A STABILIZATION PROCEDURE FOR THE TRANSFORMATION OF MAGNETIC DATA

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Abstract

Magnetic anomalies are as a result of the summation of magnetic effects produced by spatial variation of magnetic polarization. Consequently, transformations which attenuate the unwanted components of the observed anomalies have been of great interest in processing of magnetic data. Transforming data into the frequency domain has a problem caused by imperfections introduced in the data spectrum by aliasing and Gibb=s effect, which are caused by the discretization and truncation of data respectively. Stable transformation such as upward continuation, integration and reduction to the pole at high magnetic latitudes either preserves or attenuates the amplitude of these imperfection in the data spectrum. Whereas unstable transformation such as downward continuation, reduction to the pole at low latitudes and first vertical derivatives, greatly enhances the amplitude of the spectrum imperfections so that any direct application of such transformation (without stabilization) will produces poor result. Using equivalent source technique usually produce computations too large for computers to handle and the choice of a good stabilizing factor can be arbitrary. In this paper comparison is first made between the conventional filtering technique and the equivalent source technique using theoretical data and secondly a quantitative method is developed by using an algorithm which uses the correlation coefficient between successive pairs of the transformed maps.

1. Introduction

Treatment of the magnetic data is mainly concerned with three types of information. The first type is the vertical derivative map of the total field. Usually, the first and second derivatives are evaluated. Second derivative maps are found to be of much use in approximately delineating the boundary of the body causing magnetic anomalies. The second type of information is the continuation of the total field above or below the level of observation. Such continuation of the field is important in either reducing or accentuating the effects of bodies close to the earth's surface. The third information is the computation of the field with the assumption that the geological source was physically moved to the magnetic pole (reduction-to-the-pole). Transforming the magnetic fields to the earth's magnetic pole makes interpretation of magnetic anomalies easier, as the anomalies are not shifted as a result of the obliquity of the normal field.

Interpretation of magnetic anomalies is complicated by three factors. First, the magnetic field has both attractive and repulsive forces. Second, a magnetic anomaly may be generated both by induction in the direction of the earth's field and by remanent magnetism, which may be oriented differently. Park (1968) showed that remanence can be a significant or even dominant contributor to the anomalous field. The third complication is the inclination and declination of the earth's inducing field. This causes distortion of the magnetic anomaly, with the degree of distortion being a function of the magnetic latitude.

Gunn (1975) showed that magnetic data can be treated by a linear transformation applied to the original field either in the space or wavenumber domain. In this respect, the transformation is viewed as a filtering operation. However, the problem of numerical instability arising from aliasing and Gibb's effect was not addressed in Gunn's paper. Silva (1986) showed that the transformation of magnetic fields can be solved by formulating the problem as an inverse problem, in which stable solutions in the presence of noise could be found using well-known methods, such as that of Hoerl and Kennard (1970).

2. Methodology

Initially, the magnetization of an equivalent layer of doublets is computed from the observed data. All magnetic doublets are assumed parallel to the magnetization vector whose direction is supposedly constant throughout the sources. Once strengths of the doublets are found, the transformation is done using the appropriate Green's function. The first stage involves the solution of an inverse problem, while the second is a forward problem. Only the first stage presents instability so that a stabilizing procedure must be applied only there.

Let h_i be the total magnetic field observed at point $P(x_i, y_i, z_i)$. The magnetic field due to arbitrary sources can be exactly fitted by the magnetic field due to a horizontal equivalent layer of dipoles at depth $z = d$. For each depth there is a unique distribution of magnetization. The equivalent layer of dipoles can be approximated by an equivalent layer of doublets at depth d. Each doublet has length

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b and is parallel to the unit magnetization vector. The magnetization direction α is assumed constant throughout the sources.

The magnetic field at point P (x_i, y_i, z_i) due to the doublet centred at point Q (x',y', d) is:

$$h_{ii} = a_{ii} P_j \qquad (1)$$

where P_j = strength of magnetization of the j^{th} doublet.

$$a_{ij} = \left[\frac{x_i - x_j' - (b/2)\cos I \cos D}{F_1} - \frac{x_i - x_j' + (b/2)\cos I \cos D}{F_2}\right].\cos I_o \cos D_o$$

$$+\left[\frac{y_i - y_j' - (b/2)\cos I \sin D}{F_1} - \frac{y_i - y_j' + (b/2)\cos I \sin D}{F_2}\right]\cos I_o \sin D_o$$

$$+\left[\frac{z_i - d - (b/2)\sin I}{F_1} - \frac{z_i - d + (b/2)\sin I}{F_2}\right]\sin I_o$$

$$F_1 = \left[x_i - x_j' - (b/2)\cos I \cos D)^2 + \left(y_i - y_j' - (b/2)\cos I \cos D)^2\right) + z_i - d - (b/2)\sin I)^2\right]^{3/2}$$

$$F_2 = \left[\left(x_i - x_j' + (b/2)\cos I \cos D\right)^2 + \left(y_i - y_j' + (b/2)\cos I \sin D\right)^2 + \left(z_i - d + (b/2)\sin I\right)^2\right]^{3/2}$$

D and D_o=declination of polarizing field and declination of earth's normal total field, respectively.

I and I_o= inclination of polarizing field and earth's normal total field, respectively.

The total magnetic field h_i at point P(x_i, y_i, z_i) due to all doublets is the sum of all the magnetic fields at point P(x_j, y_j, z_j) due to each doublet. That is,

$$h_i = \sum_{j=i}^{M} a_{ij} P_j \qquad (2)$$

for i = 1,2,.......N

where:

N.= number of observations

M= number of doublets

The above equation can be written in matrix notation as:

$$h = Ap \qquad (3)$$

where:

h= vector containing the computed magnetic field at the observation points due to a layer of doublets.

A = matrix whose a_{ij} elements are given by equation (1)

p = vector containing doublet strengths.

The smaller the doublet length b relative to the sampling interval, the better the approximation by the equivalent sources. However, if the doublet length is too small relative to the sampling interval, the matrix A becomes ill-conditioned. That is, large changes in the solution may result from small changes in the vector of observations. This is because the coefficients a_{ij} in equation (1) tend to very small values. Silva (1986) showed that a value of b between one-half and one-fifth of the sampling interval produced good results.

From equation (3), the solution for p is given by:

$$p = A^{-1}h \qquad (4)$$

where A^{-1} is a suitable inverse. The inverse A^{-1} can be expressed as:

$$A^{-1} = D\left(DA^T AD + \lambda I\right)^{-1} DA^T \qquad (5)$$

where D is a diagonal scaling matrix given by:

$$d_{ii} = \frac{1}{\left(\sum_{j=1}^{N} a_{ji}^2\right)^{1/2}}$$

A^T is the transpose of A and λ is a real positive number usually between 0 and 1. I is an identity matrix. The criterion for selection of the best value of λ is given in Hoerl and Kennard (1970) based on the analysis of the Aridge trace@ . In this method the estimate p in equation (4) is plotted against values of λ between 0 and 1, the best value of λ occurs where all the coefficients seem to settle down.

Once the vector P of strengths is estimated, the transformed field h_T is computed by performing the following matrix operation.

$$h_T = Bp \qquad (6)$$

where B is the matrix of the Green=s function necessary for the transformation. Therefore:

$$h_T = BD\left(DA^T AD + \lambda I\right)^{-1} DA^T h \qquad (7)$$

(a) Comparison with the Filtering technique

In this section we demonstrate the advantage of the proposed technique over the conventional filtering technique. The method of Bhattacharyya (1964) was used to generate theoretical magnetic field due to a vertical prism uniformly magnetized in the direction of the geomagnetic field. In terms of grid units the prism is 2 units long (x-direction, assumed parallel to the north-south direction) 2 units wide (y-direction, assumed parallel to the east-west direction) and 4 units thick (z- direction). Its top is located 3 units below the plane of measurements and the magnetic susceptibility is 0.001 SI units. The field is computed at discrete points on the plane z = 0 using a 10 x 10 square grid with the magnetization vector and the geomagnetic field having inclinations of 90 degrees and azimuth of 25 degrees with respect to the x-axis. The magnetic field produced by this prism is shown in Figure 1 and it is equivalent to that produced at the magnetic pole. The inscribed solid line represent the boundary of the upper face of the prism and it can be observed that the anomaly is centred directly on the prism. Figure 2 shows the results obtained by applying the both the proposed technique and the filtering technique when the source is situated at different magnetic latitudes. In Figure 2, A1, A2 and A3 shows the magnetic field of the prism at magnetic latitudes 0, 10 and 60 degrees respectively, while B1, B2, B3 and C1, C2, C3 shows the corresponding results of applying filtering and the proposed techniques respectively. It can be seen in Figure 2 that a high magnetic latitude (A3 and C3) both techniques gave satisfactory results, but as the magnetic inclination is reduced the superiority of the inverse method over the filtering method is clearly seen , especially at zero degrees where the filtering technique gave a completely meaningless result. Also there is almost a perfect resemblance of C1, C2 and C3 with the magnetic field at the pole (Figure 1), which is an indication of good performance of the technique. It is important to note that the data in Figure 2 is assumed to be noise-free since the prism/source is isolated and field is generated theoretically. It is therefore expected that a value of zero be used for λ, but instead a very small value (10^{-5}) was added due to computational round-off errors.

When processing data for large areas (which is usually the case in practice), the number of entries in the matrix to be inverted increases with the square of the total number of observations. A 30 x 30 area (in grid units), for example requires the inversion of a 900 x 900 matrix. Attempts to divide a large area into smaller subareas and process them individually incur undesirable edge effects, despite the adoption of some strategies such as overlapping adjacent subareas (von Frese *et al.,* 1988). However Leao and Silva (1989) presented a method which can be used to process areas with large amount of data. An N x N data window is inverted using an M x M equivalent layer, with M greater than N (an under-determined least squares problem) so that the equivalent layer extends beyond the data window. Only the transformed field at the centre of the data window is computed by pre-multiplying the equivalent source matrix by the row of the matrix B in equation (7) corresponding to the data window. As a result, a grid operator for the desired transformation is obtained which is applied to the data by a procedure similar to discrete convolution. In this method the damping factor λ is selected by producing transformed fields for increasing values of λ and choosing the one for which there is no qualitative change in the transformed field with respect to that for the previous value.

Computation of the damping factor as proposed by Leao and Silva (1989) can be time consuming, expensive and also subjective since it involves a visual inspection of the transformed fields. In this work an algorithm which utilizes the correlation coefficients between successive transformed fields is developed thereby making the choice of λ more objective and easier to compute.

(b) The algorithm

The best choice of the damping parameter λ can be obtained by computing the correlation coefficients between successive transformed fields, using the following algorithm:

(i) Select an interval for λ. For normalized matrices, it should be between 10^{-5} (slightly above the machine precision) and 1.

(ii) Initialize with $\lambda = 10^{-5}$

(iii) Compute the reduced-to-the-pole field with the selected value of λ

(iv) If this is the first value of λ go to step (f); otherwise, go to step (e)

(v) Obtain the cross correlation between the current reduced-to-the-pole field and the one obtained with the previous value of λ

(vi) Multiply λ by a factor 5 and go to step (c)

The optimum value of λ will be the smallest value producing a cross correlation which does not deviate much from the cross correlation between the previous pair of transformed fields.

(c) Application to multiple sources

Equation (7) was tested on theoretical total field anomalies (Figure 3) produced by vertical prisms using the method of Bhatacharyya (1964). whose plan views are also displayed. The magnetic field

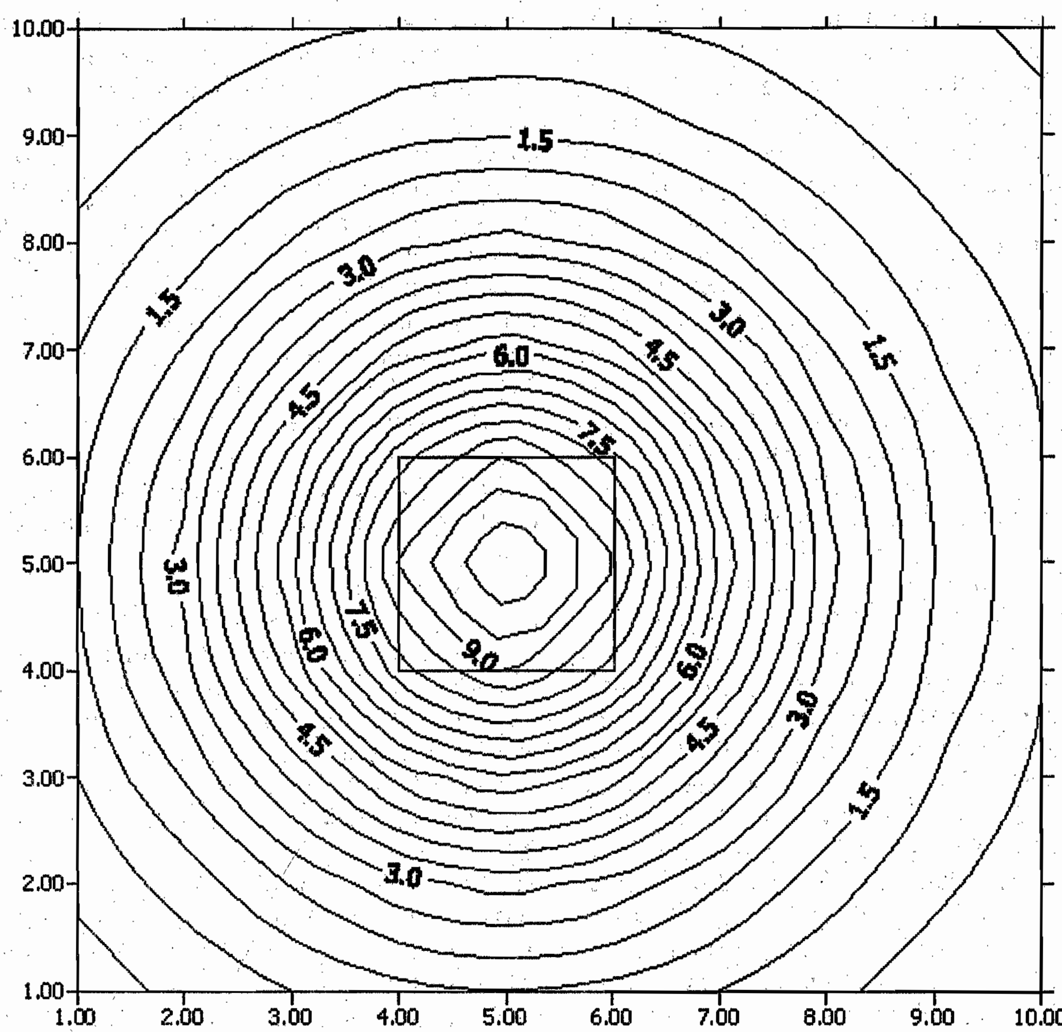

Figure 1: Map showing the field generated by a prism situated at the earth=s magnetic pole with $I_0 = 90°$. Contour interval is 0.5 nT.

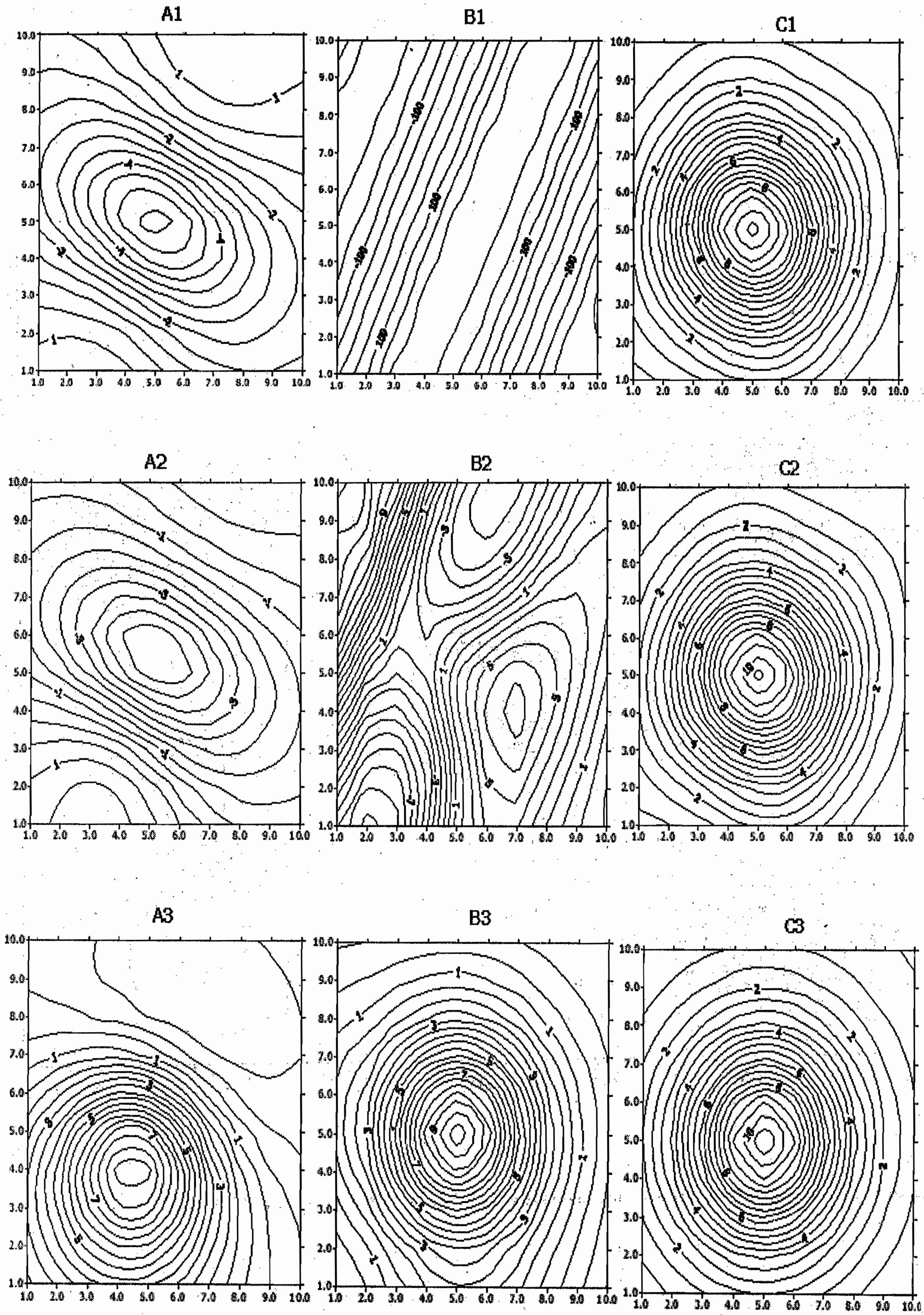

Figure 2: Maps showing results obtained by applying the filtering technique and the proposed technique. A1, A2 and A3 shows the magnetic field of the prism at magnetic latitudes 0?,10?and 60?degrees respectively, while B1, B2, B3 and C1, C2, C3 shows the corresponding results of applying filtering and the proposed techniques respectively.

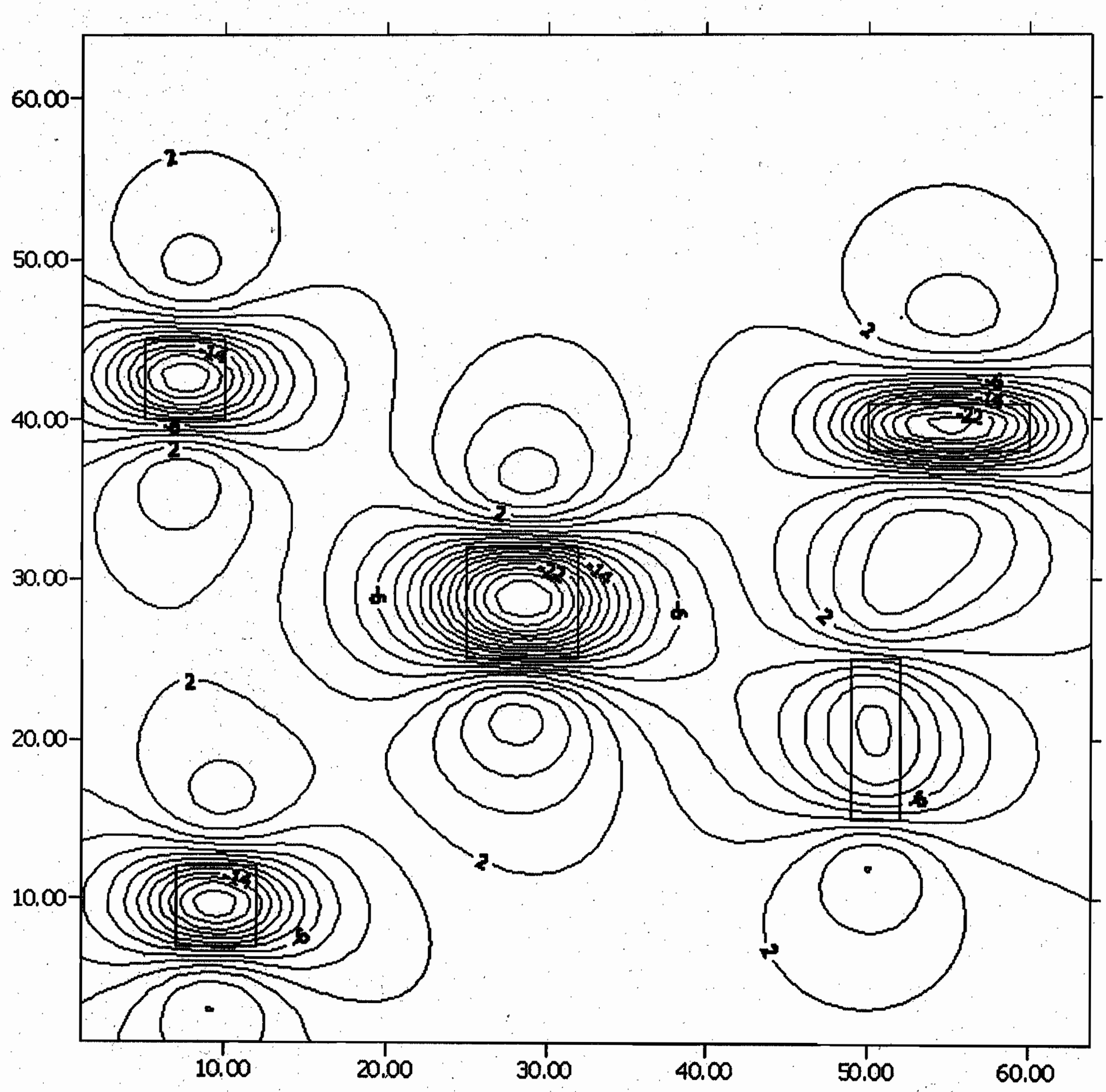

Figure 3: Total magnetic field for prisms situated at magnetic inclination of 5°. Contour interval is 2 nT

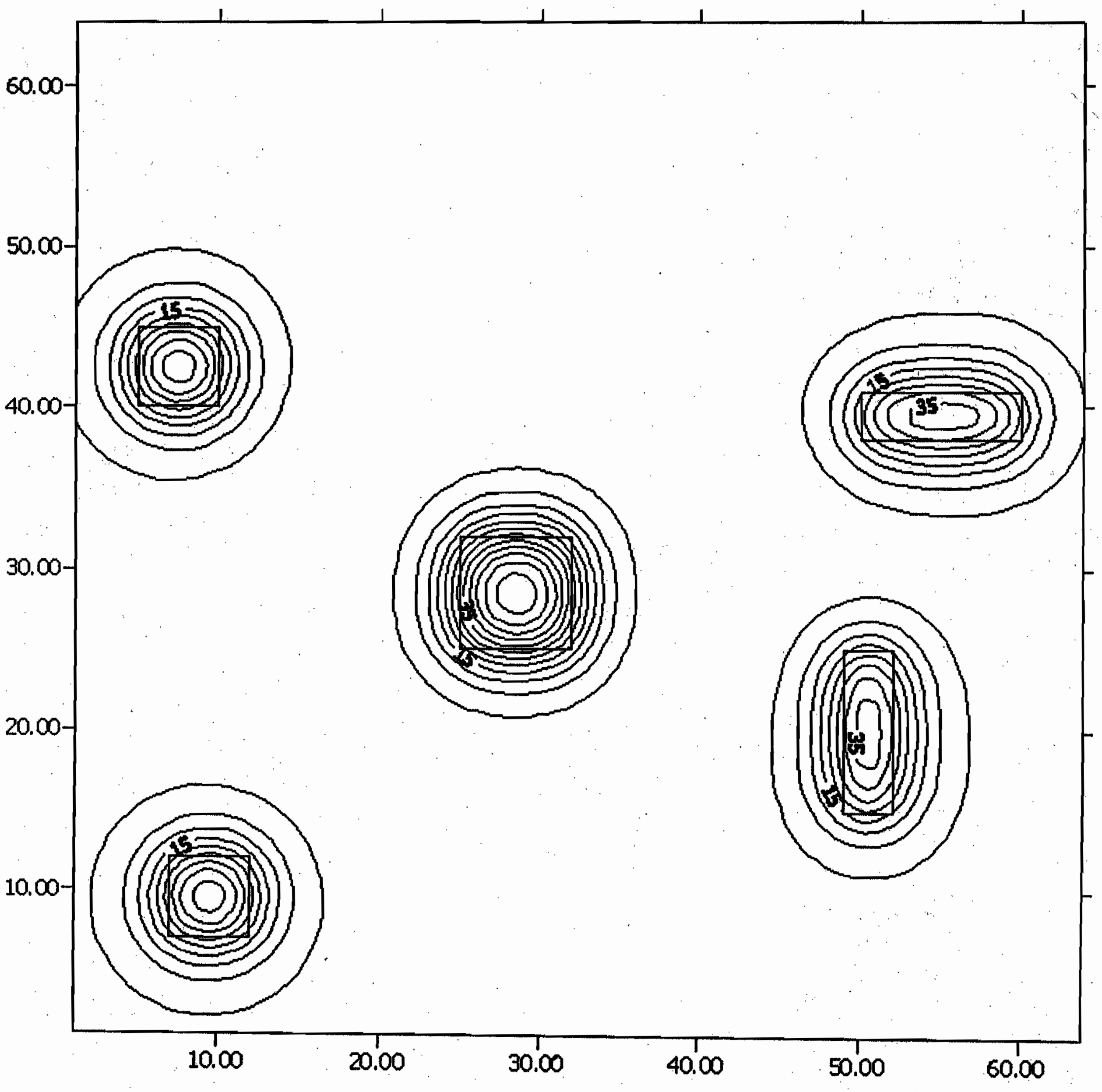

Figure 4: Total magnetic field for prisms situated at the pole (90°). Contour interval is 5 nT

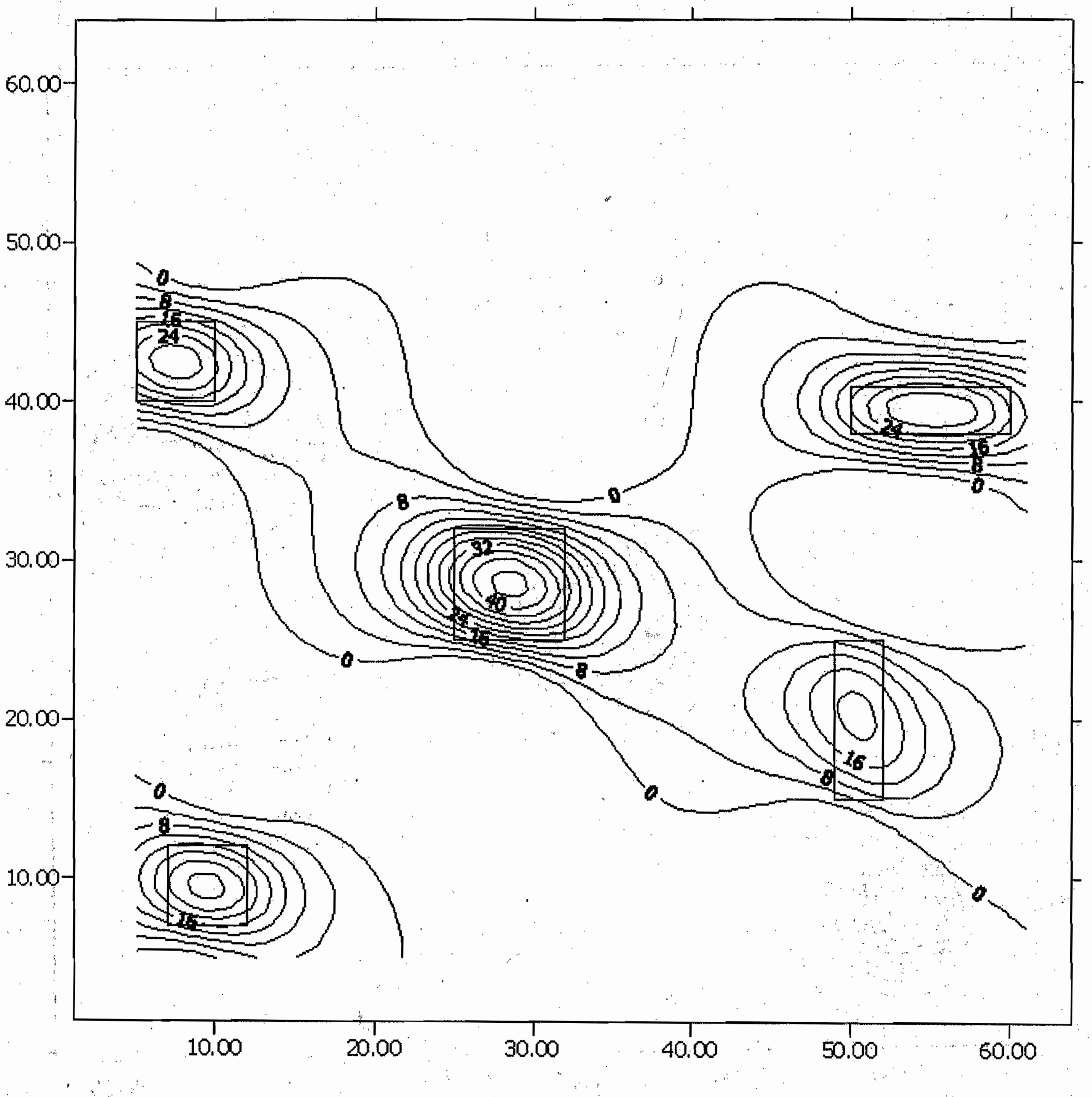

Figure 5: Total magnetic field for prisms reduced to the pole. Contour interval is 4 nT

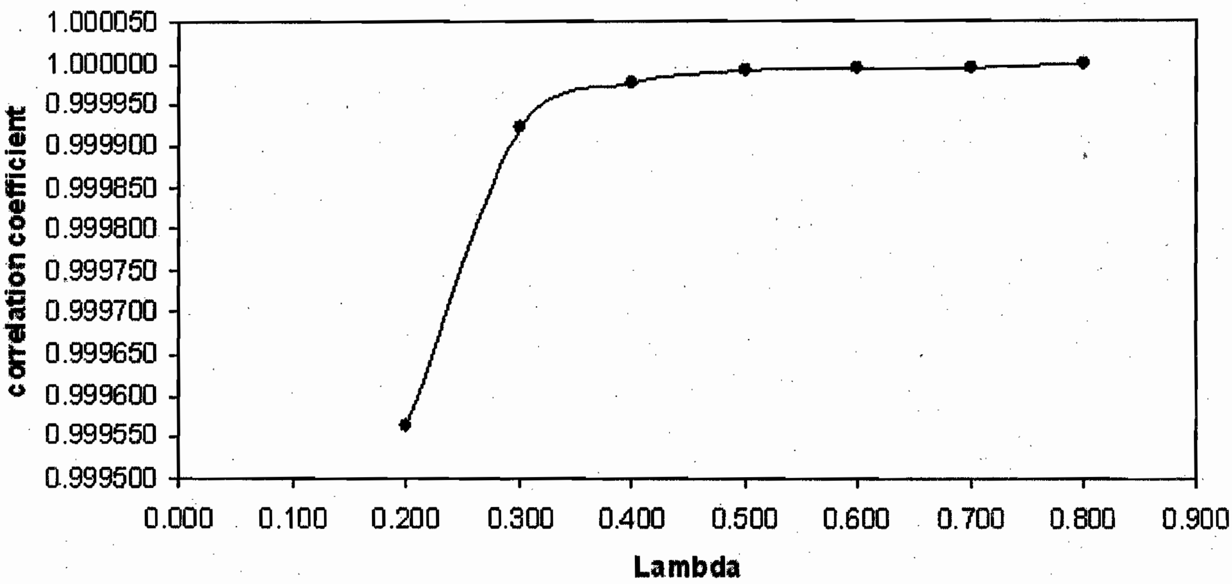

Figure 6: Graph showing the variation of correlation coefficient with lambda (λ). Optimum value of lambda chosen is 0.5.

was computed at points on a 64 by 64 grid. All prisms have a magnetization intensities of 3.3×10^{-8}A/m, thickness of 5.0 km, depth to top of 3.0 km, and the geomagnetic field has an inclination of 5 degrees and declination of 12 degrees. Figure 4 shows the theoretical anomaly at the earth=s magnetic pole. Using an equivalent layer of 225 (15 by 15) point sources having unit grid spacing and grid points located directly below the observations with data window of 49 (7 by 7) points, the anomalies were then reduced-to-the-pole (using the above algorithm) and the result is shown in Figure 5. Figure 6 is a graph showing the variation of the correlation coefficient between successive fields with λ, and an optimum value of 0.5 was chosen for the value of λ. Comparing Figure 5 with the theoretical anomaly at the pole (Figure 4) we see that the apex of the anomalies are shifted to the centers of the bodies, thereby indicating good performance of the method.

3. Conclusion

The transformation of magnetic anomalies is usually done in the frequency domain. This produces unstable results because of aliasing and Gibb=s effect arising from the discretization of the data .On the other hand, by formulating the problem as an inverse problem, the stabilization is applied to the method itself, leaving the data unaltered. The superiority of the inverse technique over the conventional filtering technique was demonstrated using theoretical data, and quick and cheap method of stabilizing the solutions has been achieved by computing the correlation coefficients of the transformed fields for various values of the damping factor.

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