https://dx.doi.org/10.4314/ijs.v20i1.9

ON A ONE-STEP ADMISSION BATCH SIZE MODEL FOR DEGREE AWARDING INSTITUTIONS IN NIGERIA

***Ekhosuehi, V. U.[†], Oyegue, F. O.**
Department of Mathematics, University of Benin, P.M.B. 1154, Benin City, Nigeria
Tel. +234 806 499 0117; Email: virtue.ekhosuehi@uniben.edu
Department of Mathematics, University of Benin, P.M.B. 1154, Benin City, Nigeria
Tel. +234 8038799995; Email: fooyegue@yahoo.com
*Corresponding author: Tel. +234 806 499 0117; Email: virtue.ekhosuehi@uniben.edu
(Received: 21st September, 2017; Accepted: 23rd January, 2018)

ABSTRACT

This study considers the admission batch size for the undergraduate programme in a degree awarding institution in Nigeria as akin to the transition model based on fractional flows. The study provides an objective tool to serve as a benchmark for the number of new entrants. Rather than relying only on quota admission figures, the model utilises the capacity requirements and the transition rates. The model is formulated and solved by assuming that the expected enrolment at each level of entry equates the capacity requirement. The results obtained provide a guide on the number of new entrants that should be admitted into the system in the subsequent session.

Keywords: admission; capacity requirement; fractional flow model; quota system; transition rate.

INTRODUCTION

This paper makes a case for admission into degree awarding institutions in Nigeria to be done by considering the rate at which students repeat the level of entry vis-à-vis the actual carrying capacity of the system. The carrying capacity of an institution may be measured using the available manpower resources or physical facilities. These variables are not always constant over time. For instance, a new classroom/lecture theatre may be built and equipped so much so that it can accommodate more students than the existing ones, or the infrastructure in a system may depreciate so much so that it no longer accommodate as many students as the approved quota. Loosely speaking admission quota is seen as the same as the carrying capacity in the short-run. However, this is not entirely so given that the carrying capacity of a system may expand or contract over time. Here a short-run scenario is considered where the capacity requirement is constant. Even so the possibility of students repeating a level coupled with a fixed quota of new intake added to those that repeat the level of entry may lead to excess enrolment at that level. When the carrying capacity of the system equates the quota, there are no perturbing challenges as the admission quotas are usually filled up one way or the other. Suppose the carrying capacity of the system is different from the approved quota. Then the problem of optimal admission batch size becomes necessary so that the system does not admit students that would cause undue strain on the available resources in an attempt to achieve the quota requirements or under-admit students which may lead to underutilisation of resources.

This study is aimed at developing a formula to determine the size of new entrants into an undergraduate programme for a degree awarding institution (system hereafter) in the Nigerian setting. Quite frankly, a system is that part of reality being studied. More precisely, a system as used in this study refers to the levels of entry into a degree awarding institution and not the institution in its entirety. The aim of this study is consistent with the subject of educational planning. There is a considerable body of literature on this subject stemming from the work of Gani (1963). Earlier studies have been concerned with educational systems where the flow probabilities, or rates, are constant (Gani, 1963; Nicholls, 2009; Ekhosuehi and Osagiede, 2013). This is often the case because the models for students' enrolment provide a means not only to evaluate, plan and benchmark the (expected) enrolment structure, but also to project the structure of the system. There is a two-way flow between the educational system and the outside world, viz. admission and attrition. However, the control of enrolment stocks is exercised through admission. This is because attrition from the system largely depends on the individual student's performance.

An undergraduate degree programme in Nigerian higher institutions consists of a minimum of four levels. New entrants into the system are admitted into the first two lower levels of the system. These levels will be referred to as 100 Level and 200 Level hereafter. New entrants admitted in the same session form a cohort, and they are identified by their matriculation/ registration number. Earlier on, the survivor rates of candidates admitted into either 100 Level or 200 Level was considered (Ekhosuehi and Oyegue, 2016). The present study attempts to answer the question: 'How many candidates should be admitted into an undergraduate degree programme in order to avoid excess enrolments at the levels of entry?' This question is fundamental to the system if undue strain on facilities should be evaded. This study is relevant to academic planning because the admission of new intake into an undergraduate degree programme for the subsequent session is often done before the current session ends.

Regulating agencies such as the National Universities Commission (NUC), Medical and Dental Council of Nigeria (MDCN), National Institute of Science Laboratory Technology (NISLT), etc. may fix a quota (i.e., an upper bound) on the number of new entrants for the undergraduate degree programmes within their jurisdiction. This quota varies among higher institutions. On top of this the system may encounter excess enrolment at the levels of entry. This is because admissions are often done regardless of the transitions. That is the number of students on probation at 100 Level and 200 Level, transfers and those who are promoted from 100 Level to 200 Level are not considered. By transfers, we refer to those students who are relocated from other courses of study into the programme of interest. There is a need to clarify what is meant by the terms 'promotion' and 'probation'. By promotion, we mean that the students have satisfied the requirements to move to the next higher level, and by probation we refer to a student who has not satisfied the requirements to move to the next higher level and is to remain in the course of study by repeating all the courses registered in the previous session. Thus a student on probation did not meet a specified benchmark and may be advised to withdraw from the programme at the end of the next session if such a student is still unable to meet the benchmark. In the University of Benin and several other universities which the authors are acquainted, students may be promoted with some carry-over courses provided the minimum credits load for promotion is at least accumulated, while a student on probation repeats the previous level by retaking all the courses earlier registered and this is allowed to do so at most once throughout his stay in the institution, except for the final year. Although a student on probation may go on to graduate if he is able to pass all his courses registered for, yet he is unable to graduate within the stipulated time, as an additional session has been added to the time he will spend in the programme.

This study is apt in the sense that it builds a transition model based on the concept of 'fractional flow' to address the problem of excess enrolment at the levels of entry. The term 'fractional flow' was earlier coined by Grinold and Stanford (1974). The study is centred on the art of applying 'simple and easy-to-follow' mathematical techniques to find the size of new entrants into the system. The paper is intentionally so, as academic planners may not be too interested in sophisticated models. However this is not a compelling justification, as the advent of computer programs has taken care of greater sophistication needed to study complex systems. It is important to mention here that the limitation of the model constructed in this paper may be attributed to the assumptions made in developing the model and not the level of sophistication.

THEORETICAL FRAMEWORK

Consider a programme of study in a degree awarding institution, wherein there are two levels of entry, $i=1,2$. Here $i=1\equiv$ 100 Level and $i=2\equiv$ 200 Level. We assume that the expected enrolment at each level of entry equates the capacity requirement. Let CR_i be the capacity requirement for level i of the programme. We assume that $CR_1 \leq CR_2$. We examine a short-run scenario where the capacity requirements, CR_1, are constant. We derive a formula to determine the number of new entrants by reference to Figure 1.

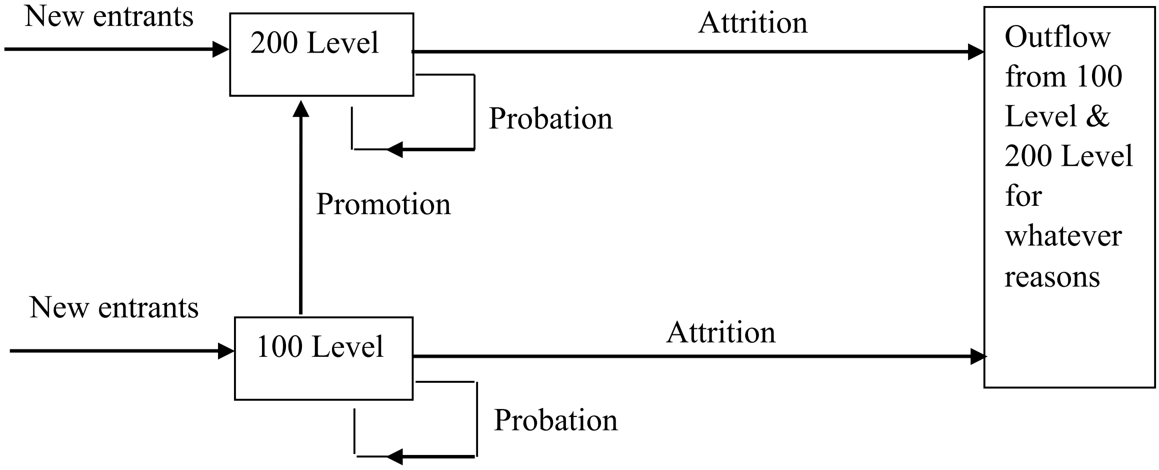

Figure 1: Transition diagram for students at the levels of entry.

In Figure 1, the three rectangles represent the states that a student may occupy and the arrows indicate the changes of state which can occur. New entrants into the system may be admitted into either 100 Level or 200 Level. A student initially at the level of entry may repeat that level by serving a probation period of one session, be promoted to the next higher level if sufficient credit load for promotion has been accumulated or dropout from the system due to academic deficiency, financial insolvency, ill-health, death, etc. Notice that Figure 1 does not include progression from 200 Level to 300 Level and so on. This is because the part of the degree awarding institution under study is the levels in which admission is done, higher levels beyond 200 Level are outside the scope of the study.

Let $n_{ij}(t)$ denote the number of students moving from level i to level j at the end of a session t, j=1,2. With $n_{0j}(t+1)$ being the number of students admitted into level j at the beginning of a session $t+1$, the capacity requirements are expressed as

$$CR_1 = n_{11}(t) + n_{01}(t+1) \qquad (1)$$

$$CR_2 = n_{12}(t) + n_{22}(t) + n_{02}(t+1) \qquad (2)$$

We use the notation $n_i(t)$ to represent the number of students enrolled in level i in session t and $A(t+1)$ to denote the admission batch size (i.e., the total number of new entrants) in session $t+1$. The number of students, $n_i(t)$, does not equate the admission batch size, $A(t+1)$. This is because the former includes students who repeat level and those who are promoted from the lower level in addition to the admission batch size. We assume that the flows, $n_{ij}(t)$ and $n_{0j}(t+1)$, are proportional to the stocks, $n_i(t)$ and $A(t+1)$, from which they come, respectively. This assumption is consistent with the class of Markov chain models in the literature (Bartholomew et al., 1991). Arising from this assumption is a constant of proportionality called the transition rates, p_{ij} or p_{0j}, according to whether the transitions are within the system or between the outside world and the system, respectively. The transition rates satisfy the relations: $p_{11} + p_{12} \leq 1$ and $p_{22} \leq 1$. The shortfall in the transition rates is attributed to wastage. The foregoing assumption leads us to write

$$n_{ij}(t) = p_{ij} n_i(t) \qquad (3)$$

$$n_{0j}(t+1) = p_{0j} A(t+1) \qquad (4)$$

The notations used in equations (3) and (4) are consistent with the ones in Bartholomew et al. (1991). Substituting equations (3) and (4) into equations (1) and (2), we obtain

$$CR_1 = p_{11} n_1(t) + p_{01} A(t+1) \qquad (5)$$

$$CR_2 = p_{12} n_1(t) + p_{22} n_2(t) + p_{02} A(t+1) \qquad (6)$$

The time epochs used in the accounting equations (5) and (6) are consistent with that of Gani (1963) and Bartholomew et al. (1991). In matrix form, equations (5) and (6) are expressed as

$$\begin{bmatrix} CR_1 \\ CR_2 \end{bmatrix} = \begin{bmatrix} p_{11} & 0 \\ p_{12} & p_{22} \end{bmatrix} \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix} + \begin{bmatrix} p_{01} \\ p_{02} \end{bmatrix} A(t+1) \qquad (7)$$

The task is to find an expression for $A(t+1)$ in terms of the transition rates and the previous stocks given the capacity requirements. The admission batch size, $A(t+1)$, is obtained by simplifying equation (7) to get

$$A(t+1) = [\theta_{01} \quad \theta_{02}]\left(\begin{bmatrix} CR_1 \\ CR_2 \end{bmatrix} - \begin{bmatrix} p_{11} & 0 \\ p_{12} & p_{22} \end{bmatrix}\begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix}\right) \quad (8)$$

where $\theta_{0j} = \dfrac{p_{0j}}{p_{01}^2 + p_{02}^2}$.

There is no guarantee that the $A(t+1)$ obtained from equation (8) will be a nonnegative integer. However, $A(t+1) \geq 0$ if $p_{12} \leq \frac{1}{2}$ and $p_{22} \leq \frac{1}{2}$.

This is easy to see as $n_1(t) \leq CR_1$, $n_2(t) \leq CR_2$ and $CR_1 \leq CR_2$, implies that $\frac{1}{2}n_1(t) + \frac{1}{2}n_2(t) \leq CR_2$.

For $A(t+1) \geq 0$, the admission batch size is taken as $\lceil A(t+1) \rceil$, where $\lceil \bullet \rceil$ is a ceiling function that ensures that $A(t+1)$ is an integer. If $A(t+1)$ is negative, then no admission should be done in session $t+1$.

The transition rates, p_{ij} and p_{0j}, are estimated from historical data on the stocks and flows for $t=1, 2, \cdots, T$, as the observed proportions over time. That is

$$p_{ij} = \frac{\sum_{t=1}^{T} n_{ij}(t)}{\sum_{t=1}^{T} n_i(t)} \quad (9)$$

and

$$p_{0j} = \frac{\sum_{t=1}^{T} n_{0j}(t)}{\sum_{t=1}^{T} A(t)} \quad (10)$$

Suppose a quota of size Q is given. Then we suggest the following rules as a guide towards making a decision on the admission batch size, $A(t+1)$.

— For $0 < A(t+1) \leq Q$, admit $A(t+1)$ regardless of Q. Do not contest the quota, Q, on admission.
— For $Q < A(t+1)$, admit Q if $p_{10}A(t+1) \leq Q$; otherwise the quota should be contested for $p_{10}A(t+1)$ to be admitted into 100 Level as the admission batch size.

In either case, any attrition from the admission batch size should be replaced by transfers or relocation from other programmes.

NUMERICAL ILLUSTRATION

An illustration of the use of the fractional flow model on the B.Sc. (Industrial Mathematics) programme of the University of Benin is presented in this section. The programme is offered on a full-time study mode and comprises four levels. The stock and flow data for three academic sessions are extracted from the results approved by Senate of the university and presented in Table 1 for ease of reference. Notice in Table 1 that the sum of flows satisfies the relation:

$\sum_{j=1}^{4} n_{ij}(t) \leq n_i(t)$ for each i. This is attributed to wastage (or dropout) from the programme.

Suppose the quota for new entrants into the programme is $Q = 120$. Then we determine what should be the admission batch size in the 2016/2017 session, i.e., $A(1)$, using the 2015/2016 session as the base year. The transition rates are computed as

$p_{11} = 0.0268$, $p_{12} = 0.9375$, $p_{22} = 0.0471$, $p_{01} = 0.9686$, $p_{02} = 0.0314$.

With $CR_1 = 120$, $CR_2 = 150$, say, the admission batch size for the 2016/2017 session should be

Table 1: Enrolment data from 2013/2014 to 2015/2016 at the end of each session

	i/j	1	2	3	4	$n_i(t)$
2013/2014	n_{0j}	47	2	0	0	49
	1	2	50	0	0	52
	2	0	4	67	0	79
2014/2015	n_{0j}	42	4	0	0	46
	1	0	37	0	0	43
	2	0	0	38	0	54
2015/2016	n_{0j}	127	1	0	0	128
	1	4	123	0	0	129
	2	0	4	32	0	37

$$A(1) = \begin{bmatrix} 1.0313 & 0.0334 \end{bmatrix} \left(\begin{bmatrix} 120 \\ 150 \end{bmatrix} - \begin{bmatrix} 0.0268 & 0 \\ 0.9375 & 0.0471 \end{bmatrix} \begin{bmatrix} 129 \\ 37 \end{bmatrix} \right) = 121.10 > Q = 120$$

However, $p_{01}A(1) = 117.30 < Q = 120$. In the totality of the foregoing, the decision rule is to admit $\lceil p_{01}A(t+1) \rceil = 118$ students into 100 Level and 2 students into 200 Level in the 2016/2017 session, summing up to a batch size of 120 new students. Nonetheless, if $CR_1 = CR_2 = 150$, the decision would have been to contest the admission quota as $p_{01}A(1) = 147.27 > Q = 120$.

CONCLUDING REMARKS

This paper has attempted to provide an answer to the problem of admission batch size for undergraduate degree programme. A model was derived. The model utilises the concept of fractional flow as its theoretical underpinning. However, the model is not meant to replace managerial intuition, but to guide the management of the system on the batch size based on the capacity requirements and the transition rates. It also offers a valuable insight when a decision is to be made on a quota admission system vis-à-vis the transition rates and the actual carrying capacity of the system. It is possible to improve the performance of the model for the educational system if the system operates such that the assumptions of the model are more nearly satisfied.

ACKNOWLEDGEMENT

We acknowledge the Editor-in-Chief (Physical Sciences) and the anonymous reviewers for their helpful comments which have greatly improved the quality of the earlier manuscript.

REFERENCES

Bartholomew, D.J., Forbes, A.F. and McClean, S.I. 1991. *Statistical techniques for manpower planning*, 2nd Ed. John Wiley & Sons, Chichester.

Ekhosuehi, V. U. and Osagiede, A. A. 2013. A proposed Markov model for predicting the structure of a multi-echelon educational system in Nigeria. *Monografias Matematicas Garcia de Galdeano*, 38: 65-74.

Ekhosuehi, V. U. and Oyegue, F. O. 2016. Survivor rates of new entrants into an undergraduate degree programme under clearance uncertainty. *African Journal of Applied Statistics*, 3(1): 59 – 67.

Gani, J. 1963. Formulae for projecting enrolments and degrees awarded in universities. *Journal of the Royal Statistical Society, Series A (General)*, 126(3): 400-409.

Nicholls, M. G. 2009. The use of Markov models as an aid to the evaluation, planning and benchmarking of doctoral programs. *Journal of the Operational Research Society*, 60: 1183-1190.

Grinold, R. C. and Stanford, R. E. 1974. Optimal control of a graded manpower system. *Management Science*, 20(8): 1201-1216.