1} + X_{t-2} + X_{t-3}$$
$$Z_{2} = X_{t-1} + 2X_{t-2} + 3X_{t-3}$$
$$Z_{3} = X_{t-1} + 4X_{t-2} + 9X_{t-3}$$

Where  $Z_i$  are constructed from the original lagged variable  $X_p$ ,  $X_{i-p}$ ,  $X_{i-2}$  and  $X_{i-3}$ . Therefore Y is regressed on the constructed variable  $Z_i$  and not on the original variable X. OLS method is used to estimate the coefficient of the model since the assumptions of the disturbance term is satisfied.

The coefficients of 
$$d_0$$
,  $d_1$ ,  $d_2$  can be estimated by di  
=  $(ZZ)^1 ZY$  (6)

Thereafter the estimate the coefficients of  $\beta$  can be estimated from the original model by equation 3.

# Autoregressive Polynomial Distributed Lag Model

The model can be defined as:

$$Y_{t} = \varphi_{1}Y_{t-1} + \varphi_{2}Y_{t-2} + \dots + \varphi_{p}Y_{t-p} + \beta_{0}X_{t} + \beta_{1}X_{t-1} + \beta_{2}X_{t-2} + \dots + \beta_{j}X_{t-j} + \varepsilon_{t}$$
(7)

where  $\beta_{j}$  is approximated by polynomial in the lag k as

$$\beta_{j} = d_{o} + d_{1}j + d_{2}j^{2} + d_{3}j^{3} + \cdots$$
$$d_{k}j^{k} = \sum_{i=0}^{k} d_{k}j^{k}$$
(8)

where "j" is the number of periods away from the current period "t" and "k" is the degree of polynomial. Assuming j=3 and k=2 and obtaining a new equation from 7 and by substituting  $\beta_i$  into the new equation and factorizing the equation, we obtain:

$$Y_{t} = \varphi_{1}Y_{t-1} + \varphi_{2}Y_{t-2} + d_{0}Z_{1} + d_{1}Z_{2} + d_{2}Z_{3} + \varepsilon_{t}$$
(9)  
where

$$Z_{1} = X_{t} + X_{t-1} + X_{t-2} + X_{t-3}$$
  

$$Z_{2} = X_{t-1} + 2X_{t-2} + 3X_{t-3}$$
  

$$Z_{3} = X_{t-1} + 4X_{t-2} + 9X_{t-3}.$$

The sequence of random deviation  $(e_t)$  can be estimated by:

$$\varepsilon_t = Y_t - \varphi_1 Y_{t-1} - \varphi_2 Y_{t-2} - d_0 Z_1 - d_1 Z_2 - d_2 Z_3(10)$$

To obtain the unknown parameters of the model, we make some assumption that random error is independently and identically distributed with mean zero and variance of  $\sigma^2$ .

Minimizing the likelihood function, with respect to the parameters  $(\phi_1, \phi_2, d_0, d_1, d_2)$  we can obtain estimate of the parameters of the model using least squares method; Chen (2010) and subsequently, we obtain the parameters of  $\beta$ .

# Autoregressive Moving Average Polynomial **Distributed Lag Model**

The model is defined as

$$Y_{t} = \varphi_{1}Y_{t-1} + \dots + \varphi_{p}Y_{t-p} + \beta_{0}X_{t} + \beta_{1}X_{t-1} + \dots + \beta_{k}X_{t-k} - \phi_{1}\varepsilon_{t-1} - \dots - \phi_{q}\varepsilon_{t-q} + \nu_{t},$$
(11)  
where

 $\varphi_1, \dots, \varphi_p$  are the parameters of the autoregressive component,  $\phi_1, \dots, \phi_q$  are the parameters of the moving average component,  $\beta_0 \dots \beta_i$  are the parameters of the polynomial distributed lag model,  $Y_t$  and  $X_t$  are the dependent and independent variable respectively,  $v_t$  is the error term and is assumed to be normally distributed with mean zero and variance  $\sigma^2$ .

# Estimation of parameters of Autoregressive Moving Average Polynomial Distributed Lag Model

$$Y_{t} = \varphi_{1}Y_{t-1} + \dots + \varphi_{p}Y_{t-p} + \beta_{0}X_{t} + \beta_{1}X_{t-1} + \dots$$

 $\dots + \beta_j X_{t-j} - \phi_1 \varepsilon_{t-1} - \dots - \phi_q \varepsilon_{t-q} + V_t.$ We consider Newton Raphson iterative method

using the approach of Ojo (2009) and Pascal (2001) to estimate the parameters of the model. Representing the mean response as  $f_i$  the error term becomes

$$v_t = Y_t - f_i. \tag{12}$$

The least square estimator of G of G which minimizes the sum of the square of residual is

$$S(G) = \sum_{t=1}^{n} (v_t)^2$$

We differentiate S(G) with respect to the parameter G ( $\varphi_1, \varphi_2, \dots, \varphi_p, \beta_0, \beta_1, \dots, \beta_j, \phi_1, \phi_2, \dots, \phi_q$ ) We shall write  $G_1 = \varphi_1, G_2 = \varphi_2, \dots, G_R = \varphi_q$ . The partial derivatives of  $\Im(G)$  are

$$G_i = \frac{\partial S(G)}{\partial G} = -2\sum_{i} (\mathbf{v}_i) \frac{\partial (\mathbf{v}_i)}{\partial G_i}$$

where i =1, 2,  $\dots$ , R and R= p+j+q  $H = \frac{\partial^2 S(G)}{\partial G_i G_m} = -2 \left[ \sum_{t=1}^n (V_t) \frac{\partial^2 (v_t)}{\partial G_i \partial G_m} + \frac{\partial (v_t)}{\partial G_i} \frac{\partial (v_t)}{\partial G_m} \right]$ 

where 
$$i = 1, 2, ..., R, m = 1, 2, ..., R.$$
 (14)

Where the partial derivatives satisfy the recursive equations

$$\frac{\partial S(G)}{\partial \varphi_{1}} = -2\sum_{i} (v_{t})Y_{t-1}$$

$$\frac{\partial S(G)}{\partial \varphi_{i}} = -2\sum_{i} (v_{t})Y_{t-i} \quad \text{where } i = 2, \dots, p \quad (15)$$

$$\frac{\partial S(G)}{\partial \beta_{0}} = -2\sum_{i} (v_{t})X_{t}$$

$$\frac{\partial S(G)}{\partial \beta_{k}} = -2\sum_{i} (v_{t})X_{t-i} \quad \text{where } k = 1, 2, \dots, j \quad (16)$$

$$\frac{\partial S(G)}{\partial \varphi_{1}} = -2\sum_{i} (v_{t}) \in_{t-1}$$

$$\frac{\partial S(G)}{\partial \varphi_{r}} = -2\sum_{i} (v_{t}) \in_{t-r} \quad \text{where } r = 2, \dots, q. \quad (17)$$

The second derivative is given as

$$\frac{\partial^2 S(G)}{\partial \phi_1 \phi_1'} = -2 \left[ \sum_{i=1}^n (v_i)(0) + (Y_{t-1})(-\sum Y_{t-1}) \right] = 2 \sum Y_{t-1}^2$$

$$\frac{\partial^2 S(G)}{\partial \phi_i \phi_i'} = -2 \left[ \sum_{i=1}^n (v_i)(0) + (Y_{t-1})(-\sum Y_{t-i}) \right] = 2 \sum Y_{t-i}^2$$
where i =2, ..., p. (18)

$$\frac{\partial^2 S(G)}{\partial \beta_0 \beta'_0} = -2 \left[ \sum_{i=1}^n (v_i)(0) + (X_{i-1})(-\sum X_{i-1}) \right] = 2 \sum X_{i-1}^2$$
$$\frac{\partial^2 S(G)}{\partial \beta_k \beta'_k} = -2 \left[ \sum_{i=1}^n (v_i)(0) + (X_{i-1})(-\sum X_{i-k}) \right] = 2 \sum X_{i-k}^2$$
where k = 1, 2, ..., j. (19)

$$\frac{\partial^2 S(G)}{\partial \phi_1 \phi_1'} = -2 \left[ \sum_{i=1}^n (v_i)(0) + (\epsilon_{t-1})(-\sum \epsilon_{t-1}) \right] = 2 \sum \epsilon_{t-1}^2$$
$$\frac{\partial^2 S(G)}{\partial \phi_r \phi_r'} = -2 \left[ \sum_{i=1}^n (v_i)(0) + (\epsilon_{t-1})(-\sum \epsilon_{t-r}) \right] = 2 \sum \epsilon_{t-r}^2$$

where 
$$r = 2, ..., q$$
. (20)

$$\frac{\partial^2 S(G)}{\partial \phi_1 \beta_0} = -2 \left[ \sum_{i=1}^n (v_i)(0) + (Y_{i-1})(-\sum X_i) \right] = 2 \sum X_i Y_{i-1}$$

$$\frac{\partial^2 S(G)}{\partial \phi_i \beta_k} = -2 \left[ \sum_{i=1}^n (v_i)(0) + (Y_{i-1})(-\sum Y_{i-i}) \right] = 2 \sum X_{i-k} Y_{i-k}$$
where  $i = 2$ , p and  $k = 1, 2$ ,  $i$  (21)

where 
$$r = 2, ..., p$$
 and  $k = 1, 2, ..., j$ . (21)  
 $\frac{\partial^2 S(G)}{\partial x^2} = -2 \left[ \sum_{k=1}^{n} (y_k)(0) + (Y_k)(-\sum_{k=1}^{n} y_k) \right] = 2 \sum_{k=1}^{n} Y_k \in \mathbb{R}$ 

$$\frac{\partial \varphi_{1} \varphi_{1}}{\partial \varphi_{i} \varphi_{r}} = -2 \left[ \sum_{i=1}^{n} (v_{i})(0) + (Y_{i-1})(-\sum \varepsilon_{i-1}) \right] = 2 \sum Y_{i-1} \varepsilon_{i-1}$$
  
$$\frac{\partial^{2} S(G)}{\partial \varphi_{i} \varphi_{r}} = -2 \left[ \sum_{i=1}^{n} (v_{i})(0) + (Y_{i-i})(-\sum \varepsilon_{i-r}) \right] = 2 \sum Y_{i-i} \varepsilon_{i-r}$$
  
where i = 2, ..., p and r = 2, ..., q. (22)

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$$\frac{\partial^2 S(G)}{\partial \beta_0 \phi_1} = -2 \left[ \sum_{i=1}^n (v_i)(0) + (X_i)(-\sum_{t=1}) \right] = 2 \sum X_i \in_{t-1}$$

$$\frac{\partial^2 S(G)}{\partial \beta_k \phi_r} = -2 \left[ \sum_{i=1}^n (v_i)(0) + (X_{i-k})(-\sum_{t=1}) \right] = 2 \sum X_{t-k} \in_{t-1}$$
where k = 1, 2, ..., j and r = 2, ..., q. (23)
$$\frac{\partial S(G)}{\partial G_i} = 0,$$

$$\frac{\partial^2 S(G)}{\partial G_i \partial G_m} = 0.$$

Set the gradient to be V(G) where

$$VG \ ) = \frac{\partial S(G)}{\partial G_1}, \frac{\partial S(G)}{\partial G_2}, \dots, \frac{\partial S(G)}{\partial G_R}$$

And the Hessian is represented by H

$$H = \frac{\partial^2 S(G)}{\partial G_i \partial G_j}.$$

The approximate mean responses  $f(X_i,G)$  for the n cases by the linear term in the Taylor series expansion we obtain

 $V(G^{1}) \approx V(G^{0}) + H(G^{0})(G^{1}-G^{0}) = 0$  $G^{1}-G^{0} = -H^{-1}(G^{0})V(G^{0})$ 

thereby obtaining the iterative equation given by  $G^{(k+1)}=G^{k}-H^{-1}(G^{k})V(G^{k}).$ 

 $G^k$  is the set of estimates obtained at the k<sup>th</sup> stage of iteration. The estimates obtained by the above iterative equation usually converge. For starting the iteration, we need to have good sets of initial values of the parameters. This can be obtained by fitting the best autoregressive moving average model.

## Performance of the Model Indicator

### **Residual Variance**

Residual variance or unexplained variance is part of the variance of any residual. In analysis of variance and regression analysis, residual variance is that part of the variance which cannot be attributed to specific causes. The unexplained variance can be divided into two parts. First, the part related to random, everyday, normal, free will differences in a population or sample. Among any aggregation of data these conditions equal out. Second, the part that comes from some condition that has not been identified, but that is systematic. That part introduces a bias and if not identified can lead to a false conclusion (Ojo *et al.*, 2008).

#### Selection of the Length of the Lag

Numerous procedures have been suggested for selecting the length *n* of a finite distributed lag in Judge *et al.* (2000). Two goodness-of-fit measures that are more appropriate are Akaike's Information Criterion (AIC)

$$AIC = \ln \frac{SSE_n}{T - N} + \frac{2(n+2)}{T - N}.$$
 (24)

Schwarz criterion known as Bayesian Information Criterion (BIC)

$$SC(n) = \ln \frac{SSE_n}{T - N} + \frac{(n+2)\ln(T - N)}{T - N}.$$
 (25)

For each of these measures we seek that lag length n that minimizes the criterion can be used. Since adding more lagged variables reduces *SSE*, the second part of each of the criteria is a penalty function for adding additional lags. These measures weigh reductions in sum of squared errors obtained by adding additional lags against the penalty imposed by each. They are useful for comparing lag lengths of alternative models estimated using the same number of observations Ojo (2013). In this study we shall use AIC and BIC criteria for selecting best order for the models under study.

## **RESULTS AND DISCUSSION**

#### Numerical Example

To present the application of these models we will use a real time series dataset, monthly rainfall and temperature series between 1979 and 2008 obtained from Forestry Research Institute of Nigeria (FRIN), Ibadan, Nigeria (see the appendix). Rainfall series is the endogenous variable while temperature series is the exogenous variable. For the fitted model, the estimation technique in the previous section were used

#### Fitted Distributed Lag Model

 $\hat{Y}_{t} = 0.434900 - 8.060110X_{t} - 3.430149X_{t-1} + 0.126389X_{t-2} + 2.731801X_{t-3} + 5.413812X_{t-4} + 6.769427X_{t-5}$ 

Fitted Polynomial Distributed Lag Model

 $\hat{Y}_{t}$ =-7.9488X<sub>t</sub>-3.59344X<sub>t-1</sub>+0.05829X<sub>t-2</sub>+3.00639X<sub>t-3</sub>+ 5.25086X<sub>t-4</sub>+6.79170X<sub>t-5</sub>

Fitted Autoregressive Polynomial Distributed Lag Model  $\hat{Y}_{t} = 0.392048Y_{t-1} - 0.152001Y_{t-2} - 0.069668Y_{t-3} - 0.163191Y_{t-4} - 6.31189X_{t} - 3.12314X_{t-1} - 0.25566X_{t-2} + 2.29056X_{t-3} + 4.51657X_{t-4} + 6.41919X_{t-5}$ 

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Fitted Autoregressive Moving Average Polynomial Distributed Lag Model  $\hat{Y}_{t} = 1.179189Y_{t-1} - 0.400669Y_{t-2} - 0.826939\varepsilon_{t-1} - 5.73789X_{t} - 2.91633X_{t-1} - 0.31444X_{t-2} + 2.06779X_{t-3} + 4.23038X_{t-4} + 6.17331X_{t-5}$ 

#### Table 1: Model Performance

Model	Order Determination	AIC	BIC	Residual Variance
DL	5	11.6777	11.7540	6655.2833
PDL	(5,2)	11.6552	11.6879	6655.8739
ARPDL	(4,5,2)	11.5076	11.5845	5611.8556
ARMAPDL	(2,1,5,2)	11.4913	11.5570	5553.7937

The inclusion of Moving Average term in Autoregressive Polynomial Distributed Lagmodel yielded reduction in the value of the residual variance which made ARMAPDL the best model.

## CONCLUSION

In this study, four types of distributed lag models were considered namely: Distributed Lag (DL) model, Polynomial Distributed Lag (PDL) model, Autoregressive Polynomial Distributed Lag (ARPDL) model and Autoregressive Moving Average Polynomial Distributed Lag (ARMAPDL) model. These models were studied with a view to determining the best among them. The parameters of these models were estimated using least squares and Newton Raphson iterative methods. Selection criteria were used to determine the order of these models. The residual variances attached to these models were studied using a numerical example and it was found out that the residual variance attached toAutoregressive Moving Average Polynomial Distributed Lag Model (ARMAPDL) was the least. It implied that ARMAPDL model was the best among these models. We suggest that ARMAPDL model be used in further studies when fitting distributed lag model.

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240.9

108.6

225.1

180.4

14.2

0

Jul.

Aug.

Sept.

Oct.

Nov.

Dec.

210.6

275

145.6

160.2

6

0

293.6

60.3

164.6

255.4

0

30

306.6

118.4

115.2

217.6

1.5

6.4

147.2

29.9

275.4

276.3

47.9

0

261

237.7

255.5

200.3

56.7

17.2

169.7

83.5

236

148.8

19.9

0

125.3

304.2

113.2

155.7

25.8

3.7

175.5

224.7

304.1

171.7

0

0

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# APPENDIX

### Rainfall Statistics (January, 1979 – December, 2008)

	1979	1980	1981	1982	1983	1984	1985	1986	1987
Jan.	0	3.7	0	0	0	0	0	0	2.1
Feb.	5.5	60	0	78.4	5.5	3.5	0	45.8	26.1
Mar.	79.6	21.8	66.2	181.9	0	148.7	29	127.3	35.1
Apr.	136.6	116.6	172.9	185.2	105.8	70	118.2	101.5	0
May	123.8	123.2	115.7	141	250.7	223	181.8	146.8	122.1
Jun.	162.8	306	184.3	180.7	172.9	233.6	200.6	312.9	195.8
Jul.	291.2	176.7	75.4	112.8	114.9	136.8	307.2	174.7	246.8
Aug.	280.1	427.4	62.1	21.5	21.1	156.6	232.2	52.7	357.1
Sept.	269	333.5	233.9	96.3	219	112.9	214.7	374.1	252.5
Oct.	223.6	196.8	225.1	134.1	45.5	157.5	132.3	216.7	200.9
Nov.	261.4	44	60	1.5	44	30.7	49	14.3	10
Dec.	0	0	0	0	75.5	2.5	0	0	23
	1988	1989	1990	1991	1992	1993	1994	1995	1996
Jan.	0	0	32	3.8	0	0	1.3	0	0
Feb.	51.7	18.4	40.3	47.6	0	60.1	3	0	61.1
Mar.	180.9	57	11.7	21	30.6	80.4	15.4	105.9	107.9
Apr.	173.2	97.8	233.8	108.9	112.7	48.8	73.8	142.7	153.3
May	121.1	259.2	123.6	258.2	67.4	153.2	214.7	334.3	114.9
Iun	242.9	338.7	118.3	191.1	168.2	203.9	129.8	162.3	193.3

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	1997	1998	1999	2000	2001	2002
Jan.	41	0	0	14.2	0	0
Feb.	0	2	68.7	0	0	0
Mar.	122.2	12.7	67.9	48.8	15	61.3
Apr.	261.7	136	185	87.7	98	140.7
May	184.9	245.4	129.7	101.2	265	122.7
Jun	160.5	135.8	278.3	135.4	178	112
Jul.	70	95.7	300.6	220.4	139.3	118
Aug.	122	65	154.5	263.8	62.4	95.2
Sept.	179.8	259.3	157.1	155.1	275.2	187.8
Oct.	154.6	131.4	268.4	151.8	80.8	265
Nov.	19.4	40	30.3	30	19.9	93.7
Dec.	35.3	24.8	0	0	0	0
						-
	2003	2004	2005	2006	2007	2008
Jan.			<b>2005</b> 0	<b>2006</b> 0	<b>2007</b> 0	<b>2008</b> 0
Jan. Feb.	2003	2004				
	<b>2003</b> 16.8	<b>2004</b> 0	0	0	0	0
Feb.	<b>2003</b> 16.8 40.5	<b>2004</b> 0 84.1	0 43.8	0 43.8	0	0 12.3
Feb. Mar.	<b>2003</b> 16.8 40.5 20	2004 0 84.1 1.5	0 43.8 67.7	0 43.8 67.7	0 0 2	0 12.3 73
Feb. Mar. Apr.	<b>2003</b> 16.8 40.5 20 110	2004 0 84.1 1.5 176.4	0 43.8 67.7 124.7	0 43.8 67.7 55.5	0 0 2 28.6	0 12.3 73 108
Feb. Mar. Apr. May	<b>2003</b> 16.8 40.5 20 110 69	2004 0 84.1 1.5 176.4 181.4	0 43.8 67.7 124.7 186.8	0 43.8 67.7 55.5 68.7	0 0 2 28.6 178.8	0 12.3 73 108 129.9
Feb. Mar. Apr. May Jun	<b>2003</b> 16.8 40.5 20 110 69 275.3	2004 0 84.1 1.5 176.4 181.4 146.1	0 43.8 67.7 124.7 186.8 238	0 43.8 67.7 55.5 68.7 130	0 0 2 28.6 178.8 174.6	0 12.3 73 108 129.9 234.9
Feb. Mar. Apr. May Jun Jun	2003 16.8 40.5 20 110 69 275.3 164.6	2004 0 84.1 1.5 176.4 181.4 146.1 92.1	0 43.8 67.7 124.7 186.8 238 207.7	0 43.8 67.7 55.5 68.7 130 190.3	0 0 2 28.6 178.8 174.6 177.6	0 12.3 73 108 129.9 234.9 177.7
Feb. Mar. Apr. May Jun Jul. Aug.	2003 16.8 40.5 20 110 69 275.3 164.6 28.2	2004 0 84.1 1.5 176.4 181.4 146.1 92.1 68.8	0 43.8 67.7 124.7 186.8 238 207.7 9	0 43.8 67.7 55.5 68.7 130 190.3 143.1	0 0 2 28.6 178.8 174.6 177.6 65.9	0 12.3 73 108 129.9 234.9 177.7 224.8
Feb. Mar. Apr. May Jun Jul. Aug. Sept.	2003           16.8           40.5           20           110           69           275.3           164.6           28.2           226	2004 0 84.1 1.5 176.4 181.4 146.1 92.1 68.8 75.7	0 43.8 67.7 124.7 186.8 238 207.7 9 304.3	0 43.8 67.7 55.5 68.7 130 190.3 143.1 250.8	0 0 2 28.6 178.8 174.6 177.6 65.9 159.5	0 12.3 73 108 129.9 234.9 177.7 224.8 289.9

Temperature Statistics (January, 1979 – December, 2008)

	1979	1980	1981	1982	1983	1984	1985	1986	1987
Jan.	32	32	33	33	35	33	35	34	34
Feb.	36	35	37	34	37	35	36	35	36
Mar.	35	34	34	33	37	36	34	33	35
Apr.	33	35	31	33	34	33	33	34	35
May	30	31	32	31	31	32	32	33	34
Jun	30	31	32	30	30	30	31	31	32
Jul.	29	28	31	28	28	30	30	27	30
Aug.	29	28	29	26	28	32	30	28	30
Sept.	29	29	30	28	29	29	30	29	31
Oct.	31	30	31	30	31	29	31	30	31
Nov.	32	31	32	33	33	32	33	35	35
Dec.	30	32	33	33	32	29	31	33	33

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	1988	1989	1990	1991	1992	1993	1994	1995	1996
Jan.	34	33	34	34	34	34	33	34	34
Feb.	35	35	35	35	36	33	35	36	36
Mar.	34	35	37	35	35	34	36	34	34
Apr.	33	34	33	32	34	34	34	33	33
May	33	32	32	32	32	32	32	31	31
Jun	30	31	31	31	30	30	31	31	31
Jul.	29	29	29	29	28	29	28	28	28
Aug.	28	29	28	28	27	29	29	29	29
Sept.	30	29	30	30	29	30	30	30	30
Oct.	31	31	31	30	31	31	31	31	31
Nov.	33	34	32	32	32	32	32	32	32
Dec.	32	34	32	33	33	32	34	33	33

	1997	1998	1999	2000	2001	2002
Jan.	34	34	34	34	35	32
Feb.	36	36	35	36	37	37
Mar.	35	37	34	37	37	36
Apr.	32	36	34	34	35	37
May	32	34	33	33	33	35
Jun	30	32	31	31	31	32
Jul.	29	33	30	29	31	31
Aug.	28	32	29	29	28	30
Sept.	31	34	29	31	29	30
Oct.	31	32	30	31	31	29
Nov.	33	34	33	33	34	33
Dec.	33	33	34	34	34	32

	2003	2004	2005	2006	2007	2008
Jan.	33.8	34.3	21.8	34.9	21	22.3
Feb.	35.1	34.9	27.8	36.5	24.7	24
Mar.	33.7	35.8	26.8	35	25.1	24.8
Apr.	33.1	33.7	28.3	34	24.9	24.6
May	33.8	32.3	27.2	30.8	24	27.8
Jun	30.6	31	25.3	30.6	23.5	23.9
Jul.	29.7	30	25.1	29.1	22.8	24
Aug.	29.4	28.7	23.7	30.1	23	23.5
Sept.	30.8	30.7	21.5	30.4	23.2	23.9
Oct.	32	31	26.3	31.1	23.2	23.6
Nov.	34	32.1	26	33.1	24.5	26.5
Dec.	33.9	33.9	25.6	34.6	24	24.7

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