https://dx.doi.org/10.4314/ijs.v24i3.7

Ife Journal of Science vol. 25, no. 3 (2023)

REAL LIFE APPLICATIONS OF MATHEMATICAL MODEL OF KIFILIDEEN TRINOMIAL THEOREM DISTRIBUTION OF POSITIVE POWERS OF n WITH MATHEMATICAL INDUCTION OF THE CONSTITUENTS OF ITS NEGATIVE POWERS OF n COUNTERPART

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(Received: 14th June, 2023; Accepted: 24th November, 2023)

ABSTRACT

Binomial system is a system of analysis which involves two chances or possible outcomes of event which are the extreme ends of such event. In real life, there exist the mid or intermediary chances or possible outcomes of some events, if not all events, making the possible outcomes of such events to be three. In any such scenario or instance of three possible outcomes, a trinomial system of analysis would be applicable. There is need to develop a model to analysis such scenarios of trinomial system. This research work presents real life applications of mathematical model of Kifilideen trinomial theorem distribution of positive powers of n with mathematical induction of negative powers of n counterpart. A mathematical model of the Kifilideen trinomial theorem distribution was formulated. This research work also develops alternate Kifilideen power combination formula, alternate Kifilideen term formula and alternate Kifilideen position formula for the negative powers of n, which conform but not the same, with the one developed for the positive powers of n counterpart. The formulated mathematical model of the Kifilideen trinomial theorem distribution was utilized to analysis real life events having three possible outcomes. The model invented in this paper can be used to determine the probability of the combination of different n outputs involving three possible categories of outcomes of events. The mathematical induction helps to support and prove that the developed Kifilideen formulas for the constituents of Kifilideen trinomial theorem of negative powers of n are valid and true.

Keywords: Binomial system, Mathematical model, Kifilideen trinomial theorem, Theorem distribution.

INTRODUCTION

Binomial system is system of analysis which involves two chances or possible outcomes of event which are the extreme ends of such event (Goss, 2011; Flusser and Francia, 2020; Tatira, 2021; Campos and Decano, 2022). Binomial system is applied in binomial distribution to solve problems related to probability problems (Tsokos and Wooten, 2016; Mohr *et al.*, 2021). Probability is a branch of applied mathematics which deals with random experiments (Siegel and Wagner, 2022; Trunkenwald *et al.*, 2022). Two of the pioneering mathematicians in the field of probability are French mathematician, Blaise Pascal (1623 – 1662) and Russian mathematician, Andrei Nikolaevitch Kolmogorov (1903 – 1987) (Parthasarathy, 1998; Bingham, 2009; Akyıldırım and Soner, 2014; Jahic, 2019). According to Debnath (2015), in binomial distribution, for any particular event there are two possible chances which are success and failure or pass and fail of such event. So, if the probability of passing an event is p and the probability of failing the event is q from a large population then for certain n number of people or objects or entities selected at random from the large population the binomial distribution is presented as:

$$[p+q]^n = {}_{n,0}^{\ n}Cp^nq^0 + {}_{n-1,1}^{\ \ n}Cp^{n-1}q^1 + {}_{n-2,2}^{\ \ n}Cp^{n-2}q^2 + {}_{n-3,3}^{\ \ n}Cp^{n-3}q^3 + \cdots + {}_{3,n-3}^{\ \ n}Cp^3q^{n-3} + {}_{2,n-2}^{\ \ n}Cp^2q^{n-2} + {}_{1,n-1}^{\ \ n}Cp^1q^{n-1} + {}_{0,n}^{\ n}Cp^0q^n \quad (1)$$

Such that $p+q=1$ (2)

That is the probability of passing an event $+$ Probability of failing the event $=1$ (3)

$P_r\ (Pass) + P_r\ (Fail) = 1$ (4)

For Term T_1 for $t=1$ in (1) indicates the probability of n pass and 0 fail for the random selection of the n people or objects or entities of the population. For Term T_2 for $t=2$ in Equation 1 indicates the probability of $(n-1)$

pass and 1 fail for the random selection of the n people or objects or entities of the population. For Term T_3 for $t = 3$ in Equation 1 indicates the probability of $(n - 2)$ pass and 2 fail for the random selection of the n people or objects or entities of the population. The trend above for Term T_1 for $t = 1$, Term T_2 for $t = 2$ and Term T_3 for $t = 3$ is also applicable to all other terms in the binomial distribution.

In real life there exist the mid or intermediary chances or possible outcomes of some events, if not all events, making the possible outcomes of such events to three. In any such scenario or instance of three possible outcomes, a trinomial system of analysis would be applicable. There is need to develop a model to analysis such scenarios of trinomial system. In this paper, Kifilideen trinomial theorem distribution model was used in analysing trinomial systems. Kifilideen trinomial theorem distribution model is applicable and utilized in the field and area of Mathematics, Physics, Chemistry, Biology, Agriculture, Transportation, Social Science, Engineering, Finance, Sport, Industry, and Computer Science. Kifilideen trinomial theorem distribution model extends the scope of analysis of binomial theorem distribution model to three possible outcomes of event. The mathematical model of the Kifilideen trinomial distribution invented in this paper can be used to determine the probability of the combination of different n output involving three possible categories of outcomes of event. The Kifilideen matrix visualizes and helps to indicate all possible power combinations present in any particular Kifilideen trinomial theorem distribution of positive power of n. The Kifilideen matrix can be fully utilized in the analysis of any Kifilideen trinomial theorem distribution (Osanyinpeju, 2021a). The Kifilideen trinomial system is varied along three parameters of two extreme ends (say, either pass, p or fail, q) and an intermediary, r (which is neither pass nor fail). The power combination of these variables (p, r and q) indices tells more about the confidence level ascertained.

Sighting various examples of binomial and trinomial systems; examinational system can be used as a case study where students can either be categorized as above average students $(+)$ scoring (from $50 - 100$ marks) or below average students $(-)$ scoring (from $0 - 49$ marks) in a situation of binomial system while for trinomial system, the students are divided into three categories which are above average students $(+)$scoring (from $60 - 100$ marks), below average students $(-)$ scoring (from $0 - 30$ marks) and average students $(/)$ scoring (from $40 - 59$ marks). In case of the trinomial system of analysis, lesser effort is put in for the average students to migrate from that level to above average level when compare to the below average students which require two steps and massive effort to migrate to the above average level. Any little relaxation can easily make the average students to fall to below average level unlike the above average students which take time to drop to below average level when relaxed.

More so, in case of defect of manufacture products, products are classified into two categories which are good $(+)$or bad $(-)$products for binomial system where the good products are useful and the bad products cannot be utilized at all. It should be known that in manufacturing of products, some products can be categories as semi – good or undecided whether good or bad $(/)$ that is they have features of both good and bad. These kinds of products are not totally bad and can be useful and manageable in some area. These kinds of products cannot be grouped under good or bad but can be categorized as semi – good with efficiency from $40\ to\ 60\ \%$. Under the Kifilideen trinomial theorem distribution model, there are three categories of possible outcomes of products which are good $(+)$, semi – good $(/)$ and bad $(-)$ products. For example, in laptop computer system production one category of the laptops produced has good appearance and all the hardware and software are working optimally. This category is classified under good $(+)$ product. Another categories of the laptops produced has bad appearance but all the hardware and software are working optimally. This category can be classified under semi –

good or undecided $(/)$ products. The last category of the laptops produced has bad appearance and all the hardware and software are not working properly. This category can be classified under bad products. In production of substance, substance can be categories as solid and liquid when dealing with binomial system but in some cases products manufacture can come in form of semi – solid (for examples tooth paste, gel, and body cream). So for Kifilideen trinomial distribution analysis model, products of substances from a company can be analysed under the categories of solid, liquid and semi – solid. Kifilideen trinomial theorem distribution model gives room for mid – way.

In agricultural banking sector there exist trinomial system. Agricultural banking sector gives loan to farmers to boost agricultural produces. In order for the bank not to run at lost they do ask farmer to produce collateral which replace the loan if the farmer is unable to pay back the loan. First category of farmers is farmers that have enough or adequate collateral $(+)$ to obtain the loan. Second category of farmers is farmers that lack or do not have any collateral $(-)$neither in form of land, car nor any other assets to present so they are not qualified at all for the loan. The third category of farmers is farmers that have collaterals but are inadequate $(/)$ or not enough to obtain loan. This category of farmers is given partial consideration, they may be given the loan or not. They are under the category maybe. In obtaining loan from bank as regard collateral, there are three sets of farmers which are those that have adequate collateral, inadequate collateral and lack collateral. When agricultural banks are dealing with farmers in area of loan, there are trinomial systems of farmers they encountered with.

More so, analysis of binomial distribution in social classes, people are categories into rich $(+)$ or poor $(-)$ but there are also some categories of people that are middle class $(/)$ which these people cannot be categories as been rich or poor because they have some attribute of rich and poor. The rich people $(+)$ live extravagent life and have more than enough. The poor people $(-)$ lack basic needs and live below standard of living. The middle class $(/)$ lives a moderate life. So, Kifilideen trinomial theroem distribution helps to analysis these three categories of classes which are rich $(+)$, middle $(/)$ and poor $(-)$ classes.

In classification of materials based on conductivity, materials are classified into conductors and insulators for which analysis of binomial distribution would be useful. It is known that there are also existence of some materials which cannot be classified as conductors $(+)$ or insulators $(-)$ which are called semi – conductors or metalloids $(/)$ examples are silicon and germanium. Semi – conductors are materials which electrical and thermal conductivity lies between conductors and insulators. In putting semi – conductors into analysis with conductors and insulators, Kifilideen trinomial theorem distribution model would be useful. Kifilideen trinomial theorem distribution model gives room for mid – way.

In secondary school and tertiary institutions, students end up into three categories. The first categories of students are students who get to the last level of education, succeed and obtain certificates with high grade $(+)$. The second categories of students are students who manage to get to the last level of education, although fail out but obtain certificates containing low grades $(-)$. Certificates were given to the second categories of students because they were able to meet up with the minimum requirements. The last categories of students are students who started the program or admitted but latter on drop out or were unable to get to the last stage $(/)$ of the program. These sets of students would not have certificate that they attended the institution although they have acquired one skill or the other from the institution. The school has also benefited from such students for the school fees paid by the students for the time been they are in the institution. These set of students are also valuable to the institution because the little knowledge acquire in the institution can be used by such students to add value to the society at large. This logic indicate that secondary school and tertiary institutions build trinomial system of students.

Furthermore, in transportation system, raw and finished goods such as crude oil, electronics, gold, limestone, cotton and cocoa can be imported or exported by country through three different modes of transportation which are land, sea and air. In analysis of importation and exportation of raw and finished goods by a country Kifilideen trinomial theorem distribution model would be needed to carry out such analysis.

Primary colours are trinomial system of colours in nature which when mixed together gives white light. The primary colours are Red $(+)$, Green $(/)$ and Blue $(-)$. For any random selection of the primary colours there are three possible outcomes which are red, green and blue which are trinomial in nature. To analysis random selection from a set of primary colours, Kifilideen trinomial theorem distribution model can be utilized. More so, secondary colours are also trinomial system of colours which are cyan (Green and Blue), Magneta (Blue and Red) and Yellow (Green and Red).

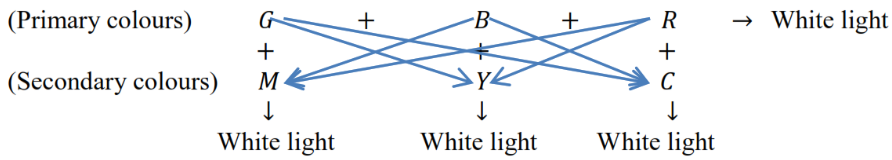


Figure 1: The analysis of the types of colours into trinomial system

The analysis of the types of colours into trinomial system in Figure 1 above can be understood and memorized using the code General Manager (GM), BuY (BY), RiCe (RC) which is presented in the Figure. Where G is green colour, M is magenta, B is blue, Y is yellow, R is Red and C is Cyan. Addition of Green and Magenta gives white light making green to be complementary colour to magenta, addition of Blue and Yellow gives white light making blue a complementary colour to yellow and Red and Cyan gives white light making red complementary colour to cyan.

More so, substances can come in three forms which are white, coloured and black substances. White substances are substance which contain all colours or spectra (red, orange, yellow, blue. green, indigo and violet). Black substances are substances which lack colours or without presents of colours while coloured substances are substances which have at least one colour or spectrum but not all colours are presents in it. To analysis these sets of substances, Kifilideen trinomial theorem distribution model would be appropriate.

In wiring system, some connections are done using binomial system where two wires are used which are live having colour red $(+)$ and neutral having colour blue $(-)$ while some connections are done using trinomial system where three wires are used which are live having colour red $(+)$, earth having colour green $(/)$ and neutral having colour blue. For three wiring system of connections, Kifilideen trinomial theorem distribution model is applicable and appropriate in the analysis of the wiring system.

All chemical substances are in categories of trinomial system and are categories into three substances which are acidic substances $(+)$, neutral susbtances $(/)$ and basic substances $(-)$. So, any chemical substance produces in chemical industry can be classified under the categories of these three chemical substances. Acidic substance turns blue litmus paper red, neutral substance has no effect on litmus paper and basic substances turn red litmus paper blue. For analysis of these three substances, Kifilideen trinomial theorem distribution model would be useful. This research work presents real life applications of mathematical model of Kifilideen trinomial theorem distribution of positive powers of n.

Mathematical induction method was adopted in developing the formulas of the components of the Kifilideen theorem of negative powers of n.

The constituents of the Kifilideen trinomial theorem are power combination, group, row, column, position, period and terms (Osanyinpeju, 2020a; Osanynipeju, 2020b; Osanyinpeju, 2020c, Osanyinpeju, 2021b, Osanyinpeju, 2022). Mathematical induction is not a method of making discoveries but intensive and meticulous method of proving a fact or discoveries that already been established. There is need to present mathematical induction of the mathematical fact of this trinomial theorem of negative powers of n so as to inbuilt more confidence in the established fact. The mathematical induction would help to buttress and show how the Kifilideen formulas of the negative powers of n are generated. The mathematical induction would also helps to support and prove that the developed formulas for the constituents of Kifilideen trinomial theorem of negative powers of n are valid and true. This study makes available thematathematical induction of Kifilideen trinomial theorem of negative powers of n adopting series and sequence ideology. This research work presents real life applications of mathematical model of Kifilideen trinomial theorem distribution of positive powers of n. This research work also develops alternate general Kifilideen power combination formula, alternate general Kifilideen term formula and alternate general Kifilideen position formula for the negative powers of n which conformed but not the same with the one developed for the positive powers of $n$counterpart.

MATERIALS AND METHODS

The Kifilideen matrix was fully utilized in the analysis of the Kifilideen trinomial theorem distribution model. The material used for this research work in formulating the Kifilideen formulas and equations of the constituents of the Kifilideen trinomial theorem of negative powers of n is the adoption of series and sequence ideology

Mathematical Model of Kifilideen Trinomial Theorem Distribution of Positive Powers of n

Taking the total number of objects/items/people/entities of an event as $n(T)$, if there exists three possible categories of outcomes for the event where the number of occurrence of the first category (say, pass category) of outcome in the total number of population is $n\ (P)$, the number of occurrence of the second category (say, indecisive or intermediary category) of outcome in the total number of population is $n\ (R)$, the number of occurrence of the third category (say, fail category) of outcome in the total number of population is $n\ (Q)$, then Kifilideen trinomial theorem distribution model would be useful or applicable.

Table 1: Breakdown of categories of trinomial system in various areas.

S/N	First Category	Second Category	Third Category
1.	True	indecisive	False
2.	Pass	Inconclusive or withdraw	Fail
3.	On	Intermediate	Off
4.	Solid	Semi – Solid	Liquid
5.	Win	Draw	Loss
6.	Metal	Metalloid	Non – Metal
7.	Accelerate	Constant motion	Decelerate or Retardate
8.	Conductor	Semi - Conductor	Insulator
9.	Land	Sea	Air
10.	Solid	Liquid	Gas
11.	Constructive	Neutral/No change or effect	Destructive
12.	Elevation	Stationary	Depression
13.	Supersaturated	Saturated	Understaturated
14.	Good	Semi – Good	Bad
15.	Above Average	Average	Below Average
16.	Rich Class	Middle Class	Poor Class
17.	Up	At rest	Down
18.	Positive	Neutral	Negative
19.	Live	Earth	Neutral
20.	Acid	Neutral	Base
21.	Uniform	Indecisive	Non – Uniform
22.	Alive	Coma (Between live and death)	Dead

Let the probability of first category, say pass $= P_r(pass) = p$ (5)

Let the probability of second category, say indecisive $= P_r\ (indecisive) = r$ (6)

Let the probability of third category, say fail $= P_r\ (fail) = q$ (7)

Then,

$$p = \frac{n(P)}{n(T)}\ ,\quad r = \frac{n(R)}{n(T)}\ \text{and}\ q = \frac{n(Q)}{n(T)} \quad (8)$$

Note: $p + r + q = 1$ and $n(P) + n(R) + n(Q) = n(T)$ (9)

Say: p –true/pass/win/rich class/good/conductor/on/uniform acceleration

r –intermediate/indecisive/draw/middle class/median/inconclusive/uniform velocity

q –false/fail/loss/poor class/insulator/bad/off/uniform retardation

Table 1 above shows the break down categories of trinomial system in various areas. Examples of first category of a trinomial system are true, pass, on, solid, win, metal, accelerate, conductor, supersaturated, constructive, up, alive, positive, elevation and rich class. Examples of the second category of the trinomial system are draw, intermediate, indecisive, average, inconclusive, withdraw, semi – solid, metalloid, saturated, at rest, coma, middle class, stationary, and semi – conductor while examples of third category of trinomial system are false, fail, off, loss, non – metal, decelerate, destructive, depression, poor class, down, dead, negative and below average.

If n objects/people/entities/items are selected from the total population of objects/items/people/entities of an event, then, the mathematical model to represent the trinomial system of all possible combination of the n objects/items/people/entities selected is given as:

$$[p + r + q]^n \quad (10)$$

Using Kifilideen trinomial theorem distribution model of positive powers of n we have:

$$[p + r + q]^n = {}_{n,0,0}^{\ \ n}Cp^n r^0 q^0 + {}_{n-1,1,0}^{\ \ \ \ n}Cp^{n-1} r^1 q^0 + {}_{n-2,2,0}^{\ \ \ \ n}Cp^{n-2} r^2 q^0 + {}_{n-3,3,0}^{\ \ \ \ n}Cp^{n-3} r^3 q^0 + \cdots + {}_{n-1,0,1}^{\ \ \ \ n}Cp^{n-1} r^0 q^1 + {}_{n-2,1,1}^{\ \ \ \ n}Cp^{n-2} r^1 q^1 + {}_{n-3,2,1}^{\ \ \ \ n}Cp^{n-3} r^2 q^1 + {}_{n-4,3,1}^{\ \ \ \ n}Cp^{n-4} r^3 q^1 + \cdots + {}_{0,n-1,1}^{\ \ \ \ n}Cp^0 r^{n-1} q^1 + \cdots + \cdots + \cdots + {}_{0,0,n}^{\ \ n}Cp^0 r^0 q^n \quad (11)$$

So, if the probability of passing an event is p, the probability of indecisive of the event is r and the probability of failing the event is q from a large population then for certain n number of people or object or items or entities selected at random from the large population the Kifilideen trinomial distribution model is presented as (11). The summation of the series of (11) would give 1.

Such that $p + r + q = 1$ (12)

That is, probability of pass of event $+$ probability of indecisive of event $+$ Probability of fail of the event $= 1$ (13)

$P_r\ (Pass) + P_r\ (Indecisive) + P_r\ (Fail) = \ 1$ (14)

For Term $T_1$for $t = 1$ in (11) indicates the probability of n pass, 0 indecisive and 0 fail for the random selection of the n people or objects or items or entities of the population. For Term $T_2$for $t = 2$ in (11) indicates the probability of $(n - 1)$ pass,1 indecisive0 fail for the random selection of the n people or objects or entities of the population. For Term $T_3$for $t = 3$ in (11) indicates the probability of $(n - 2)$ pass and 2 indecisive and 0fail for the random selection of the n people or objects or entities of the population. For last Term $T_t$for $t = t$ in (11) indicates the probability of 0 pass, 0 indecisive n fail for the random selection of the n people or objects or entities of the population. The trend above for Term T_1 for $t = 1$, Term T_2 for $t = 2$, Term T_3 for $t = 3$ and last Term T_t for $t = t$ is also applicable to all other terms in the Kifilideen trinomial theorem distribution model.

Mathematical Induction of Kifilideen Power Combination of Negative Powers of n of Kifilideen Trinomial Theorem

Kifilideen power combination formula 1

$C_P = 9t - 110a - 9m + n00$ (15)

Where C_P is the power combination of term, t, t is the term of the power combination , a and m are the migration column factor and migration row factor respectively and n is the value of the negative power of Kifilideen trinomial theorem.

Figure 2 below shows Kifilideen matrix of negative power of n (where $n = -6$ for the sample Figure below), of Kifilideen trinomial theorem while Figure 3 indicates the arrangement of the terms in each group for negative power of n (where $n = -6$ for the sample Figure 3 below), of Kifilideen trinomial theorem. From Figure 2 it is observed that the difference in value between the first member of power combination in one group and another first member of power combination in the next or proceeding group is 110. For example; for group 1, the first power combination, fm_{g_1} is $-\ 600$ and for group 2, the first power combination, $fm_{g_2}$is $-\ 710$. So, the common difference in power combination, D is $-710 - (-600) = -110$

That is, $D =$ Common difference in power combination $= fm_{g_2} - fm_{g_1} = -700 - (-600) = -110$ (16)

Also, for group 2, the first power combination, fm_{g_2} is $-\ 710$ and for group 3, the first power combination, fm_{g_3} is $-\ 820$. So, the common difference in power combination, D is $-820 - (-710) = -110$

That is, $D =$ Common difference in power combination $= fm_{g_3} - fm_{g_2} = -820 - (-710) = -110$ (17)

g_1	g_2	g_3	g_4	g_q
kif	kif	kif	kif	
-600				
	-710			
	-701	-820		
		-811	-930	
		-802	-921	.
			-912	..
			-903	…
				…
				…
				…

Figure 2: Kifilideen matrix of negative power of 6 of Kifilideen trinomial theorem

g_1	g_2	g_3	g_4	g_q
T_1				
	T_2			
	T_3	T_4		
		T_5	T_7	
		T_6	T_8	
			T_9	.
			T_{10}	..
				...
				...

Figure 3: Arrangement of the terms in each group for negative power of 6 of Kifilideen trinomial theorem.

More so, the difference in value between one power combination and the proceeding power combination down a particular group is 9. For illustration; considering group 4, the members of the power combination serially are -930, -921, -912 and -903. The common difference, d down the group is:

$$d = -921 - (-930) = -912 - (-921) = -903 - (-912) = 9 \tag{18}$$

This information procured in (17) and (18) were used to formulate the mathematical induction of the power combination formula of the negative power of 6 of the Kifilideen trinomial theorem. The mathematical induction of the power combination of the negative power of 6 of the Kifilideen trinomial theorem is given as follow:

Group 1 Term T_1 for $t = 1$,

$$C_P = -600 = -600 - 110 \times 0 + 9 \times 0 \tag{19}$$

$$C_P = -600 = -600 - 110 \times 0 + 9 \times (1 - 1) \tag{20}$$

$$C_P = -600 = 9 \times 1 - 110 \times 0 - 9 \times 1 - 600 \tag{21}$$

Group 2 Term T_2 for $t = 2$,

$$C_P = -710 = -600 - 110 \times 1 + 9 \times 0 \tag{22}$$

$$C_P = -710 = -600 - 110 \times 1 + 9 \times (2 - 2) \tag{23}$$

$$C_P = -710 = 9 \times 2 - 110 \times 1 - 9 \times 2 - 600 \tag{24}$$

Term T_3 for $t = 3$,

$$C_P = -701 = -600 - 110 \times 1 + 9 \times 1 \tag{25}$$

$$C_P = -701 = -600 - 110 \times 1 + 9 \times (3 - 2) \tag{26}$$

$$C_P = -701 = 9 \times 3 - 110 \times 1 - 9 \times 2 - 600 \tag{27}$$

Group 3 Term T_4 for $t = 4$,

$$C_P = -820 = -600 - 110 \times 2 + 9 \times 0 \tag{28}$$

$$C_P = -820 = -600 - 110 \times 2 + 9 \times (4 - 4) \tag{29}$$

$$C_P = -820 = 9 \times 4 - 110 \times 2 - 9 \times 4 - 600 \tag{30}$$

Term T_5 for $t = 5$,

$$C_P = -811 = -600 - 110 \times 2 + 9 \times 1 \tag{31}$$

$$C_P = -811 = -600 - 110 \times 2 + 9 \times (5 - 4) \quad (32)$$
$$C_P = -811 = 9 \times 5 - 110 \times 2 - 9 \times 4 - 600 \quad (33)$$

Term T_6 for $t = 6$,

$$C_P = -802 = -600 - 110 \times 2 + 9 \times 2 \quad (34)$$
$$C_P = -802 = -600 - 110 \times 2 + 9 \times (6 - 4) \quad (35)$$
$$C_P = -802 = 9 \times 6 - 110 \times 2 - 9 \times 4 - 600 \quad (36)$$

In Summary; the mathematical induction of the power combination of the negative power of 6 of the Kifilideen trinomial theorem is given as follow;

Group 1, Term T_1 for $t = 1$, $\quad C_P = -600 = 9 \times 1 - 110 \times 0 - 9 \times 1 - 600 \quad (37)$

Group 2, Term T_2 for $t = 2$, $\quad C_P = -710 = 9 \times 2 - 110 \times 1 - 9 \times 2 - 600 \quad (38)$

Term T_3 for $t = 3$, $\quad C_P = -701 = 9 \times 3 - 110 \times 1 - 9 \times 2 - 600 \quad (39)$

Group 3, Term T_4 for $t = 4$, $\quad C_P = -820 = 9 \times 4 - 110 \times 2 - 9 \times 4 - 600 \quad (40)$

Term T_5 for $t = 5$, $\quad C_P = -811 = 9 \times 5 - 110 \times 2 - 9 \times 4 - 600 \quad (41)$

Term T_6 for $t = 6$, $\quad C_P = -802 = 9 \times 6 - 110 \times 2 - 9 \times 4 - 600 \quad (42)$

. . .

. . .

. . .

Group g; Term T_t for $t = t$; $\quad C_P = kif = 9 \times t - 110 \times a - 9 \times m - 600 \quad (43)$

Following the trend in (37) to (42), we can deduce that the mathematical induction of the power combination of the negative powers of n of the Kifilideen trinomial theorem is given as:

For Term T_t for $t = t$ $\quad C_P = kif = 9t - 110a - 9m + n00 \quad (44)$

Where C_P is the power combination of Term T_t for $t = t$; t is the term of the power combination, a and m are the migration column factor and migration row factor respectively; k, i and f are the first, second and third components of the power combination; and n is the negative power of Kifilideen trinomial theorem.

To derive an equation for a and m, we have the mathematical induction of the power combination of the negative power of 6 of the Kifilideen trinomial theorem as given below;

$$C_P = 9 \times \boldsymbol{t} - 110 \times a - 9 \times \boldsymbol{m} + n00 = kif \quad (45)$$

Group 1; Term T_1 for $t = 1$, $\quad C_P = 9 \times \mathbf{1} - 110 \times 0 - 9 \times \mathbf{1} - 600 = -60\mathbf{0} \quad (46)$

Group 2; Term T_2 for $t = 2$, $\quad C_P = 9 \times \mathbf{2} - 110 \times 1 - 9 \times \mathbf{2} - 600 = -71\mathbf{0} \quad (47)$

Term T_3 for $t = 3$, $\quad C_P = 9 \times \mathbf{3} - 110 \times 1 - 9 \times \mathbf{2} - 600 = -70\mathbf{1} \quad (48)$

Group 3; Term T_4 for $t = 4$, $\quad C_P = 9 \times \mathbf{4} - 110 \times 2 - 9 \times \mathbf{4} - 600 = -82\mathbf{0} \quad (49)$

Term T_5 for $t = 5$, $\quad C_P = 9 \times \mathbf{5} - 110 \times 2 - 9 \times \mathbf{4} - 600 = -81\mathbf{1} \quad (50)$

Term T_6 for $t = 6$, $\quad C_P = 9 \times \mathbf{6} - 110 \times 2 - 9 \times \mathbf{4} - 600 = -80\mathbf{2} \quad (51)$

Minus **Gives**

$\boldsymbol{t} \quad - \quad \boldsymbol{m} \quad = \quad \boldsymbol{f}$

Taking the value of the power combination as kif. Where k, i and f are the first, second and third digits of the power combination. From the mathematical induction in (46) to (51) we can deduce that:

$$t - m = f \quad (52)$$

Where t is the term of the power combination, m is the migration row factor and f is the third component of the power combination.

Also, from the mathematical induction in (46) to (51); Table 1 is worked out below. Table 1 shows the values of a and m for group, g for negative power of 6 or any $-n$ of Kifilideen trinomial theorem. It can be noticed from Table 1 that: $a = g - 1$ (53)

To evaluate m, we have:

$m = 1 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + \cdots + a$ (54)

Let $\theta = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + \cdots + a$ (55)

So, $m = 1 + \theta$ (56)

From (55), 1^{st} term $= v = 1$, $d = T_2 - T_1 = 2 - 1 = 1$ and $l = a$ and $w = a$ (57)

Where v is the first term, d is the common difference, T_1 is the first term, T_2 is the second term, l is the last term, w is the number of terms and a is the migration column factor. The series of θ in (55) is Arithmetic progression A.P. From the sum of A.P. given by Irmak and Alp (2013) we have:

$$S_w = \frac{w}{2}(v + l) \quad (58)$$

Where v and l are the first and last terms of the series, θ respectively in (56), S_w is the series of θ and w is the number of terms of θ.

$$S_w = \theta = \frac{a}{2}(1 + a) \quad (59)$$

Table 1: Values of a and m for group, g for negative power of 6 or any $-n$ of Kifilideen trinomial theorem

g	a	m
1	0	$1 = 1^{+0} = 1$
2	1	$1 + 1 = 1^{+1} = 2$
3	2	$1 + 1 + 2 = 2^{+2} = 4$
.	.	.
.	.	.
.	.	.
g	$g - 1$	$1 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + \cdots + a$

Put (59) in (55), we have:

$$m = 1 + \theta = 1 + \frac{a}{2}(1 + a) \quad (60)$$

$$m = \frac{a^2 + a + 2}{2} \quad (61)$$

Since from (53), we have

$$a = g - 1 \quad (62)$$

$$m = \frac{(g-1)^2 + (g-1) + 2}{2} \quad (63)$$

$$m = \frac{g^2 - g + 2}{2} \quad (64)$$

Generally from the mathematical induction for negative powers of n for Kifilideen trinomial theroem; we have:

$$C_P = kif = 9t - 110a - 9m + n00 \quad (65)$$

The migration column factor, $a = g - 1$ (66)

The migration row factor, $m = \frac{a^2 + a + 2}{2}$ or $m = \frac{g^2 - g + 2}{2}$ (67)

Kifilideen power combination formula 2

$$C_P = 9(t - 1) - \frac{9}{2}(n - k)^2 - \frac{229}{2}(n - k) + n00 \quad (68)$$

Where C_P is the power combination of Term T_t for $t = t$, t is the term of the power combination, k is the first component of the power combination; and n is the negative power of Kifilideen trinomial theorem.

Proof:

The mathematical induction of the power combination of the negative power of 6 of the Kifilideen trinomial theorem from (37) to (42) is given as follow;

$$C_P = 9 \times t - 110 \times \boldsymbol{a} - 9 \times m + \boldsymbol{n}00 = \boldsymbol{kif} \quad (69)$$

Group 1;Term T_1 for $t = 1$, $C_P = 9 \times 1 - 110 \times \mathbf{0} - 9 \times 1 - \mathbf{600} = -\mathbf{600}$ (70)

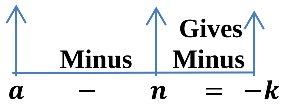
Group 2;Term T_2 for $t = 2$, $C_P = 9 \times 2 - 110 \times \mathbf{1} - 9 \times 2 - \mathbf{600} = -\mathbf{710}$ (71)

Term T_3 for $t = 3$, $C_P = 9 \times 3 - 110 \times \mathbf{1} - 9 \times 2 - \mathbf{600} = -\mathbf{7}01$ (72)

Group 3; Term T_4 for $t = 4$, $C_P = 9 \times 4 - 110 \times \mathbf{2} - 9 \times 4 - \mathbf{600} = -\mathbf{8}20$ (73)

Term T_5 for $t = 5$, $C_P = 9 \times 5 - 110 \times \mathbf{2} - 9 \times 4 - \mathbf{600} = -\mathbf{8}11$ (74)

Term T_6 for $t = 6$, $C_P = 9 \times 6 - 110 \times \mathbf{2} - 9 \times 4 - \mathbf{600} = -\mathbf{8}02$ (75)



Taking the value of the power combination as kif.Where k, i and f are the first, second and third digits of the power combination. From the mathematical induction in (70) to (75) we can deduce that;

$$a - n = -k \quad (76)$$

$$a = n - k \quad (77)$$

Where a is the migration column factor, k is the first component of the power combination and n is value of the negative powers of Kifilideen trinomial theorem.

From (67), the migration row factor,

$$m = \frac{a^2+a+2}{2} \quad (78)$$

$$m = \frac{(n-k)^2+(n-k)+2}{2} \quad (79)$$

For further deduction for the power combination for the negative powers of n for the Kifilideen trinomial theorem, we have:

From (65),

$$C_P = 9t - 110a - 9m + n00 \quad (80)$$

$$C_P = 9t - 110(n-k) - 9\left(\frac{(n-k)^2+(n-k)+2}{2}\right) + n00 \quad (81)$$

$$C_P = 9(t-1) - \frac{9}{2}(n-k)^2 - \frac{229}{2}(n-k) + n00 \quad (82)$$

Where C_P is the power combination of Term T_t for $t = t, t$ is the term of the power combination; k is the first component of the power combination; and n is the negative power of Kifilideen trinomial theorem.

Kifilideen power combination formula 3

$$C_P = -110a + 9(t-m) + n00 \quad (83)$$

$$a = \frac{-1+\sqrt{8t-7}}{2} \quad (84)$$

$$m = \frac{a^2+a+2}{2} \quad (85)$$

Where C_P is the power combination of term, t, t is the term of the power combination , a and m are the migration column factor and migration row factor respectively and n is the value of the negative powers of Kifilideen trinomial theorem.

The mathematical induction of the negative power of – 6 for the Kifilideen trinomial theorem is given as:

Group 1; Term T_1 for $t = 1$, $C_P = -600 = -110 \times 0 - 9(1-1) - 600$ (86)

Group 2;Term T_2 for $t = 2$, $C_P = -710 = -110 \times 1 - 9(2-2) - 600$ (87)

Term T_3 for $t = 3$, $C_P = -701 = -110 \times 1 - 9(3-2) - 600$ (88)

Group 3; Term T_4 for $t = 4$, $C_P = -820 = -110 \times 2 - 9(4-4) - 600$ (89)

Term T_5 for $t = 5$, $C_P = -811 = -110 \times 2 - 9(5-4) - 600$ (90)

Term T_6 for $t = 6$, $C_P = -802 = -110 \times 2 - 9(6-4) - 600$ (91)

. . .
. . .
. . .

Group g; Term T_t for $t = t$; $C_P = kif = -110 \times a - 9(t - m) - 600$ (92)

Following the trend in (86) to (91), we can deduce that the mathematical induction of the power combination of the negative powers of n of the Kifilideen trinomial theorem is given as:

For Term T_t for $t = t$; $C_P = kif = -110a - 9(t - m) + n00$ (93)

From Figure 3, Term T_1 for $t = 1$ belong to group 1, g_1; Term T_2 for $t = 2$ and Term T_3 for $t = 3$ belong to group 2, g_2; Term T_4 for $t = 4$, Term T_5 for $t = 5$, and Term T_6 for $t = 6$ belong to group 3, g_3;..., respectively. Considering only the first term of each group (m value of each group), we have:

$g = 1,\quad t = 1 = 1^{st}$ term (94)

$g = 2,\quad t = 1 + 1 = 2^{nd}$ term (95)

$g = 3,\quad t = 1 + 1 + 2 = 4^{th}$ term (96)

..

..

..

$g = g,\quad t = 1 + 1 + 2 + 3 + 4 + \cdots + (g - 1)$ (97)

Let $\mu = 1 + 2 + 3 + 4 + \cdots + (g - 1)$ (98)

Then $t = 1 + \mu$ (99)

From the (98), 1st term $= x = 1$, last term $= z = g - 1$ and $u = g - 1$ (100)

Where x is the first term of the series, μ in (99), z is the last term of μ and u is the number of terms of μ.

Using sum of arithmetic progression formula presented by Irmak and Alp (2013), we have

$S_u = \mu = \frac{u}{2}(x + z)$ (101)

$S_{g-1} = \mu = \frac{(g-1)}{2}(1 + g - 1)$ (102)

$\mu = \frac{g\,(g-1)}{2}$ (103)

Put (103) in (98),

$t = 1 + \mu = 1 + \frac{g\,(g-1)}{2}$ (104)

$g^2 - g + 2 - 2t = 0$ (105)

Using quadratic formula, $g = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(2-2t)}}{2 \times 1}$ (106)

$g = \frac{1 \pm \sqrt{8t-7}}{2}$ (107)

Since g is positive in value,

$g = \frac{1+\sqrt{8t-7}}{2}$ (108)

Recall from (53),

$a = g - 1, g = a + 1$ (109)

Put (109) in (108),

$g = a + 1 = \frac{1+\sqrt{8t-7}}{2}$ (110)

$a = \frac{-1+\sqrt{8t-7}}{2}$ (111)

Recall from (85), $m = \frac{a^2+a+2}{2}$ (112)

For group 1, putting $t = 1$ for Term T_1in (111) a whole number of 0 can be obtained for a (that is $a = 0$). Inserting $a = 0$ in (108) the value of $m = 1$ is arrived at. Group 2 contains Term T_2 and Term $T_3$where values for m and a are $m = 2$and $a = 1$, putting $t = 2$ forTerm T_2 in (111) a whole number of 1 can be achieved for a (that is $a = 1$) and putting $t = 3$ for Term T_3 in (107) a decimal value of 1.5615 can be attained for a. Since the value of a in group 2 is 1, so the whole number part of the value of a attained for $t = 3$ is considered. Inserting $a = 1$ in (112) value of $m = 2$ is arrived at. Subsequently Group 3 contains Term T_4, Term T_5 and Term $T_6$where values for m and a are $m = 4$ and $a = 2$, putting $t = 4$ for Term T_4 in (111) a whole number of 2 can be obtained for a (that is $a = 2$) and putting $t = 5$ for Term T_5, $t = 6$ for Term T_6 in (111) decimal values of 2.3722 and 2.7016 are work out respectively for a. Since the value of a in group 3 is 2, so the whole number part of the value of a attained for $t = 5$, $t = 6$ are considered. Inserting $a = 2$ in (112) value of $m = 4$is secured. The trend above as calculated for group 1, group 2 and group 3 is also applicable to all the other groups.

Generally, $a = \frac{-1+\sqrt{8t-7}}{2}$ (113)

which can be used to acquire the value of $a$for any given term of Kifilideen trinomial theorem of negative powers of n and $m = \frac{a^2+a+2}{2}$ (114)

which can be used to obtained the value of m once the value of a is known.

In summary, the following can be deduced from the mathematical induction above,

$C_P = -110a + 9(t - m) + n00$ (115)

$a = \frac{-1+\sqrt{8t-7}}{2}$ (116)

$m = \frac{a^2+a+2}{2}$ (117)

Mathematical Induction of Kifilideen General Term Formula of Negative Powers of $\boldsymbol{n}$of Kifilideen Trinomal Theorem

Kilifideen general term formula 1

$t = \frac{(n-k)^2+(n-k)+2+2f}{2}$ 118)

The mathematical induction of the Kifilideen term formula 1 of the negative powers of n of the Kifilideen trinomial theorem (using the negative power of -6) is given as follow:

$C_P = 9 \times \boldsymbol{t} - 110 \times a - 9 \times \boldsymbol{m} + n00 = kif$ (119)

Group 1; Term $T_1$for $t = 1$, $C_P = 9 \times \mathbf{1} - 110 \times 0 - 9 \times \mathbf{1} - 600 = -60\mathbf{0}$ (120)

Group 2; Term $T_2$for $t = 2$, $C_P = 9 \times \mathbf{2} - 110 \times 1 - 9 \times \mathbf{2} - 600 = -71\mathbf{0}$ (121)

Term $T_3$for $t = 3$, $C_P = 9 \times \mathbf{3} - 110 \times 1 - 9 \times \mathbf{2} - 600 = -70\mathbf{1}$ (122)

Group 3;Term $T_4$for $t = 4$, $C_P = 9 \times \mathbf{4} - 110 \times 2 - 9 \times \mathbf{4} - 600 = -82\mathbf{0}$ (123)

Term $T_5$for $t = 5$, $C_P = 9 \times \mathbf{5} - 110 \times 2 - 9 \times \mathbf{4} - 600 = -81\mathbf{1}$ (124)

Term $T_6$for $t = 6$, $C_P = 9 \times \mathbf{6} - 110 \times 2 - 9 \times \mathbf{4} - 600 = -80\mathbf{2}$ (125)

Minus **Gives**

$\boldsymbol{t} \quad - \quad \boldsymbol{m} \quad = \quad \boldsymbol{f}$

Taking the value of the power combination as kif. Where k, i and $f$are the first, second and third digits of the power combination. From the mathematical induction in (120) to (125) we can deduce that:

$t - m = f$ (126)

Where t is the term of the power combination, m is the migration row factor and f is the third component of the power combination. Table 2 illustrates the mathematical induction for the Kifilideen general term formula 1 for negative powers of n of Kifilideen trinomial theorem (using negative power of 6). Recall from (77) and (79), the migration group factor, a and migration row factor, m are given as:

$a = n - k$ (127)

$m = \frac{(n-k)^2+(n-k)+2}{2}$ (128)

Therefore, $t - m = f$ (129)

$t - \frac{(n-k)^2+(n-k)+2}{2} = f$ (130)

$t = \frac{(n-k)^2+(n-k)+2f+2}{2}$ (131)

Table 2: Mathematical induction for the Kifilideen general term formula 1 for negative powers of n of Kifilideen trinomial theorem (using negative power of 6).

Group, g	Power combination, C_P (kif)	Migration column factor, a ($a = g - 1$) Or ($a = n - k$)	Migration row factor, m $\left(m = \frac{a^2 + a + 2}{2}\right)$	Term, t ($t = m + f$)
1	-600	$1 - 1 = 0$ Or $-6 - -6 = 0$	$\frac{0^2+0+2}{2} = \mathbf{1}$	$\frac{0^2+0+2}{2} + 0 = 1 + \mathbf{0} = 1$
2	-710	$2 - 1 = 1$ Or $-6 - -7 = 1$	$\frac{1^2+1+2}{2} = \mathbf{2}$	$\frac{1^2+1+2}{2} + 0 = \mathbf{2} + \mathbf{0} = 2$
	-701	$2 - 1 = 1$ Or $-6 - -7 = 1$	$\frac{1^2+1+2}{2} = \mathbf{2}$	$\frac{1^2+1+2}{2} + 1 = \mathbf{2} + \mathbf{1} = 3$
3	-820	$3 - 1 = 2$ Or $-6 - -8 = 2$	$\frac{2^2+2+2}{2} = \mathbf{4}$	$\frac{2^2+2+2}{2} + 0 = \mathbf{4} + \mathbf{0} = 4$
	-811	$3 - 1 = 2$ Or $-6 - -8 = 2$	$\frac{2^2+2+2}{2} = \mathbf{4}$	$\frac{2^2+2+2}{2} + 1 = \mathbf{4} + \mathbf{1} = 5$
	-802	$3 - 1 = 2$ Or $-6 - -8 = 2$	$\frac{2^2+2+2}{2} = \mathbf{4}$	$\frac{2^2+2+2}{2} + \mathbf{2} = \mathbf{4} + \mathbf{2} = 6$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
g	kif	$a = g - 1$ Or $a = n - k$	$m = \frac{a^2 + a + 2}{2}$ Or $m = \frac{(n-k)^2 + (n-k) + 2}{2}$	$t = \frac{a^2 + a + 2}{2} + f$ Or $t = \frac{(n-k)^2 + (n-k) + 2}{2} + f$ Or $t = \frac{(n-k)^2 + (n-k) + 2f + 2}{2}$

Also, the mathematical induction from Table 2, the Kifilideen general term formula 1 for negative powers of n of Kifilideen trinomial theorem (using negative power of 6 or any value of negative power of n) is given as:

$$t = \frac{(n-k)^2+(n-k)+2f+2}{2} \quad (132)$$

In summary, the mathematical induction of the Kifilideen general term formula 1 for negative powers of n of Kifilideen trinomial theorem is given as:

$$t = \frac{(n-k)^2+(n-k)+2f+2}{2} \quad (133)$$

Kilifideen general term formula 2

$$t = \frac{(n-k)^2+3(n-k)-2i+2}{2} \quad (134)$$

The mathematical induction of the Kifilideen term formula 2 of the negative powers of n of the Kifilideen trinomial theorem (using the negative power of 6) is given as follow:

Minus **Result 2**

$$C_P = 9\times \boldsymbol{t} - 110\times \boldsymbol{a} - 9\times \boldsymbol{m} - n00 = kif \quad (135)$$

Group 1; Term T_1 for $t=1$, $C_P = 9\times \mathbf{1} - 110\times \mathbf{0} - 9\times \mathbf{1} - 300 = -300$ (136)

Group 2; Term T_2 for $t=2$, $C_P = 9\times \mathbf{2} - 110\times \mathbf{1} - 9\times \mathbf{2} - 300 = -410$ (137)

Term T_3 for $t=3$, $C_P = 9\times \mathbf{3} - 110\times \mathbf{1} - 9\times \mathbf{2} - 300 = -401$ (138)

Group 3; Term T_4 for $t=4$, $C_P = 9\times \mathbf{4} - 110\times \mathbf{2} - 9\times \mathbf{4} - 300 = -520$ (139)

Term T_5 for $t=5$, $C_P = 9\times \mathbf{5} - 110\times \mathbf{2} - 9\times \mathbf{4} - 300 = -511$ (140)

Term T_6 for $t=6$, $C_P = 9\times \mathbf{6} - 110\times \mathbf{2} - 9\times \mathbf{4} - 300 = -502$ (141)

Minus **Result 1**

Taking the value of the power combination as kif. Where k, i and f are the first, second and third digits of the power combination. From the mathematical induction in (135) to (141) we can deduce that:

$$Results\ 1 = Results\ 2 \quad (142)$$

$$t - m = a - i \quad (143)$$

Where t is the term of the power combination, a is the migration column factor, m is the migration row factor and i is the second component of the power combination. Table 3 demonstrates the mathematical induction for the Kifilideen general term formula 2 for negative powers of n of Kifilideen trinomial theorem.

Recall from (77) and (79), the migration column factor, a and migration row factor, m are given as:

$$a = n - k \quad (144)$$

$$m = \frac{(n-k)^2 + (n-k) + 2}{2} \quad (145)$$

Therefore, $t - m = a - i$ (146)

$$t - \frac{(n-k)^2 + (n-k) + 2}{2} = (n-k) - i \quad (147)$$

$$t = \frac{(n-k)^2 + 3(n-k) - 2i + 2}{2} \quad (148)$$

Also, the mathematical induction from Table 3, the Kifilideen general term formula 2 for negative powers of n of Kifilideen trinomial theorem (using negative power of 6 or any value of negative power of n) is given as

$$t = \frac{(n-k)^2 + 3(n-k) - 2i + 2}{2} \quad (149)$$

Table 3: Mathematical induction for the Kifilideen general term formula 2 for negative powers of n of Kifilideen trinomial theorem (using negative power of 6).

Group g	Migration column factor a ($a = g - 1$) Or ($a = n - k$)	Migration row factor m $\left(m = \frac{a^2 + a + 2}{2}\right)$	Power combination C_P (kif)	Term t ($t = m + a - i$)
1	$1 - 1 = 0$ Or $-6 - -6 = 0$	$\frac{0^2 + 0 + 2}{2} = \mathbf{1}$	-600	$\frac{0^2 + 0 + 2}{2} + 0 - 0 = \mathbf{1} + 0 - 0 = 1$
2	$2 - 1 = 1$ Or $-6 - -7 = 1$	$\frac{1^2 + 1 + 2}{2} = \mathbf{2}$	-710	$\frac{1^2 + 1 + 2}{2} + 1 - 1 = \mathbf{2} + 1 - 1 = 2$
	$2 - 1 = 1$ Or $-6 - -7 = 1$	$\frac{1^2 + 1 + 2}{2} = \mathbf{2}$	-701	$\frac{1^2 + 1 + 2}{2} + 1 - 0 = \mathbf{2} + 1 - 0 = 3$
3	$3 - 1 = 2$ Or $-6 - -8 = 2$	$\frac{2^2 + 2 + 2}{2} = \mathbf{4}$	-820	$\frac{2^2 + 2 + 2}{2} + 2 - 2 = \mathbf{4} + 2 - 2 = 4$
	$3 - 1 = 2$ Or $-6 - -8 = 2$	$\frac{2^2 + 2 + 2}{2} = \mathbf{4}$	-811	$\frac{2^2 + 2 + 2}{2} + 2 - 1 = \mathbf{4} + 2 - 1 = 5$
	$3 - 1 = 2$ Or $-6 - -8 = 2$	$\frac{2^2 + 2 + 2}{2} = \mathbf{4}$	-802	$\frac{2^2 + 2 + 2}{2} + 2 - 0 = \mathbf{4} + 2 - 0 = 6$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
g	$a = g - 1$ Or $a = n - k$	$m = \frac{a^2 + a + 2}{2}$ Or $m = \frac{(n-k)^2 + (n-k) + 2}{2}$	kif	$t = \frac{a^2 + a + 2}{2} + a - i$ Or $t = \frac{(n-k)^2 + (n-k) + 2}{2} + (n-k) - i$ Or $t = \frac{(n-k)^2 + 3(n-k) - 2i + 2}{2}$

In summary, the mathematical induction of the Kifilideen general term formula 2 for negative powers of $n$of Kifilideen trinomial theoremis given as: $t = \frac{(n-k)^2 + 3(n-k) - 2i + 2}{2}$ (150)

Mathematical Induction of Kifilideen Position Formula of Negative Powers of n Kifilideen Trinomial Theorem

Kifilideen position formula 1

$R_{member} = F_{member} + 9(p - 1)$ (151)

Where R_{member} is the power combination of the required member in which position is to be determined,F_{member} is the power combination of the first member of the group in which the required member belong and p is the position of the required member.

In Figure 2, the power combination of the first member, F_{member} in groups$1,2,3,4 \ldots$, are $-600, -710, -701, -820, \ldots$, respectively. The mathematical induction of the Kifilideen position formula for negative powers of n of Kifilideen trinomial theorem is presented as follow:

Group 1,Term T_1 for $t = 1$ $p = 1,$ $C_P = -600 = -600 + 9(1 - 1)$ (152)
Group 2, Term T_2 for $t = 2,$ $p = 1,$ $C_P = -710 = -710 + 9(1 - 1)$ (153)
Term T_3 for $t = 3,$ $p = 2,$ $C_P = -701 = -710 + 9(2 - 1)$ (154)
Group 3, Term T_4 for $t = 4,$ $p = 1,$ $C_P = -820 = -820 + 9(1 - 1)$ (155)
Term T_5 for $t = 5,$ $p = 2,$ $C_P = -811 = -820 + 9(2 - 1)$ (156)
Term T_6 for $t = 6,$ $p = 3,$ $C_P = -802 = -820 + 9(3 - 1)$ (157)
. . . .
. . . .
. . . .
Group g, Term T_t for $t = t,$ $p = p,$ $C_P = R_{member} = F_{member} + 9(p - 1)$ (158)

Where R_{member} is the power combination of the required member in which position is to be determined, F_{member} is the power combination of the first member of the group in which the required member belong and p is the position of the required member.It is observed from (152) to (157) that to obtainthe position of the power combination of the required member of a group, the power combination of the first member of that group is needed. In summary, the mathematical induction of the Kifilideen position formula for negative powers of n of Kifilideen trinomial theorem is given as: $R_{member} = F_{member} + 9(p - 1)$ (159)

Kifilideen position formula 2

$$C_P = kif = n00 - 110(a) + 9(p - 1) \quad (160)$$

The mathematical induction of the Kifilideen position formula 2 for negative power – 6 of Kifilideen trinomial theorem is presented as follow:
Group 1, Term T_1 for $t = 1,$ $p = 1,$ $C_P = -600 = -600 - 110(0) + 9(1 - 1)$ (161)
Group2, Term T_2 for $t = 2,$ $p = 1,$ $C_P = -710 = -600 - 110(1) + 9(1 - 1)$ (162)
Term T_3 for $t = 3,$ $p = 2,$ $C_P = -701 = -600 - 110(1) + 9(2 - 1)$ (163)
Group 3, Term T_4 for $t = 4,$ $p = 1,$ $C_P = -820 = -600 - 110(2) + 9(1 - 1)$ (164)
Term T_5 for $t = 5,$ $p = 2,$ $C_P = -811 = -600 - 110(2) + 9(2 - 1)$ (165)
Term T_6 for $t = 6,$ $p = 3,$ $C_P = -802 = -600 - 110(2) + 9(3 - 1)$ (166)
. . . .
. . . .
. . . .
Group g,Term T_t for $t = t,$ $p = p,$ $C_P = kif = -600 - 110(a) + 9(p - 1)$ (167)

In summary, for negative power of n of Kifilideen trinomial theorem, the Kifilideen position formula is stated as follow: $C_P = kif = n00 - 110(a) + 9(p - 1)$ (168)
Where k, i and f are the first, second and third digits of the power combination, a is the migration column factor, n is the value of the negative power of the Kifilideen trinomial theorem and p is the position of the power combination to be determined.

Mathematical Induction of Kifilideen Power combination row column Formula of Negative Power of $n$of the Kifilideen Trinomal Theorem

$CP_{rc} = kif = n00 - 110$
$(c - 1) + 9(r - c)$ (169)
$r = 2f + i + 1$ (170)
$c = f + i + 1$ (171)
$n - c + 1 = k$ (172)
$c = a + 1 = g$ (173)
$p = r - c + 1$ (174)

Where k, i and f are the first, second and third digits of the power combination, a is the migration column factor, n is the value of the negative power of the Kifilideen trinomial theorem, g is the group the power combination belong to, r and c are the row and column the power combination belong to and p is the position of the power combination to be determined.

Figure 4 shows the Kifilideen matrix for negative power of − 6 of Kifilideen trinomial theorem. For close study of the Kilifideen matrix of the negative power of -6 trinomial theorem; the migration column value is 110 and the migration row value is 9.This idea forms the basis of the mathematical induction for the Kifilideen power combination row column formula of the negative power of n of Kifilideen trinomial theorem.

The mathematical induction of the Kifilideen power combination row column formula for negative power 6 of Kifilideen trinomial theorem is illusrated as follow:

g_1 $CP_{rc} = kif$	g_2 $CP_{rc} = kif$	g_3 $CP_{rc} = kif$	... $CP_{rc} = kif$
$CP_{11} = -600$			
	$CP_{22} = -710$		
	$CP_{32} = -701$	$CP_{33} = -820$	
		$CP_{43} = -811$	
		$CP_{53} = -802$	.
			..
			...

Figure 4: the Kifilideen matrix of negative power of 6 of Kifilideen trinomial theorem

$\boldsymbol{n}$ **Minus** $-$ $(\boldsymbol{c}-\boldsymbol{1})$ **Gives** $=$ $\boldsymbol{k}$

Group 1, Term T_1 for $t = 1$ $\quad C_{11} = -600 - 110(1-1) + 9(\mathbf{1}-\mathbf{1}) = -600$ (175)

Group 2, Term T_2 for $t = 2$, $\quad C_{22} = -600 - 110(2-1) + 9(\mathbf{1}-\mathbf{1}) = -710$ (176)

Term T_3 for $t = 3$, $\quad C_{32} = -600 - 110(2-1) + 9(\mathbf{2}-\mathbf{2}) = -701$ (177)

Group 3, Term T_4 for $t = 4$, $\quad C_{33} = -600 - 110(3-1) + 9(\mathbf{3}-\mathbf{3}) = -820$ (178)

Term T_5 for $t = 5$, $\quad C_{43} = -600 - 110(3-1) + 9(\mathbf{4}-\mathbf{3}) = -811$ (179)

Term T_6 for $t = 6$, $\quad C_{53} = -600 - 110(3-1) + 9(\mathbf{5}-\mathbf{3}) = -802$ (180)

. . .

. . .

. . .

Group g,Term T_t for $t = t$, $\quad C_{rc} = -600 - 110(\boldsymbol{c}-\mathbf{1}) + 9(\boldsymbol{r}-\boldsymbol{c}) = \boldsymbol{kif}$ (181)

. .

Gives

$(\boldsymbol{r}-\boldsymbol{c}) = \boldsymbol{f}$

In summary, for negative powers of n of Kifilideen trinomial theorem, we have:

Group g,Term T_t for $t = t$, $\quad C_{rc} = n00 - 110(\boldsymbol{c}-\mathbf{1}) + 9(\boldsymbol{r}-\boldsymbol{c}) = \boldsymbol{kif}$ (182)

Also, from (175) to (180), we can deduce that:

$$n - (c-1) = k \quad (183)$$

$$n - c + 1 = k \quad (184)$$

More so,

$$r - c = f \quad (185)$$

$$r = c + f \quad (186)$$

Note always,

$$n = k + i + f \quad (187)$$

Put (187) in (184), we have:

$$k + i + f - c + 1 = k \quad (188)$$

So,

$$c = f + i + 1 \quad (189)$$

Put (189) in (186), we have: $r = f + i + 1 + f$ (190)

$r = 2f + i + 1$ (191)

Comparing (168) and 182, we have: $C_P = \quad kif = n00 - 110(a) + 9(p-1)$ (192)

$C_{r\ c} = n00 - 110(c-1) + 9(r-c) = kif$ (193)

Comparing the coefficient of -110: $a = c - 1$ (194)

$c = a + 1 = g$ (195)

Comparing the coefficient of $+9$: $p - 1 = r - c$ (196)

$p = r - c + 1$ (197)

Therefore, it can be deduced that the mathematical induction of Kifilideen power combination row column formula from (175) to (197) for negative powers of n of Kifilideen trinomial theorem is given as:

$$CP_{rc} = \quad kif = n00 - 110(c-1) + 9(r-c) \quad (198)$$

$$r = 2f + i + 1 \quad (199)$$

$$c = f + i + 1 \quad (200)$$

$$n - c + 1 = k \quad (201)$$

$$c = a + 1 = \quad (202)$$

$$p = r - c + 1 \quad (203)$$

RESULTS

Real Life Applications of Mathematical Model of Kifilideen Trinomial Theorem Distribution

The real life applications of mathematical model of Kifilideen trinomial theorem distribution of positive power of n is presented as follows:

(1) A car company produces three colours of cars which are red, green and blue. If the probability of a car selected at random is red is $\frac{1}{6}$ and the probability that the car selected at random is blue is $\frac{1}{3}$. If 4 samples of the cars are selected at random, determine the probability that three of the cars are red.

Solution

Let the probability of red car

$= P_r\,(red) = p$ (204)

Let the probability of green car

$= P_r\,(green) = r$ (205)

Let the probability of blue car

$= P_r\,(blue) = q$ (206)

$p = \frac{1}{6}$ and $q = \frac{1}{3}$ (207)

From $p + r + q = 1$ in (12), we have (208)

$\frac{1}{6} + r + \frac{1}{3} = 1$ (209)

$r = \frac{1}{2}$ (210)

For $n = 4$ number of car selected at random, the all possible power combinations are presented in the Kifilideen matrix of positive power of 4 in Figure 5. For the probability that three of the cars are red $(n(P) = 3)$ the possible power combinations from the Kifilideen matrix in Figure 5 are 310 and 301.

PRQ	PRQ	PRQ	PRQ	PRQ
400				
310				
220	**301**			
130	211			
040	121	202		
	031	112		
		022	103	
			013	
				004

Figure 5: Kifilideen matrix of positive power of 4 for $n = 4$ number of car selected at random showing all the possible power combinations.

Probability of three red cars

$= P_r(3reds, 1green\ and\ 0blue) + P_r(3reds, 0green\ and\ 1blue)$ (211)

Probability of three red cars

$= {}_{310}^{\ \ 4}Cp^3r^1q^0 + {}_{301}^{\ \ 4}Cp^3r^0q^1$ (212)

Probability of three red cars

$= \frac{4!}{3!1!0!}\left(\frac{1}{6}\right)^3\left(\frac{1}{2}\right)^1\left(\frac{1}{3}\right)^0 + \frac{4!}{3!0!1!}p^3r^0q^1$ (213)

Probability of three red cars $= 0.0154$ (214)

(2) A bulb producing company observes three categories of bulb after production which are good (of extreme quality) bulbs, semi – good (of medium quality) bulbs and bad (of very low quality) bulbs. The probability of selecting a good and semi – good bulbs at random from the bulbs produce by the company are 0.5 and 0.2 respectively. If 5 samples of bulbs are selected at random. Find the probability that at most 2 bulbs selected are good.

Solution

Let the probability of good bulbs

$= P_r\ (good) = p$ (215)

Let the probability of semi – good bulbs

$= P_r\ (semi\ good) = r$ (216)

Let the probability of bad bulbs

$= P_r\ (bad) = q$ (217)

$p = 0.5$and $r = 0.2$ (218)

From $p + r + q = 1$ in (12), we have (219)

$0.5 + 0.2 + q = 1$ (220)

$q = 0.3$ (221)

For $n = 5$ number of bulbs selected at random, the all possible power combinations are presented in the Kifilideen matrix of positive power of 5 in Figure 6. For the probability that at most 2 bulbs selected are good $(n(P) = 0, n(P) = 1\ and\ n(P) = 2),$ then the possible power combinations from the Kifilideen matrix in Figure 6 are 230, 140, 050, 221, 131, 041, 212,122, 032, 203, 113, 023, 104, 014 and 005 and for the probability that at least 3 bulbs selected are good $(n(P) = 3, n(P) = 4\ and\ n(P) = 5),$ the possible power combinations from the Kifilideen matrix in Figure 4 are 500, 410, 320, 401, 311 and 302.

PRQ	PRQ	PRQ	PRQ	PRQ	PRQ
500					
410					
320	**401**				
230	**311**				
140	221	**302**			
050	131	212			
	041	122	203		
		032	113		
			023	104	
				014	
					005

Figure 6: Kifilideen matrix of positive power of 5 for $n = 5$ number of bulbs selected at random showing all the possible power combinations.

Note: The probability of at most 2 good bulbs + the probability of at least 3 good bulbs = 1 (222)

Since the probability of at the least 3 good bulbs as lesser power combination, so we have (223)

Probability of at most 2 good bulbs $= 1 -$ the probability of at least 3 good bulbs (224)

Probability of at most 2 good bulbs

$$= 1 - ({}_{500}^{5}Cp^5r^0q^0 + {}_{410}^{5}Cp^4r^1q^0 + {}_{320}^{5}Cp^3r^2q^0 + {}_{401}^{5}Cp^4r^0q^1 + {}_{311}^{5}Cp^3r^1q^1 + {}_{302}^{5}Cp^3r^0q^2)$$ (225)

Probability of at most 2 good bulbs $= 1 - (\frac{5!}{5!0!0!}(0.5)^5(0.2)^0(0.3)^0 + \frac{5!}{4!1!0!}(0.5)^4(0.2)^1(0.3)^0 + \frac{5!}{3!2!0!}(0.5)^3(0.2)^2(0.3)^0 + \frac{5!}{4!0!1!}(0.5)^4(0.2)^0(0.3)^1 + \frac{5!}{3!1!1!}(0.5)^3(0.2)^1(0.3)^1 + \frac{5!}{3!0!2!}(0.5)^3(0.2)^0(0.3)^2) = 1 - (0.03125 + 0.0625 + 0.0500 + 0.09375 + 0.15 + 0.1125)$ (226)

Probability of at most 2 good bulbs $= 1 - 0.5000 = 0.5000$ (227)

(3) In a group of 40 people, 30 people are interested in attending a party, out of those 30 people 16 people later attended while the rest could not make it to the party. If 4 samples of people are selected at random from the group of the people, determine the probability that one person show interest and did not attend the party.

Solution

Let the total number of people in the group $= n(T)$ (228)

Let the number of people that show interest and attended $= n(P)$ (229)

Let the number of people that did not show interest at all $= n(R)$ (230)

Let the number of people that show interest but did not attend $= n(Q)$ (231)

$n(T) = 40, n(P) = 16, n(R) = 40 - 30 = 10$ and $n(Q) = 30 - 16 = 14$ (232)

Let the probability of people that show interest and attended

$= P_r\,(interested\ and\ attended) = p$ (233)

Let the probability of people that did not show interest at all $= P_r\,(not\ interested) = r$ (234)

Let the probability of people that show interest but did not attend $= P_r\,(interested\ but\ not\ attended) = q$ (235)

$p = \frac{n(P)}{n(T)} = \frac{16}{40} = \frac{2}{5}, r = \frac{n(R)}{n(T)} = \frac{10}{40} = \frac{1}{4}$ and $q = \frac{n(Q)}{n(T)} = \frac{14}{40} = \frac{7}{20}$ (236)

PRQ	PRQ	PRQ	PRQ	PRQ
400				
310				
220	**301**			
130	**211**			
040	**121**	202		
	031	112		
		022	103	
			013	
				004

Figure 7: Kifilideen matrix of positive power of 4 for $n = 4$ number of people of selected at random showing all the possible power combinations.

For $n = 4$ number of people selected at random, the all possible power combinations are presented in the Kifilideen matrix of positive power of 4 in Figure 7. For the probability that one person show interest and did not attend the party ($n(Q) = 1$) the possible power combinations from the Kifilideen matrix in Figure 7 are 301, 211, 121 and 031.

Probability that one person show interest and did not attend the party

$= P_r$ (3 $interested\ and\ attended,$ 0 $not\ interested$ and 1 $interested\ but\ not\ attended$) +

P_r (2 $interested\ and\ attended,$

$1\ not\ interested$ and
$1\ interested\ but\ not\ attended) +$
$P_r\ \ (1\ interested\ and\ attended,$
$2\ not\ interested$ and
$1\ interested\ but\ not\ attended) +$
$P_r\ \ (0\ interested\ and\ attended,$
$3\ not\ interested$ and
$1\ interested\ but\ not\ attended)$ (237)

Probability that one person show interest and did not attend the party

$$= {}_{301}^{\ 4}Cp^3r^0q^1 + {}_{211}^{\ 4}Cp^2r^1q^1 + {}_{121}^{\ 4}Cp^1r^2q^1 + {}_{031}^{\ 4}Cp^0r^3q^1 \quad (238)$$

Probability that one person show interest and did not attend the party

$$= \frac{4!}{3!0!1!}\left(\frac{2}{5}\right)^3\left(\frac{1}{4}\right)^0\left(\frac{7}{20}\right)^1 + \frac{4!}{2!1!1!}\left(\frac{2}{5}\right)^2\left(\frac{1}{4}\right)^1\left(\frac{7}{20}\right)^1 + \frac{4!}{1!2!1!}\left(\frac{2}{5}\right)^1\left(\frac{1}{4}\right)^2\left(\frac{7}{20}\right)^1 + \frac{4!}{0!3!1!}\left(\frac{2}{5}\right)^0\left(\frac{1}{4}\right)^3\left(\frac{7}{20}\right)^1 \quad (239)$$

Probability that one person show interest and did not attend the party

$$= 0.0896 + 0.168 + 0.105 + 0.021875 = 0.384475 \quad (240)$$

(4) In a community there are three set of social classes which are the poor, middle and rich classes. The probability a person pick at random from the community is a middle class is 0.2 and that of the poor class is 0.7. If three sample of people are picked from the community at random, find the probability that (i) the three sample of people picked are poor class (ii) two people are poor and one person is rich in the three sample of people picked (iii) none of the three sample of people are poor class

Solution

Let the probability of rich class people

$$= P_r\ (rich) = p \quad (241)$$

Let the probability of middle class people

$$= P_r\ (middle) = r \quad (242)$$

Let the probability of poor class people

$$= P_r\ (poor) = q \quad (243)$$

$r = 0.2$ and $q = 0.7$ (244)

From $p + r + q = 1$ in (12), we have (245)

$$p + 0.2 + 0.7 = 1 \quad (246)$$

$$p = 0.1 \quad (247)$$

For $n = 3$ number of people selected at random, the all possible power combinations are presented in the Kifilideen matrix of positive power of 3 in Figure 8.

PRQ	PRQ	PRQ	PRQ
300			
210	201		
120	111		
030	021	102	
		012	
			003

Figure 8: Kifilideen matrix of positive power of 4 for $n = 4$ number of car selected at random showing all the possible power combinations.

(i) For the probability that the three sample of people picked are poor class $(n(Q) = 3)$ the possible power combination from the Kifilideen matrix in Figure 8 is **003.**

The probability that the three sample of people picked are poor class

$$= P_r(0\ rich, 0\ middle \text{ and } 3 \text{ poor}) = {}_{003}^{\ 3}Cp^0r^0q^3 = \frac{3!}{0!0!3!}(0.1)^0(0.2)^0(0.7)^3 = 0.3430 \quad (248)$$

(ii) For the probability that the two people are poor and one person is rich in the three sample of people picked $(n(Q) = 2,\ n(P) = 1$ and $n(R) = 0$ together) the possible power combination from the Kifilideen matrix in Figure 7 is **102**.

The probability that two people are poor and one person is rich in the three samples of people picked

$$= P_r(1\ rich, 0\ middle \text{ and } 2 \text{ poor}) = {}_{102}^{\ 3}Cp^1r^0q^2 = \frac{3!}{1!0!2!}(0.1)^1(0.2)^0(0.7)^2 = 0.147 \quad (249)$$

(iii)For the probability that none of the three samples of people are poor class$(n(Q) = 0)$ the possible power combinations from the Kifilideen matrix in Figure 7 are **300**, **210**, **120** and **030**.

The probability that none of the three samples of people are poor class $= P_r(3\, rich, 0\, middle$ and 0 poor) $+ P_r(2\, rich, 1\, middle$ and 0 poor) $+ P_r(1\, rich, 2\, middle$ and 0 poor) $+P_r(0\, rich, 3\, middle$ and 0 poor) (250)

The probability that none of the three samples of people are poor class

$= {}_{300}^{\ 3}Cp^3r^0q^0 + {}_{210}^{\ 3}Cp^2r^1q^0 + {}_{120}^{\ 3}Cp^1r^2q^0 + {}_{030}^{\ 3}Cp^0r^3q^0$ (251)

The probability that none of the three samples of people are poor class =

$\frac{3!}{3!0!0!}(0.1)^3(0.2)^0(0.7)^1 + \frac{3!}{2!1!0!}(0.1)^2(0.2)^1(0.7)^0 + \frac{3!}{1!2!0!}(0.1)^1(0.2)^2(0.7)^0 + \frac{3!}{0!3!0!}(0.1)^0(0.2)^3(0.7)^0$ (252)

Probability that one person show interest and did not attend the party$= 0.0007 + 0.0060 + 0.0120 + 0.0080 = 0.0267$ (253)

(5) The probability that a club A would win, draw and loss a match with club B are $\frac{1}{9}, \frac{5}{36}$ *and* $\frac{3}{4}$ respectively.If two matches were played with club B, find (i) the probability that club A losses the two matches (ii) the probability that club A does not the loss two matches (iii) the probability that club A loss one and draw one.

Solution

(i) Let the probability of club A win

$= P_r\,(win) = p$ (254)

Let the probability of club A draw$=$

$P_r\,(draw) = r$ (255)

Let the probability of club A loss

$= P_r\,(loss) = q$ (256)

$p = \frac{1}{9}, r = \frac{5}{36}$and $q = \frac{3}{4}$ (257)

For $n = 2$ number of matches played, the all possible power combinations are presented in the Kifilideen matrix of positive power of 2 in Figure 9. For the probability that club A losses the two matches $(n(Q) = 2)$ the possible power combinations from the Kifilideen matrix in Figure 9 are **002**.

PRQ	*PRQ*	*PRQ*
200		
110		
020	**101**	
	011	
		002

Figure 9: Kifilideen matrix of positive power of 2 for $n = 2$ number of matches showing all the possible power combinations.

The probability that club A losses the two matches$= P_r(0\, win, 0\, draw\ and\ 0\, loss)$(258)

The probability that club A losses the two matches$= {}_{002}^{\ 2}Cp^0r^0q^2$ (259)

The probability that club A losses the two matches $= \frac{2!}{0!0!2!}\left(\frac{1}{9}\right)^0\left(\frac{5}{36}\right)^0\left(\frac{3}{4}\right)^2$ (260)

The probability that club A losses the two matches $= \frac{2!}{0!0!2!}\left(\frac{1}{9}\right)^0\left(\frac{5}{36}\right)^0\left(\frac{3}{4}\right)^2$ (261)

The probability that club A losses the two matches $= 0.5625$ (262)

(ii) The probability that club A does not loss the two matches $= 1 -$ the probability that club A losses the two matches

$= 1 - 0.5625 = 0.4375$ (263)

Or

For the probability that club A losses the two matches $(n(Q) = 2)$ the possible power combination from the Kifilideen matrix in Figure 4 is **002** while for the probability that club A did not loss two matches $(n(Q) = 0, n(Q) = 1)$ the possible power combinations from the Kifilideen matrix in Figure 4 are **200**, **110**, **020**, **101** and **011**.

The probability that club A does not loss the two matches $= {}_{200}^{\ 2}Cp^2r^0q^0 + {}_{110}^{\ 2}Cp^1r^1q^0 +$

${}_{020}^{\ 2}Cp^0r^2q^0 + {}_{101}^{\ 2}Cp^1r^0q^1 +$
${}_{011}^{\ 2}Cp^0r^1q^1$ (264)

The probability that club A does not loss the two matches $= \frac{2!}{2!0!0!}\left(\frac{1}{9}\right)^2\left(\frac{5}{36}\right)^0\left(\frac{3}{4}\right)^0 +$
$\frac{2!}{1!1!0!}\left(\frac{1}{9}\right)^1\left(\frac{5}{36}\right)^1\left(\frac{3}{4}\right)^0 + \frac{2!}{0!2!0!}\left(\frac{1}{9}\right)^0\left(\frac{5}{36}\right)^2\left(\frac{3}{4}\right)^0 +$
$\frac{2!}{1!0!1!}\left(\frac{1}{9}\right)^1\left(\frac{5}{36}\right)^0\left(\frac{3}{4}\right)^1 + \frac{2!}{0!1!1!}\left(\frac{1}{9}\right)^0\left(\frac{5}{36}\right)^1\left(\frac{3}{4}\right)^1$
(265)

The probability that club A does not loss the two matches $= 0.0123457 + 0.0308642 + 0.0192901 + 0.1666667 + 0.2083333$
(266)

The probability that club A does not loss the two matches $= 0.4375$
(267)

(iii) For the probability that club A losses one and draw one $(n(R) = 1, n(Q) = 1\ and\ n(P) = 0\ altogather)$ the possible power combinations from the Kifilideen matrix in Figure 4 is 011.

The probability that club A loss one and draw one= $P_r(0\ win, 1\ draw\ and\ 1\ loss)$ (268)

The probability that club A loss one and draw one $= {}_{011}^{\ 2}Cp^0r^1q^1$
(269)

The probability that club A loss one and draw one $= \frac{2!}{0!1!1!}\left(\frac{1}{9}\right)^0\left(\frac{5}{36}\right)^1\left(\frac{3}{4}\right)^1$ (270)

The probability that club A loss one and draw one $= 0.20833$ (271)

(6) The probability that a farmer has adequate collateral to obtain loan from agricultural bank is $\frac{2}{3}$ and the probability that a farmer lack collateral to obtain loan from agricultural bank is $\frac{7}{30}$. If three farmers are selected at random, determine the probability that two farmers have adequate collateral to obtain loan from the agricultural bank.

Solution

Let the probability of adequate collateral=
$P_r\ (adequate\ collateral) = p$ (272)

Let the probability of inadequate=
$P_r\ (inadequate) = r$ (273)

Let the probability of lack
$= P_r\ (lack) = q$ (274)

$p = \frac{2}{3}$and $q = \frac{7}{30}$ (275)

From $p + r + q = 1$ in (12), we have (276)

$\frac{2}{3} + r + \frac{7}{30} = 1$ (277)

$r = \frac{1}{10}$ (278)

For $n = 3$ number of farmers selected at random, the all possible power combinations are presented in the Kifilideen matrix of positive power of 3 in Figure 10. For the probability that two farmers have adequate collateral to obtain loan from the agricultural bank $(n(P) = 2)$ the possible power combinations from the Kifilideen matrix in Figure 10 is 210 and 201.

The probability that two farmers have adequate collateral to obtain loan from the agricultural bank
$= P_r(2\text{ adequate}, 1\text{ inadequate } and\ 0\text{ lack}) +$
$P_r(2\text{ adequate}, 0\text{ inadequate } and\ 1\text{ lack})$ (279)

The probability that two farmers have adequate collateral to obtain loan from the agricultural bank $= {}_{210}^{\ 3}Cp^2r^1q^0 + {}_{201}^{\ 3}Cp^2r^0q^1$ (280)

PRQ	PRQ	PRQ	PRQ
300			
210	**201**		
120	111		
030	021	102	
		012	
			003

Figure 10: Kifilideen matrix of positive power of 3 for $n = 3$ number of farmers selected at random showing all the possible power combinations

The probability that two farmers have adequate collateral to obtain loan from the agricultural bank $= \frac{3!}{2!1!0!}\left(\frac{2}{3}\right)^2\left(\frac{1}{10}\right)^1\left(\frac{7}{30}\right)^0 + \frac{3!}{2!0!1!}\left(\frac{2}{3}\right)^2\left(\frac{1}{10}\right)^0\left(\frac{7}{30}\right)^1$ (281)

The probability that two farmers have adequate collateral to obtain loan from the agricultural bank$= 0.1333333 + \ 0.3111111 = 0.4444444$ (282)

Utilization of the Kifilideen formulas developed for the Negative Power of $-n$ of Kifilideen Trinomial Theorem

(1) For a trinomial expression of $[x + y + z)^{-3}$and the 27th term of the Kifilideen trinomial expansion of the trinomial expression, determine the following:

(i) the power combination of the 27th term

(ii) the group the 27th term belong to

(iii) the row and column in which the 27th term belong

(iv) the position of the 27th term in the group it belong to

Solution

(i) Using the Kifilideen Power Combination formula for negative powers of n,

$C_P = 9t - 110a - 9m + n00$ (283)

Determining the Kifilideen migration column factor, a for 27th term, we have:

$a = \frac{-1+\sqrt{8t-7}}{2}$ (284)

$a = \frac{-1+\sqrt{8\times27-7}}{2}$ (285)

$a = 6.73$ (286)

The migration column factor, $a = 6$ (287)

The Kifilideen migration row factor, m for 27th term, we have:

$m = \frac{a^2+a+2}{2}$ (288)

$m = \frac{6^2+6+2}{2}$ (289)

$m = 22$ (290)

So, $C_P = 9t - 110a - 9m + n00$ (291)

$n =$ negative power of the trinomial expression $= -3$

$C_P = 9 \times 27 - 110 \times 6 - 9 \times 22 - 300$ (292)

$C_P = 9 \times 27 - 110 \times 6 - 9 \times 22 - 300$ (293)

$C_P = -915$ (294)

(ii) Using the Kifilideen general group formula for negative power of n, we have:

$g = \frac{1+\sqrt{8t-7}}{2}$ (295)

$g = \frac{1+\sqrt{8\times27-7}}{2}$ (296)

$g = 7.73$ (297)

Therefore 27th term belongs to group 7 in the Kifilideen matrix of the negative power of $-\ 3$

(iii) From $C_P = kif = -915,$ (298)

$k = -9, i = 1$ and $f = 5$ (299)

From $n - c + 1 = k$ (300)

$-3 - c + 1 = -9$ (301)

$c \ = 7$ (302)

Or

From $c = a + 1 = g$ (303)

$c = 6 + 1 = 7$ (304)

$c \ = 7$ (305)

From $r = 2f + i + 1$ (306)

$r = 12$ (307)

Therefore, the 27th term is found in column 7 and row 12 of the Kifilideen matrix of the negative power of $-\ 3$.

(iv) From (203), the position of the 27th term in the group it belongs to is determined as:

$p = r - c + 1$ (308)

$p = 12 - 7 + 1$ (309)

$p = 6$ (310)

Or

Using Kifilideen general position Formula for negative power of n,

$C_P = \ kif \ = n00 + -110(a) + 9(p - 1)$ (311)

The migration column factor, $a = 6,$ Power combination = $C_P = -915$and value of the negative power of the trinomial theorem = -3

$C_P = \ \ kif \ = -300 + -110(6) + 9(p - 1) = -915$ (312)

$p = 6$ (313)

Therefore, the 27th term is in position 6 in the group 7 of the Kifilideen matrix of the negative power of $-\ 3$.

(2) The power combination of the t^{th} termof Kifilideen trinomial expansion ofthe trinomial expression $\left[\frac{x^{-9}y^4}{4z^2} + 1 - yz^2\right]^{-n}$ is -732. Determine the following:

(i) the value of the power of the trinomial expression

(ii) thet^{th}term of the power combination

Solution

(i) For $C_p = kif = -732$ (314)

From, $n = k + i + f$ (315)

$k = -7, i = 3$ and $f = 2$ (316)

We have: $n = -7 + 3 + 2$ (317)

$n = -2$ (318)

(ii) Using the Kifilideen Power Combination formula for negative power of n,

$C_P = 9(t-1) - \frac{9}{2}(n-k)^2 - \frac{229}{2}(n-k) + n00$ (319)

$-732 = 9(t-1) - \frac{9}{2}\left(-2-(-7)\right)^2 - \frac{229}{2}\left(-2-(-7)\right) - 200$ (320)

$-732 = 9(t-1) - \frac{9}{2}\left(-2-(-7)\right)^2 - \frac{229}{2}\left(-2-(-7)\right) - 200$ (321)

$-1464 = 18(t-1) - 9 \times 5^2 - 229 \times 5 - 400$ (322)

$18(t-1) = 306$ (323)

$t = 18^{th}$ term (324)

(3) The power combination of the t^{th} term of Kifilideen trinomial expansion of the trinomial expression $(s + r + v)^{-6}$ is given as -811. Obtain the t^{th}term of the power combination

Solution

The Kifilideen migration column factor, a is obtained as:

$a = n - k$ (325)

$n =$ value of the negative power of the trinomial expression $= -6$

From $C_P = kif = -811$ (326)

$k = -8$ (327)

So, $a = n - k$ (328)

$a = -6 - (-8)$ (329)

$a = 2$ (330)

The Kifilideen migration row factor, m is obtained as

$m = \frac{a^2+a+2}{2}$ (331)

$m = \frac{2^2+2+2}{2}$ (332)

$m = 4$ (333)

Using the Kifilideen Power Combination formula for negative power of n,

$C_P = -110a + 9(t-m) + n00$ (334)

$-811 = -110 \times 2 + 9(t-4) - 600$ (335)

$t = 5^{th}$ term (336)

(4) The power combination of the t^{th} termof Kifilideen trinomial expansion ofthe trinomial expression $\left[4 - \sqrt{\frac{6}{5x}} - y^2\right]^{-4}$ is $-20,4,5$. Determine using the Kifilideen general term formula the t^{th} term of the power combination.

Solution

From the question,

$C_P = kif = -20,7,9$ and $n = -4$ (337)

$k = -20, i = 7$ and $f = 9$ (338)

Using Kifilideen general term formula 1 for negative powers of n, we have:

$t = \frac{(n-k)^2+(n-k)+2f+2}{2}$ (339)

$t = \frac{(-4-(20))^2+(-4-(-20))+2\times9+2}{2}$ (340)

$t = 146^{th}$ term (341)

Or

Using Kifilideen general term formula 2 for negative power of n, we have:

$t = \frac{(n-k)^2+3(n-k)-2i+2}{2}$ (342)

$t = \frac{(-4-(-20))^2+3(-4-(-20))-2\times7+2}{2}$ (343)

$t = 146^{th}$ term (344)

CONCLUSION

This research work presents real life applications of mathematical model of Kifilideen trinomial theorem distribution of positive powers of n with mathematical induction of negative powers of n counterpart. A mathematical model of the Kifilideen trinomial theorem distribution was formulated. This research work also developed alternate Kifilideen power combination formula, alternate Kifilideen term formula and alternate Kifilideen position formula for the negative powers of n which conform but not the same with the one developed for the positive powers of n counterpart. The formulated mathematical model of the Kifilideen trinomial theorem distribution was utilized to analysis real life events having three possible outcomes. The model invented in this paper was used to determine the probability of the combination of different n outputs involving three possible categories of outcomes of events. The mathematical induction presents in this research work helps to support and prove that the developed Kifilideen formulas for the constituents of Kifilideen trinomial theorem of negative powers of n are valid and true.

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