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FUZZY MODELING OF A MULTI-OBJECTIVE GREEN VEHICLE ROUTING PROBLEM CONSIDERING SPEED VARIABILITY AND TIME-EMISSION OPTIMIZATION

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Résumé

Cet article traite d'un problème de tournées de véhicules verts multi-objectifs flou (GVRP) dans lequel le décideur doit faire face à une imprécision sur le temps de parcours et sur les émissions de CO_2 (ou la consommation de carburant) pour trouver un compromis. Ces objectifs dépendent de la variabilité de la vitesse de véhicules modélisée à l'aide de nombres flous trapézoïdal. Les émissions polluantes sont estimées à partir de l'approche MEET, étendue à un cadre flou afin de prendre en compte la propagation de l'imprécision liée à la vitesse dans le calcul des émissions. L'objectif principal de cet article est de proposer un modèle bi-objectif flou de tournées de véhicules verts. La méthodologie adoptée se déroule en plusieurs étapes. Tout d'abord, la formulation mathématique du modèle flou est présentée. Ensuite, une défuzzification est réalisée à l'aide de l'approche des intervalles d'approximation les plus proches des nombres flous. La contrepartie déterministe obtenue est ensuite résolue à l'aide de l'heuristique de balayage, suivie d'une optimisation multi-objectifs utilisant la métaheuristique BicriterionAnt afin d'obtenir des solutions de compromis. Enfin, une simulation numérique est réalisée, accompagnée d'une discussion des résultats et de perspectives de recherche futures.

Mots-clés: Logique floue, métaheuristique, problème multi-objectif de tournée de véhicules verts, problème multi-objectif

Abstract

This paper addresses a fuzzy multi-objective Green Vehicle Routing Problem (GVRP) in which the decision-maker must deal with imprecision in travel time and vehicle speed in order to find a compromise between CO_2 emissions (or fuel consumption) and vehicle travel time, both of which strongly depend on the variability of vehicle speed. To capture this uncertainty, vehicle speeds are modeled using trapezoidal fuzzy numbers. Pollutant emissions are estimated using the MEET approach, which is extended to a fuzzy framework to account for the propagation of speed-related uncertainty in the emission calculation. The main objective of this paper is to propose a bi-objective fuzzy green vehicle routing model. The proposed methodology is organized as follows. First, the mathematical formulation of the fuzzy model is presented. Second, a defuzzification process is performed using the nearest approximation interval approach for fuzzy numbers. Third, the resulting deterministic counterpart is solved using the sweep heuristic, followed by a multi-objective optimization using the BicriterionAnt metaheuristic to obtain compromise solutions. A numerical simulation is conducted to evaluate the proposed approach, followed by a discussion of the results, and the paper concludes with concluding remarks and future research perspectives.

Keywords: Fuzzy logic, metaheuristic, Green vehicle routing problem, multi-objective problem

1. Introduction

Urban logistics is currently facing major challenges driven by the growing demand for transportation, increasing road congestion, and rising environmental concerns. The road transport sector accounts for a significant share of greenhouse gas (GHG) emissions and air pollutants, particularly in urban areas where distribution activities are intensive and subject to strong operational constraints. The greenhouse gas (GHG) emissions are proportional to fuel consumption, which in turn depends on several factors such as vehicle speed, acceleration, travel distance, and vehicle load [1]. From an environmental

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perspective, speed variability is directly responsible for variations in fuel consumption and pollutant emissions (CO_2, NOx, etc.) [2]. In this context, this paper focuses on the Green Vehicle Routing Problem (GVRP), where speed variability is modeled using fuzzy numbers derived from fuzzy set theory, originally introduced by Zadeh (1965) [3]. Fuzzy set theory provides an appropriate and flexible framework for modeling imprecision and ambiguity associated with uncertain information.

This problem aligns with the global shift toward ecological transportation, where the search for sustainable routing solutions has become a strategic priority, as reported by the International Energy Agency (IEA) [4]. Carbon dioxide (CO_2) constitutes a significant portion of GHGs, and recent statistics indicate that the transport sector experienced the most significant increase in CO_2 emissions, with a global surge of nearly 240 megatons in 2022 compared to 2021, and an annual average growth rate of 1.7% from 1990 to 2022, outpacing all other sectors [5]. This sector is also a major source of local air pollutants, emitting 20% of carbon monoxide (CO), 39% of nitrogen oxides (NOx), 26% of black carbon (BC), and 10% of particulate matter (PM) [6]. Given the significant contribution and rising trends in the transport sector's emissions, the urgency to decarbonize the sector and mitigate the resulting ambient air pollution has never been more critical.

Therefore, exploring innovative and effective strategies to reduce its environmental footprint is imperative to inform and guide policymakers. Countries are adopting various policies and strategies to address this challenge, some of these measures include limiting vehicle numbers and travel distances by promoting public implementing traffic control measures like speed management, traffic light coordination, and signalization; enhancing the energy efficiency of vehicles through the adoption of electric or hybrid models. Speed management policies, aimed at altering mobility patterns or driver behaviors, are acknowledged as effective for carbon abatement [7]. The relationship between speed, emissions, and fuel savings has been established for decades [8][9], this relationship is often described as U-shaped, signifying that there is an optimal speed range where deviations above or below this range lead to increased emissions and fuel consumption [10].

In this context, optimizing vehicle routing problems while integrating environmental criteria has become a central research topic in operations research. The Green Vehicle Routing Problem (GVRP) has emerged as a natural extension of the classical Vehicle Routing Problem (VRP) [11], aiming to reduce the environmental impact of transportation while maintaining acceptable economic performance. Since the pioneering work of Erdoğan and Miller-Hooks [12], numerous GVRP variants have been proposed, incorporating considerations such as fuel consumption [13], pollutant emissions [14], alternative fuel vehicles, and energy constraints. Recent literature reviews, notably those by Demir et al. [15] and Marrekchi et al. [16], indicate that most existing approaches still rely on fixed deterministic parameters, despite the inherently uncertain nature of real-world road transportation systems. Among the factors directly influencing fuel consumption and pollutant emissions, vehicle speed plays a decisive role. Several studies have highlighted the nonlinear relationship between speed, energy consumption, and emissions, particularly in widely used estimation models such as MEET [17], COPERT [16], and CMEM [18].

Moreover, travel time, which directly depends on the traveled distance and vehicle speed, constitutes a key parameter in route planning [19]. Liu, C.S. et al. [20] focus on a realistic variant of the Vehicle Routing Problem with Time Windows (VRPTW) in an urban context. Their problem is time-dependent, meaning that vehicle speeds vary according to different periods of the day (e.g., congestion hours). The main objective is to reduce delays and avoid congestion while respecting delivery time windows. Similarly, Ye Chong et al. [13], in their study Optimization of Vehicle Paths Considering Carbon Emissions in a Time-Varying Road Network (TDGVRP), model vehicle speed variation as a continuous function in order to better reflect real-world traffic conditions and promote carbon emission reduction. They proposed a hybrid Genetic Algorithm–Simulated Annealing (GA–SA) approach to solve the problem. However, in most of these works, speed is still treated as a fixed or average value, which limits the ability of models to accurately reflect real traffic conditions characterized by congestion, frequent stops, and temporal variability Demir et al [15] and Marrekchi et al. [16]. In contrast to probabilistic approaches, fuzzy set theory, introduced by Zadeh [3], provides a suitable framework for modeling imprecision and ambiguity when probability distributions are difficult to estimate. In the context of VRP, fuzziness has been used to represent parameters such as customer demand and service times [21], [22]. Nevertheless, the joint integration of fuzzy speed, fuzzy travel time, and fuzzy emissions remains largely underexplored in the context of multi-objective GVRP.

The integration of environmental considerations and uncertainty into vehicle routing problems has attracted increasing attention in recent years. In particular, fuzzy approaches applied to the Green Vehicle Routing Problem (GVRP) have been recognized as a relevant framework for modeling the inherent imprecision of real-world transportation systems, especially with regard to customer demand, travel times, and environmental impacts. Lin et al. [23] provided a comprehensive state-of-the-art review of the GVRP, including issues related to energy consumption and CO_2 emissions, emphasizing the importance of multi-objective methodologies and uncertainty modeling. Among the foundational contributions in this field, the study of Majidi et al. [24] represents a major reference on the fuzzy GVRP. The authors proposed a model that simultaneously integrates pickup and delivery operations, time windows, and fuzzy parameters such as demand, costs, and emissions. An Adaptive Large Neighborhood Search (ALNS) heuristic was employed to efficiently solve the problem. This work paved the way for the systematic use of fuzzy logic in green routing models, although it primarily relies on a single-objective formulation.

From a broader supply chain perspective, Giallanza and Li Puma [25] extended the fuzzy GVRP to a multi-echelon framework by considering the design of a three-level supply chain network. Their model incorporates fuzzy demand and environmental objectives, highlighting the relevance of fuzzy modeling to capture uncertainty at different decision-making levels. However, despite broadening the scope of the fuzzy GVRP, transportation-related parameters such as vehicle speed and travel time are still treated in a relatively simplified manner, without explicit modeling of their uncertainty. More recently,

Gupta (2022) [22] proposed a capacitated multi-objective Green Vehicle Routing Problem in which travel times and customer demands are modeled as fuzzy variables. The study is distinguished by the explicit integration of environmental objectives within a multi-objective framework, as well as by the consideration of uncertainty in travel times.

However, fuzzy travel times are treated as exogenous parameters, without an explicit functional relationship with vehicle speed. This assumption limits the analysis of uncertainty propagation in the emission estimation process, particularly given the strong dependence of fuel consumption and pollutant emissions on speed variability. P. Victor Paul et al. (2025) [19] proposed a GVRP variant referred to as the Speed Limit Constraint-based Green Vehicle Routing Problem (SL-GVRP), in which they explicitly addressed the relationship between time, speed, and distance within a purely deterministic and single-objective framework. Building upon the work of Gupta (2022) [22], Priyanka and Kakkar (2026) [26] proposed a metaheuristic approach for a Green Vehicle Routing Problem with Time Windows (GVRPTW) under fuzzy customer demand. Their model emphasizes the resolution of large-scale instances in the presence of uncertainty and environmental objectives. The results indicate that incorporating fuzziness leads to more robust solutions with respect to parameter variability. Nevertheless, as in most existing studies, uncertainty is primarily restricted to customer demand, while vehicle speed and travel time are treated in a deterministic or simplified manner. This modeling choice limits the capacity of the framework to fully capture the dynamic and uncertain nature of real-world transportation systems.

At a broader level, the recent review by Mohammed et al. [27] provides a comprehensive synthesis of fuzzy vehicle routing problems, covering a wide range of variants, including those incorporating environmental objectives. The review highlights a significant increase in studies combining fuzzy logic and routing optimization, while emphasizing that green and multi-objective extensions remain relatively limited. Some notable contributions, however, deserve particular attention. For instance, Wang et al. [28] investigated a multi-objective framework combining two sources of uncertainty fuzzy travel time and fuzzy fuel consumption (energy cost), with the aim of simultaneously minimizing logistics cost and environmental impact. Their approach transforms the fuzzy constraints using credibility measures and solves the resulting model through an improved hybrid algorithm. Nevertheless, the authors also point out a lack of research addressing the propagation of uncertainty between interdependent parameters, such as vehicle speed and travel time. This gap underscores the need for integrated modeling approaches capable of capturing functional dependencies among uncertain transportation variables.

The macroscopic approach known as MEET (Methodologies for Estimating Air Pollutant Emissions from Transport), reviewed by Hickman [17] and employed in early emission minimization studies such as that of Figliozzi [29], is generally based on aggregate functions that relate fuel consumption to average speed, travel distance, or vehicle type. While these models are relatively simple to implement, they often neglect the dynamic variability of real driving conditions, including acceleration patterns, frequent stops, and variations in road gradient [30]. The analysis of these studies highlights that, despite significant advances in the field of fuzzy and multi-objective GVRPs, several limitations remain. In particular, most existing models focus on uncertainties related to demand or time windows, whereas vehicle speed which is critical for calculating travel times and emissions is rarely modeled as an uncertain or imprecise variable. Moreover, few studies consider the functional dependence between speed, travel time, and emissions within a multi-objective fuzzy framework.

These observations justify the need to develop multi-objective fuzzy GVRP models that explicitly incorporate speed and travel time as interconnected fuzzy variables, along with a fuzzy estimation of emissions, in order to better capture the complexity and uncertainty of real-world transportation systems. This fuzzy approach enables the modeling of imprecision associated with actual traffic conditions. Accordingly, speed is no longer treated as a fixed parameter but as a flexible variable whose value can vary across multiple categories. This uncertainty is typically epistemic in nature, arising from incomplete information or imprecise data rather than purely random phenomena. Deterministic methods, which assume constant speeds, fail to capture this variability and may thus produce solutions with limited robustness. While stochastic models allow uncertainty to be explicitly accounted for, they require reliable probability distributions and large volumes of historical data, conditions that are rarely met in urban freight transport contexts. In contrast, the fuzzy approach provides a flexible modeling framework capable of representing speed fluctuations through fuzzy numbers without relying on restrictive probabilistic assumptions.

In this context, the present paper proposes a multi-objective fuzzy GVRP in which vehicle speed is modeled using fuzzy numbers, and travel time is defined as a fuzzy function of distance and fuzzy speed. Pollutant emissions are estimated using the MEET approach extended to a fuzzy framework, in order to capture the impact of speed-related uncertainty on emission levels. The problem aims to simultaneously optimize fuzzy economic and environmental objectives while satisfying classical operational constraints. In this study, the defuzzification process relies on the Nearest Interval of Approximation (NIA) of a fuzzy number, after which the sweep algorithm and the BicriterionAnt algorithm are applied, respectively, for customer clustering (potential routes) and constructing the problem's solutions.

The main contributions of this paper are as follows: the explicit modeling of speed and travel time as fuzzy variables in a multi-objective GVRP; the integration of the MEET approach within a fuzzy framework for emissions estimation; and a solution methodology combining fuzzy analysis and multi-objective optimization. The remainder of the paper is organized as follows: Section 2 presents the preliminary concepts that will be used throughout the paper. Section 3 describes the applied methodology. Section 4 is devoted to numerical simulations, and Section 5 is related to the discussions. Section 6 presents the conclusion.

2. Preliminaries

2.1. Fuzzy sets

Let X be a non-empty discourse universe. A fuzzy subset $\tilde{A} \in X$ is defined as follows: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$,

where $\mu_{\tilde{A}} : X \to [0,1]$ is the membership function of the fuzzy set under consideration [3]. A level set α of $\tilde{\mathrm{A}}$, denoted by $\tilde{\mathrm{A}}_{\alpha}$ is the set $\tilde{\mathrm{A}} = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}$[3]. In addition to a level subset, a fuzzy set can be characterized by its core, height, and support. Let $H(\mathrm{IR})$ the set of all normal, convex fuzzy numbers with compact support defined on IR, with membership function $\mu_{\tilde{A}} : \mathrm{IR} \to [0,1]$. If $\tilde{a} \in \mathrm{H(IR)}$, then the level set α of $\tilde{a}$ is $[a_{\alpha}^{L}, a_{\alpha}^{U}]$ with a_{α}^{L} et a_{α}^{U} are continuous real valued functions on X. The approximation interval operator of a fuzzy number given by $\mathrm{C} : \mathrm{H(IR)} \to \wp(\mathrm{IR})$, the metric $\mathrm{d} : \mathrm{H(IR)} \to [0,+\infty]$ and the Nearest Interval of Approximation (NIA) of $\tilde{a}$ in the sense of the metric d is given by algorithm 1 below. For more details, readers can consult [31, 32].

Algorithm 1: NIA

1. Read $\mu_{\tilde{a}}(x)$
2. Find $\inf\{x \in IR : \mu_{\tilde{a}}(x) \geq \alpha\} = a_{\alpha}^{L}$ and $\sup\{x \in IR : \mu_{\tilde{a}}(x) \geq \alpha\} = a_{\alpha}^{U}, \alpha \in [0,1]$
3. Calculate $\int_0^1 a^{L}(\alpha)d\alpha = N_{\alpha}^{L}$ and $\int_0^1 a^{U}(\alpha)d\alpha = N_{\alpha}^{U}$
4. Write $C_d(\tilde{a}) = [N_{\alpha}^{L}, N_{\alpha}^{U}]$

2.2. Combinatorial optimization problem

A multi-objective combinatorial optimization problem is a decision-making problem that consists of jointly optimizing a set of p often conflicting objective functions subject to a set of constraints with discrete (binary) variables, where : $F(x) = (f_1(x), f_2(x),..., f_p(x))$ is a vector of objective functions [33]. The decision vector is $x = (x_1, x_2,..., x_n)$ and we note

$$D = \{x \in \{0,1\}^n : g_i(x) \leq 0, h_j(x) = 0; i = 1,..., m; j = 1,..., k\}$$

the set of feasible solutions in the decision space. In the multi-objective framework, the decision-maker thinks rather in terms of evaluating a solution for each objective, and naturally places himself in the objective space O. The image of a solution $x \in D$ in the objective space O is the point $(y_1, y_2,..., y_n)$ with $y_i = f(x_i)$ $i = 1,..., p$; $Y = F(D)$ represents the solution in the objective space. The most widely accepted notion of optimality is the Pareto optimum [33]. From the above, this notion is extended to the fuzzy multi-objective combinatorial problem, assuming that:

(i) The set of feasible solutions D is nonempty, closed, bounded, and therefore compact.

(ii) Let $\tilde{F}(x) = (\tilde{f}_1(x), \tilde{f}_2(x),..., \tilde{f}_p(x))$ be a vector of fuzzy objective functions, where each $\tilde{f}_k \in H(IR)$.

(iii) For any $x \in D$ and any $\alpha \in [0,1]$, level set of α is given by $(\tilde{f}_k(x))_{\alpha} = [f_{k,\alpha}^{L}(x), f_{k,\alpha}^{U}(x)]$, $k \in \{1,..., p\}$, where $f_{k,\alpha}^{L}$ and $f_{k,\alpha}^{U}$ are continuous real-valued functions on D.

(iv) The fuzzy partial order relation $\preccurlyeq$ on $H(\mathrm{IR})$ is defined by $\tilde{\mathrm{A}} \preccurlyeq \tilde{\mathrm{B}} \Leftrightarrow \mathrm{A}_{\alpha}^{\mathrm{L}} \leq \mathrm{B}_{\alpha}^{\mathrm{L}}$ and $\mathrm{A}_{\alpha}^{\mathrm{U}} \leq \mathrm{B}_{\alpha}^{\mathrm{U}}$ $\forall \alpha \in [0,1]$. The fuzzy Pareto dominance relation $\prec_P$ is the Pareto dominance induced by $\preccurlyeq$. For any $x, y \in D$, there exists at least one objective $i \in \{1,..., p\}$ such that fuzzy Pareto dominance is defined as $x \prec_P y$ if and only if $\tilde{f}_i(x) \preccurlyeq \tilde{f}_k(y)$ and $\tilde{f}_k(x) \prec \tilde{f}_k(y)$.

2.3. Conceptual overview of the interdependence between speed, travel Time, and distance

The fundamental relationship between speed (v), distance (d), and time (t) is given by the classical kinematic formula [19]:

$$t = \frac{d}{v} \tag{1}$$

Equation (1) clearly shows that travel time is directly proportional to distance and inversely proportional to speed. In other words, for a fixed distance, the higher the speed, the shorter the travel time; conversely, for a fixed speed, the greater the distance, the longer the travel time. In the transportation sector, particularly in the distribution of goods by vehicles, speed is not constant. This is especially true in urban environments, where the actual vehicle speed varies depending on traffic congestion, speed limits, frequent stops (traffic lights and intersections), vehicle characteristics, and weather conditions [34]. Fan et al. [35] proposed that the relationship between vehicle speed (v) and time (t) can be expressed by the trigonometric function $v(t) = a\sin(\gamma t) + \delta$, where a, γ and δ are coefficients related to road conditions. In an urban area, vehicle speeds can be partitioned into periods T_1, T_2, T_3 and T_4 during the day.

2.4. Presentation of the MEET Model

Freight transportation is recognized as a significant source of greenhouse gas (GHG) emissions. GHG emissions are proportional to fuel consumption, which in turn depends on several factors, including vehicle speed, travel distance, and vehicle weight [17]. For this reason, this paper adopts the MEET (Methodologies for Estimating Air Pollutant Emissions from Transport) model, which is formulated as a function dependent on vehicle speed and accounts for vehicle type. In addition, several correction factors can be incorporated to consider the effects of road gradient, vehicle load, and travel

distance in the calculation of greenhouse gas emissions. The model can be written as follows:

$$E(v,\gamma,d) = E_0(v) \times \eta(v) \times L(\gamma,v) \times d \quad (2)$$

E_0 : Rate of emission when the vehicle is unloaded in [g/km]
$L(\gamma,v)$: Load correction factor function
γ : Gradient of road in [%]
v : Speed of the vehicle in [km/h]
d : distance between two points [km].

Where:

$$E_0(v) = e_0 + e_1 v + e_2 v^2 + e_3 v^3 + \frac{e_4}{v} + \frac{e_5}{v^2} + \frac{e_6}{v^3} \quad (3)$$

e_0 : is a constant ; $e_1,...,e_6$: The coefficients vary according to the vehicle category considered [17]. To account for the effect of road gradient, for each vehicle category, pollutant type, and gradient class, the MEET model proposes the following road gradient correction factor:

$$\eta(v) = \omega_0 + \omega_1 v + \omega_2 v^2 + \omega_3 v^3 + \omega_4 v^4 + \omega_5 v^5 + \omega_6 v^6 \quad (4)$$

$\omega_0,...,\omega_6$: These coefficients correspond to the pollutant used and vary according to the vehicle category and gradient class [17]. To account for vehicle load, the MEET model applies the following load correction factor:

$$L(\gamma,v) = l_0 + l_1\gamma + l_2\gamma^2 + l_3\gamma^3 + l_4 v + l_5 v^2 + l_6 v^3 + \frac{l_7}{v} \quad (5)$$

l_0 : is a constant ; $l_1,...,l_7$: The coefficients of the load correction function vary according to the vehicle category considered [17].

3. Methodology

In this section, the mathematical model will be covered explicitly along with the list of assumptions needed to build the model and emissions calculations. For modeling routing and transportation processes characterized by subjectivity, ambiguity, uncertainty, and imprecision, the fuzzy logic approach happens to be a very promising mathematical approach.

3.1. Mathematical model development

The Green vehicle routing problem (GVRP) is a natural extension of the Vehicle Routing Problem (VRP), proposed by Dantzig and Ramser [11], an emblematic example of a combinatorial problem, recognized for its NP-hard complexity. Here, we present the formulation of FMOGVRP (Fuzzy Multi-Objective Green Vehicle Routing Problem), which can be summarized as follows:

Let $G = (V,E)$ be an undirected graph, where $V = \{0\} \cup V_0$ represents a set of all nodes. Node 0 is the central repository from which all vehicles $k \in K = \{1,2,...,m\}$ depart; $V_0 = \{1,2,...,n\}$ denotes the set of customer nodes. The set $E = \{(i,j) : i \in V, j \in V\}$ is the edge set, d_{ij} represents the distance between nodes i and j. All customers have a specific demand $l_i \geq 0$ to be served by a vehicle k. All vehicles $k \in \{1,...,m\}$ of the fleet have homogeneous capacity Q. The speed and travel time of each vehicle is expressed by trapezoidal fuzzy numbers $v = (v^1, v^2, v^3, v^4)$ and $t = (t_1, t_2, t_3, t_4)$. The binary decision variables x_{ij}^k are equal to 1 if vehicle k visits node j directly after node i, and 0 otherwise. The feasible solution x_{ij}^k corresponds to the vehicle route k represented as a Hamiltonian cycle in $G = (V,E)$. The main objectives of the problem are to minimize simultaneously the total travel time of all routes driven by vehicles k and the total fuel consumption or CO_2 emissions under fuzzy considerations.
Based on the above, we formulate the following assumptions:

H1: The vehicles used belong to the HDV (Heavy-Duty Vehicle) category, with a gross vehicle weight rating between 3.5 and 7.5 tons.
H2: The road gradient is assumed to be 0%.
H3 : Vehicle speed is modeled as a trapezoidal fuzzy number $v = (v^1, v^2, v^3, v^4)$ describing vehicle travel speed with the linguistic terms {slow speed (SS), moderate speed (MS), fast speed (FS)}.
H4 : Vehicle travel time is modeled as a trapezoidal fuzzy number $t = (t_1, t_2, t_3, t_4)$ describing vehicle travel time, the linguistic terms of t is {Short time (ST), moderate time (MT), long time (LT)}

3.1.1. Development of objective functions of the problem

Within a purely decision-making framework, the decision-maker must strike a compromise between the environmental objective and the economic objective. In this paper, we consider two objective functions. On the one hand, the first objective seeks to minimize CO_2 emissions from Heavy-Duty Vehicles (HDVs). However, treating vehicle speed as a deterministic parameter makes the model less realistic, given the inherent variability of speed. For this reason, in this study, speed is modeled as a fuzzy variable. On the other hand, the second objective aims to minimize the travel time of each vehicle. Since travel time depends on fuzzy speed, it consequently becomes a fuzzy variable as well.

a) Calculation of CO_2 emissions

As vehicle speed is modeled as a trapezoidal fuzzy number $v = (v^1, v^2, v^3, v^4)$, the performance of the fuzzy modeling framework largely depends on the specification of the membership functions associated with fuzzy speeds, which are based on empirical observations and expert knowledge as reported in the literature [24]. Inadequate calibration of these functions may introduce subjective bias and undermine the reliability of the resulting outcomes. Whose membership

functions are defined as follows:

$$\mu_{\tilde{v}}(x)=\begin{cases}0 & if \quad x\le v^1\\ \dfrac{x-v^1}{v^2-v^1} & if \quad v^1<x\le v^2\\ 1 & if \quad v^2<x\le v^3\\ \dfrac{v^4-x}{v^4-v^3} & if \quad v^3<x\le v^4\\ 0 & if \quad x>v^4\end{cases} \tag{6}$$

where v^1, v^2, v^3, v^4 are real numbers and $v^1 < v^2 < v^3 < v^4$. In this formulation, the interval $[v^2, v^3]$ corresponds to the most plausible operational speeds under typical traffic conditions, whereas $[v^1, v^4]$ defines the range of extreme, yet still feasible speed values.

Under assumptions H1–H2, the classical MEET model becomes fuzzy due to the propagation of speed imprecision into the emission calculations. Consequently, Equation (5) becomes fuzzy and can be expressed as follows:

$$E=\left(110+0.000375v^3+\frac{8702}{v}\right)\times\left(1{,}27-0{,}0235v-\frac{1{,}33}{v}\right)\times d \tag{7}$$

Expression (7) can be formulated as $E = M(v)\times N(v)\times d$ with $M(v)=110+0.000375v^3+\dfrac{8702}{v}$ and $N(v)=1{,}27-0{,}0235v-\dfrac{1{,}33}{v}$

By using load and slope correction factors, we perform a preliminary analysis of Equation (7).

We can see that $M(v)>0$ for any $v>0$ and

$$N(v)=0 \Rightarrow 1{,}27-0{,}0235v-\frac{1{,}33}{v}=0$$
$$\Rightarrow 1{,}27v-0{,}0235v^2-1{,}33=0 \Rightarrow v=53{,}6 km/h$$

Hence, if $v<53{,}6 \Rightarrow N(v)>0$ the emission is positive, if $v=53{,}6 \Rightarrow N(v)=0$ implies that the emission is zero, but if $v>53{,}6 \Rightarrow N(v)<0$,

we observe that the model accepts a specific range of speeds that can be used. Exceeding this range may lead to negative values. This does not imply that emissions are physically negative; rather, it indicates that the model's estimation produces a value lower than the reference scenario. This can be explained by the fact that the MEET model is designed for an urban context, where vehicles typically do not operate at such high speeds [17]. In the remainder of the paper, we will restrict our analysis to the speed range supported by the model.

By incorporating H3 into Equation (7), the classical MEET model becomes fuzzy due to the propagation of speed imprecision in the emission calculations. Consequently, Equation (7) becomes fuzzy and represents the objective function:

$$\tilde{E}=\left(110+0.000375\tilde{v}^3+\frac{8702}{\tilde{v}}\right)\times\left(1{,}27-0{,}0235\tilde{v}-\frac{1{,}33}{\tilde{v}}\right)\times d \tag{8}$$

As a trapezoidal fuzzy number $\tilde{E}$, it can be written as : $\tilde{E}=\left(s(v^1), s(v^2), s(v^2), s(v^4)\right)$ with the membership function defined by:

$$\mu_{\tilde{E}}(x)=\begin{cases}0 & if \quad x\le s(v^1)\\ \dfrac{x-s(v^1)}{s(v^2)-s(v^1)} & if \quad s(v^1)<x\le s(v^2)\\ 1 & if \quad s(v^2)<x\le s(v^3)\\ \dfrac{s(v^4)-x}{s(v^4)-s(v^3)} & if \quad s(v^3)<x\le s(v^4)\\ 0 & if \quad x>s(v^4)\end{cases} \tag{9}$$

The expression (8) will be used as the second objective function in our new mathematical formulation developed in subsection 3.1.2.

b) Calculation of Travel Time

Under assumption H4, Equation (1) also acquires a fuzzy nature and represents the objective function:

$$\tilde{t}=\frac{d}{\tilde{v}}=d\frac{1}{\tilde{v}}=d\tilde{v}^{-1} \tag{10}$$

However, by the arithmetic property

$$\tilde{v}^{-1}=\left(\frac{1}{v_4},\frac{1}{v_3},\frac{1}{v_2},\frac{1}{v_1}\right)\Rightarrow \tilde{t}=\left(\frac{d}{v_4},\frac{d}{v_3},\frac{d}{v_2},\frac{d}{v_1}\right)$$
$$=(t_1,t_2,t_3,t_4)$$

such that $t_1 < t_2 < t_3 < t_4$ are real numbers. Given the inverse relationship between speed and travel time, each linguistic term associated to speed is naturally paired with an opposite linguistic term for travel time (i.e. slow speed implies long travel time, moderate speed implies medium travel time and fast speed implies short travel time.

Whose membership functions are defined as follows:

$$\mu_t(x)=\begin{cases}0 & if \quad x\le t_1\\ \dfrac{x-t_1}{t_2-t_1} & if \quad t_1<x\le t_2\\ 1 & if \quad t_2<x\le t_3\\ \dfrac{t_4-x}{t_4-t_3} & if \quad t_3<x\le t_4\\ 0 & if \quad x>t_4\end{cases} \tag{11}$$

The expression (10) will be used as the first objective function in our new mathematical formulation developed in subsection 3.1.2.

3.1.2. New mathematical formulation of FMOGVRP

From the above, we can formulate our fuzzy bi-objective problem (P1) as follows:

$$\tilde{f}_1 = \min \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{t}_{ij} x_{ij}^k = \min \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{d_{ij}}{\tilde{v}_{ij}} x_{ij}^k \tag{12}$$

$$\tilde{f}_2 = \min \sum_{k\in K}\sum_{i\in V}\sum_{j\in V} \left(110 + 0.000375\tilde{v}_{ij}^{\,3} + \frac{8702}{\tilde{v}_{ij}}\right) \times \left(1{,}27 - 0{,}0235\tilde{v}_{ij} - \frac{1{,}33}{\tilde{v}_{ij}}\right) \times d_{ij} x_{ij}^k \tag{13}$$

Subject to:

$$\begin{cases} \sum_{k=1}^{m}\sum_{j=0}^{n} x_{ij}^k = 1 \quad , i \in V - \{0\} & (14) \\ \sum_{i=0}^{n} x_{ij}^k = \sum_{j=0}^{n} x_{ji}^k \, , i \in V - \{0\} \quad k = 1,\ldots, m & (15) \\ \sum_{j=1}^{n} x_{0j}^k = 1 \quad , \quad k = 1,\ldots, m & (16) \\ \sum_{i=1}^{n} x_{i0}^k = 1 \quad , \quad k = 1,\ldots, m & (17) \\ \sum_{i=0}^{n}\sum_{j=0}^{n} s_i x_{ij}^k + \sum_{i=0}^{n}\sum_{j=0}^{n} t_i x_{ij}^k \le T \quad , \quad k = 1,\ldots, m & (18) \\ \sum_{i\in U}\sum_{j\in U} x_{ij}^k \ge \sum_{j=1}^{n} x_{ij}^k \quad , U \subset V - \{0\} \quad , k = 1,\ldots, m & (19) \\ \sum_{i=1}^{n} q_i \left(\sum_{j=0}^{n} x_{ij}^k\right) \le Q \quad , \quad k = 1,\ldots, m & (20) \\ x_{ij}^k \in \{0,1\} & (21) \end{cases}$$

The set of feasible solutions is given by $D = \{x_{ij}^k \in \{0,1\} : (13) - (19)$ are satisfied, $i = 1, \ldots, n ; j = 1, \ldots, n , k = 1, \ldots m\}$.

Function (12) defines the objective function aimed at minimizing fuzzy total travel time, while function (13) defines the objective function targeting the minimization of fuzzy total CO_2 emissions.The constraints can be interpreted as follows: Expression (14) ensures that each customer is visited exactly once, (15) stipulates that if a vehicle k arrives at customer j it must depart from j once finish to server, (16) and (17) require each vehicle k to return to the depot at the end of its route, (18) enforces the maximum allowed route duration T, (19) eliminates sub-tours to guarantee route connectivity, (20) there is also the vehicle capacity constraint and equation (21) specifies that the feasible solutions in D are binary i.e. $x_{ij}^k = 1$ implies that vehicle k travels from node i to node j and $x_{ij}^k = 0$ otherwise.

Theorem 1 [34]: Under assumptions (i)–(iv), the fuzzy multi-objective problem $\min_{x\in D} \tilde{F}(x)$ defined by (P1) is mathematically well posed and admits at least one fuzzy Pareto-optimal solution.

3.2. Defuzzification process

In this paper, we have chosen an interval-based defuzzification approach, namely the Nearest Interval Approximation (NIA), because, compared to single-point defuzzification methods such as the Centre of Sums (COS), Centre of Gravity (COG), or Weighted Average, the interval approximation preserves partial information about uncertainty by maintaining lower and upper bounds at each confidence level [34].

Let us apply Algorithm 1 to the membership function $\mu_{\tilde{v}_{ij}}(x)$ given by expression (5).

Let us consider $\alpha \in [0,1]$

$$\frac{x - v^1}{v^2 - v^1} = \alpha \Rightarrow x - v^1 = \alpha(v^2 - v^1)$$

$$\Rightarrow x = \alpha(v^2 - v^1) + v^1 \tag{22}$$

$$\frac{v^4 - x}{v^4 - v^3} = \alpha \Rightarrow v^4 - x = \alpha(v - v^3)$$

$$\Rightarrow x = v^4 - \alpha(v^4 - v^3) \tag{23}$$

Hence, from (22) and (23), we respectively have

$${}^{v}N_{\alpha}^{L} = \int_0^1 \left(\alpha(v^2 - v^1) + v^1\right) d\alpha = \int_0^1 \alpha(v^2 - v^1) d\alpha + \int_0^1 v^1 d\alpha$$

$$= (v^2 - v^1)\int_0^1 \alpha d\alpha + v^1 \int_0^1 d\alpha \quad = \frac{1}{2}(v^2 - v^1) + v^1 \tag{24}$$

And

$$ {}^{v}N_{\alpha}^{U} = \int_0^1 \left(v^4 - \alpha(v^4 - v^3)\right)d\alpha = \int_0^1 v^4 d\alpha - \int_0^1 \alpha(v^4 - v^3)d\alpha $$

$$ = v^4 \int_0^1 d\alpha - (v^4 - v^3)\int_0^1 \alpha d\alpha = v^4 - \frac{1}{2}(v^4 - v^3) \quad (25) $$

Thus, the nearest interval of approximation is:

$$ C_d(\tilde{v}) = \left[\frac{1}{2}(v^2 - v^1) + v^1, v^4 - \frac{1}{2}(v^4 - v^3)\right] \quad (26) $$

Thus, the mean of the expression (25) is given by:

$$ Av\left(C_d(\tilde{v})\right) = \frac{\frac{1}{2}(v^2 - v^1) + v^1 + v^4 - \frac{1}{2}(v^4 - v^3)}{2} \quad (27) $$

By the same reasoning, for the membership function $\mu_{\tilde{t}_{ij}}(x)$ given by Equation (11), we obtain:

$$ {}^{t}N_{\alpha}^{L} = \frac{1}{2}(t_2 - t_1) + t_1 \quad (28) $$

$$ {}^{t}N_{\alpha}^{U} = t_4 - \frac{1}{2}(t_4 - t_3) \quad (29) $$

Thus, the nearest interval of approximation obtained by (27) and (28):

$$ C_d(\tilde{t}) = \left[\frac{1}{2}(t_2 - t_1) + t_1, t_4 - \frac{1}{2}(t_4 - t_3)\right] \quad (30) $$

Thus, the mean of the expression (27) is given by:

$$ Av\left(C_d(\tilde{t})\right) = \frac{\frac{1}{2}(t_2 - t_1) + t_1 + t_4 - \frac{1}{2}(t_4 - t_3)}{2} \quad (31) $$

In this work, we will consider three points of analysis of the NIA, the first one is the optimistic policy given by $\left(N_{\alpha}^{L}\right)$, the second one is the mean $Av\left(C_d(\bullet)\right)$, and the last point of view is the pessimistic $\left(N_{\alpha}^{U}\right)$ option regarding the speed of vehicle. This choice is made in order to study the NIA completely according to the decision maker challenge.

By substituting Equations (31) and (27) into Equations (13) and (12), respectively, we obtain a deterministic counterpart (P2) of the fuzzy objective function, expressed as follows:

$$ (P2) \equiv \begin{cases} f_1 = \min \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} Av\left(C_d(\tilde{t}_{ij})\right)x_{ij}^{k} = \min \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{d_{ij}}{Av\left(C_d(\tilde{v}_{ij})\right)}x_{ij}^{k} \\ f_2 = \min \sum_{k\in K}\sum_{i\in V}\sum_{j\in V}\left(110 + 0{,}000375 Av\left(C_d(\tilde{v}_{ij})\right)^3 + \frac{8702}{Av\left(C_d(\tilde{v}_{ij})\right)}\right)\times\left(1{,}27 - 0{,}0235 Av\left(C_d(\tilde{v}_{ij})\right) - \frac{1{,}33}{Av\left(C_d(\tilde{v}_{ij})\right)}\right)d_{ij}x_{ij}^{k} \end{cases} $$

By substituting (28) and (24) into (13) and (12) respectively, we obtain a (P3) deterministic counterpart of the fuzzy objective function expressed by:

$$ (P3) \equiv \begin{cases} f_1 = \min \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} {}^{t}N_{\alpha}^{L}x_{ij}^{k} = \min \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{d_{ij}}{{}^{t}N_{\alpha}^{L}}x_{ij}^{k} \\ f_2 = \min \sum_{k\in K}\sum_{i\in V}\sum_{j\in V}\left(110 + 0{,}00037\left({}^{t}N_{\alpha}^{L}\right)^3 + \frac{8702}{{}^{t}N_{\alpha}^{L}}\right)\times\left(1{,}27 - 0{,}0235\,{}^{t}N_{\alpha}^{L} - \frac{1{,}33}{{}^{t}N_{\alpha}^{L}}\right)d_{ij}x_{ij}^{k} \end{cases} $$

By substituting (29) and (25) into (13) and (12) respectively, we obtain a (P4) deterministic counterpart of the fuzzy objective function expressed by:

$$ (P4) \equiv \begin{cases} f_1 = \min \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} {}^{t}N_{\alpha}^{U}x_{ij}^{k} = \min \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{d_{ij}}{{}^{t}N_{\alpha}^{U}}x_{ij}^{k} \\ f_2 = \min \sum_{k\in K}\sum_{i\in V}\sum_{j\in V}\left(110 + 0{,}00037\left({}^{t}N_{\alpha}^{U}\right)^3 + \frac{8702}{{}^{t}N_{\alpha}^{U}}\right)\times\left(1{,}27 - 0{,}0235\,{}^{t}N_{\alpha}^{U} - \frac{1{,}33}{{}^{t}N_{\alpha}^{U}}\right)d_{ij}x_{ij}^{k} \end{cases} $$

The problems (P2), (P3), and (P4) used the same constraint system given by:

$$\begin{cases} \sum_{k=1}^{v}\sum_{j=0}^{n} x_{ij}^{k} = 1 \quad , i \in V - \{0\} \\ \sum_{i=0}^{n} x_{ij}^{k} = \sum_{j=0}^{n} x_{ji}^{k} , i \in V - \{0\} \quad k = 1,..., m \\ \sum_{j=1}^{n} x_{0j}^{k} = 1 \quad , \quad k = 1,..., m \\ \sum_{i=1}^{n} x_{i0}^{k} = 1 \quad , \quad k = 1,..., m \\ \sum_{i=0}^{n}\sum_{j=0}^{n} s_i x_{ij}^{k} + \sum_{i=0}^{n}\sum_{j=0}^{n} t_i x_{ij}^{k} \le T \quad , \quad k = 1,..., m \\ \sum_{i\in U}\sum_{j\in U} x_{ij}^{k} \ge \sum_{j=1}^{n} x_{ij}^{k} \quad , U \subset V - \{0\} \quad , k = 1,..., m \\ \sum_{i=1}^{n} q_i \left(\sum_{j=0}^{n} x_{ij}^{k} \right) \le Q \quad , \quad k = 1,..., m \\ x_{ij}^{k} \in \{0,1\} \end{cases}$$

However, NIA entails a certain loss of information, as it reduces the full structure of fuzzy parameters to real intervals. This effect can influence the interpretation of Pareto dominance

within a strictly fuzzy framework, particularly in multi-objective decision-making contexts. Nevertheless, employing this approximation represents a deliberate trade-off between uncertainty representation and computational tractability.

Theorem 2 [34]: (Characterization of efficient solutions to the deterministic DMOGVRP using the reference frame)

Let O be the objective space respectively of problem (P2), (P3), and (P4), $y^* \in O$ is the solution if the following statements are equivalent:

a) $y^* \in O$ is an efficient solution (Pareto optimal solution) of (P2), (P3) and (P4) respectively.

b) There is no $y^* \in O$ such that $y^* \in O$ is located in the dominated solution area of the reference frame of y.

c) There is no $y \in O$ such that z is located in the dominant solution area of the reference frame of y^*.

Even though the DMOGVRP given by (P2), (P3), and (P4) are solved deterministically, the resulting Pareto solutions are directly derived from the initial fuzzy formulation. The resulting Pareto front can therefore be interpreted as a robust approximation of the fuzzy Pareto front, where dominance relationships are assessed with respect to the actual value intervals rather than fixed predetermined values.

3.3. Choice of optimization algorithms

For the equivalent deterministic model (P2), (P3) and (P4) the necessary conditions of optimality given by Karush-Kuhn-Tucker (KKT), unfortunately do not apply directly to a combinatorial problem, because the latter contains binary variables (NP-Hard) and due to the non-convex nature of the feasible solution set, which causes exact methods to stagnate at a local optimum, hence the need to use heuristics and metaheuristics to find solutions to the problem [34].

In the following lines, this paper relies on the consecutive application of the sweep heuristic for partitioning customers $i = 1,..., n$ into different clusters (potential routes) G^k $k = 1,..., m$ based on their polar angle relative to the central depot [36]. The objective of minimizing the number of vehicles k employed is achieved by assigning them as follows:

$$G_i^k = \begin{cases} 1 \quad if \quad i \in G^k \\ \\ 0 \quad if \quad i \notin G^k \end{cases}$$

Our choice falls on the sweep heuristic because, in addition to its simplicity, it has a lower computational complexity $O(n \log n)$ compared to Clarke-Wright $O(n^2 \log n)$ and insertion heuristics $O(n^2)$ [37], making it better suited for initializing solutions in a GVRP.

And by applying the BicriterionAnt metaheuristic according parameters in Table 1 for optimizing the route costs according to the two objective functions f_1 and f_2, directly applying the concept of Pareto dominance in the selection of generated solutions [38].

Table 1. Parameters description for the BicriterionAnt Algorithm

Parameters	Value	Description	Source
m	7	The number of ants	[33][34]
α	1	Weight for pheromone level	
β	2	Weight for heuristics information	
ρ	0.2	Pheromone evaporation constant	
iteration	100	Number of iterations	

Our methodology can be summarized as follows:

Step 1: Inputs1.1. *Write the objective functions of the FMOGVRP**1.2**. Write the deterministic VRP constraints*
Step 2: *Express the fuzzy variables as trapezoidal numbers.*
Step 3: Defuzzification3.1*. Apply the NIA algorithm to the fuzzy variables**3.2**. Formulate the resulting NIA**3.3**. Extract the real values* N_{α}^{L}, Av, *and* N_{α}^{U} *from (3.2)*
Step 4: *Formulate the deterministic counterpart (DMOGVRP) respectively for* N_{α}^{L}, Av, *and* N_{α}^{U}
Step 5: *Solve the optimization problem of the DMOGVRP**5.1.*** *Apply the sweep heuristic to cluster customers under the deterministic VRP constraints**5.2.*** *Apply the BicriterionAnt metaheuristic to the DMOGVRP for* N_{α}^{L}, Av, *and* N_{α}^{U} *based on the solutions obtained in (5.1).*

4. Numerical simulation

4.1. Presentation of data used

In this paper, three types of vehicle speed are considered, as shown in Table 2 below, with information adapted from [34].

Table 2. Presentation and description of the speed categories used

Types of speed	Description	Trapezoidal fuzzy number
Slow (S)	20 to 35 km/h	(20, 25, 30, 35)
Moderate (M)	30 to 45 km/h	(30, 35, 40, 45)
Fast (F)	40 to 60 km/h	(40, 45, 50, 60)

At present, there is no benchmark instance of FMOGVRP. That is why in this paper, we consider simulation data. Let a matrix for 19 nodes where the node labeled 0 is assumed as the depot, and the demand of each customer, the maximum vehicle capacity is $Q = 7.5\ Tonnes$. As we can see in Table 3 below.

Table 3. Customers data

Node	Coord X	Coord Y	demand	Node	Coord X	Coord Y	Demand
C0	36	26	0	**C10**	35	51	1500
C1	21	45	90	**C11**	62	35	450
C2	45	35	150	**C12**	62	57	150
C3	55	20	300	**C13**	21	24	780
C4	33	34	300	**C14**	21	36	300
C5	50	50	450	**C15**	33	44	600
C6	55	45	600	**C16**	9	56	960
C7	26	59	900	**C17**	62	48	900
C8	40	66	1200	**C18**	66	14	1410
C9	55	65	1500	**C19**	44	13	1500

The algorithms of Sweep and BicriterionAnt have been implemented in Python language on a HP Laptop with CPU Intel® Core™i5-8365U, 1.90 GHz with 8 GB of RAM.

4.2. Résultats

The graph in Fig.1 below illustrates the spatial distribution of 19 clients across a two-dimensional geographic area. Each customer $i \in \{1,2,...,19\}$ is represented by a point identified by its Cartesian coordinates (x_i, y_i). A central depot (x_0, y_0) is also shown. The graph serves as a visual basis for applying the vehicle routing and clustering algorithm.

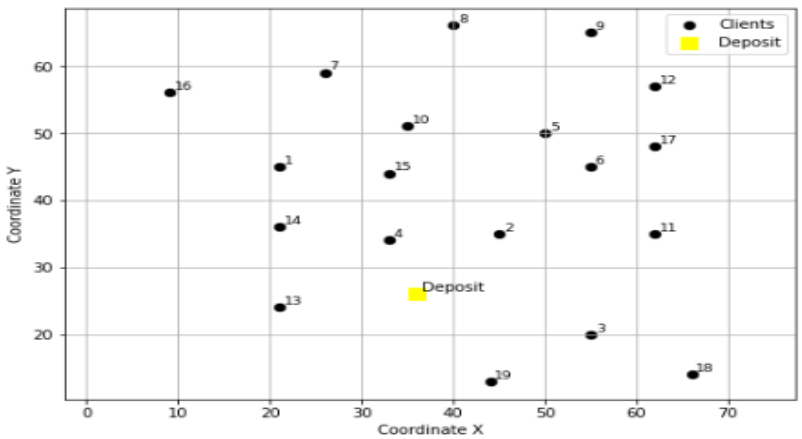

Figure 1. *Spatial distribution of clients*

As (P1) is an NP-hard problem, the resolution of the system of constraints (14) - (20) such that $x_{ij}^{k} \in \{0,1\}$ was done by applying the sweep algorithm, as shown in Fig.2 below, which illustrates how clients belonging to the same group (potential route) are represented by a distinct color. This allows for a clear visualization of the different service zones assigned to each k vehicle.

The representation highlights the geographic logic of the algorithm: clients that are close in angular position are grouped together, helping to minimize travel distance while respecting the vehicles' capacity constraints Q.

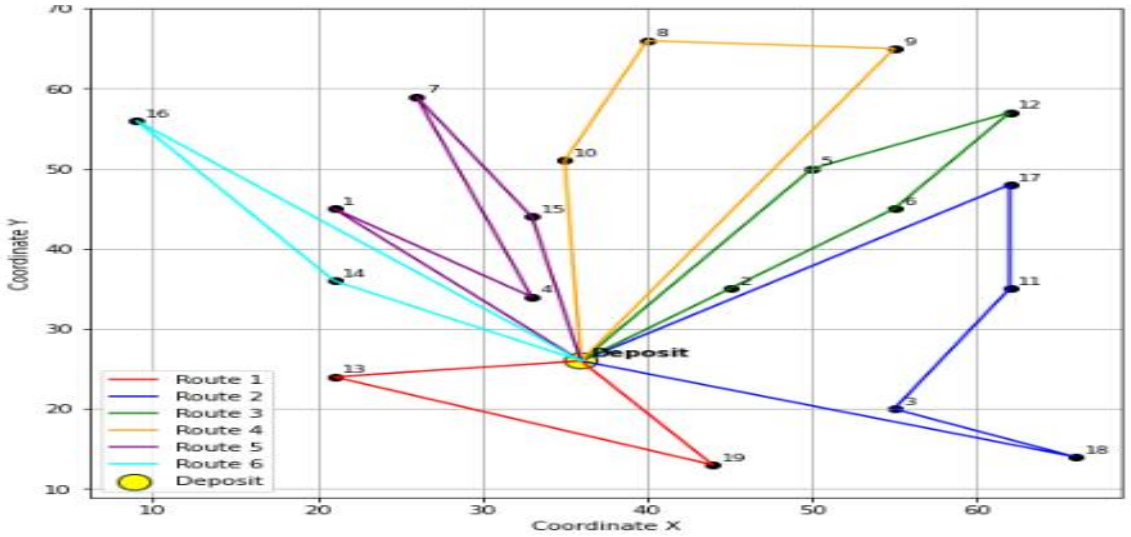

Figure 2*.Result of the sweep algorithm.*

In the context of modeling vehicle behavior on a road network, we associate a characteristic speed type to each road segment. Due to the lack of real data, we use a controlled random assignment of speed types through a Python Code. Thus, each road segment is randomly assigned a speed belonging to one of the three linguistic classes considered, as we can see in Table 4. This random distribution is performed according to a uniform probability law because all possible outcomes of a random experiment have the same probability of being chosen. The use of this method allows us to introduce variability into speed profiles and simulate realistic scenarios where speed is not uniform across the entire network.

Table 4. Speed distribution by road segment

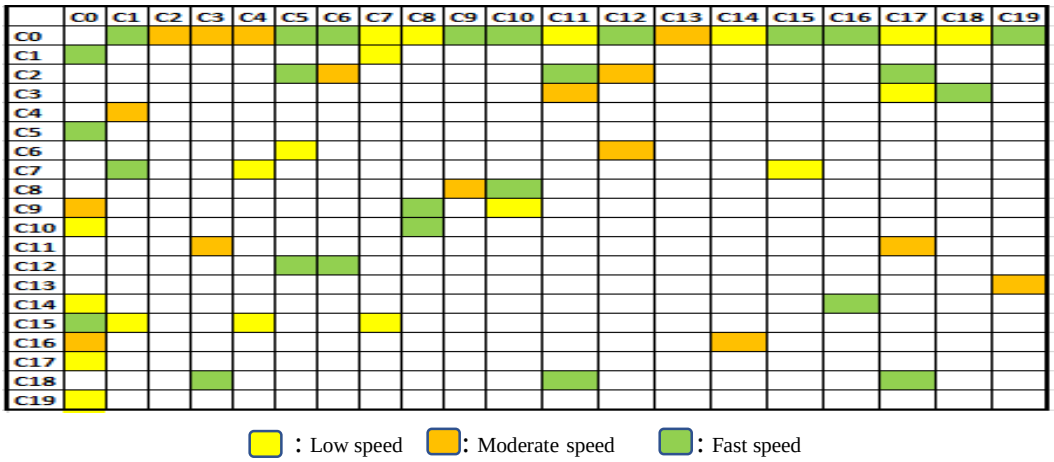

Table 5. Optimal routes obtained by BicriterionAnt

	(P2): Average speed			(P3): Optimistic speed			(P4): Pessimistic speed		
Vehicles	**Routes**	f_1	f_2	**Routes**	f_1	f_2	**Routes**	f_1	f_2
1	0-13-19-0	0,6208	5685,346652	0-13-19-0	11, 9026	8643,198388	0-13-19-0	0,9183	6123,720753
2	0-3-18-11-17-0	2,3389	13811,24419	0-11-17-18-3-0	3,571	23260,6164	0-11-17-18-3-0	1,0741	11700,12721
3	0-2-12-6-5-0	1,8392	8290,95789	0-6-12-5-2-0	1,5455	19196,38387	0-6-12-5-2-0	0, 9248	8062,085013
4	0-10-8-9-0	1,3653	10080,79608	0-10-8-9-0	1,3772	27100,84063	0-10-8-9-0	0, 9124	10666,70253
5	0-4-1-7-15-0	2,3602	13597,75167	0-4-1-15-7-0	1,3187	20822,24371	0-4-1-7-15-0	0,8812	9261,553059
6	0-14-16-0	3,3949	8215,42935	0-14-16-0	0, 9623	12535,94028	0-14-16-0	0, 9893	8526,702964

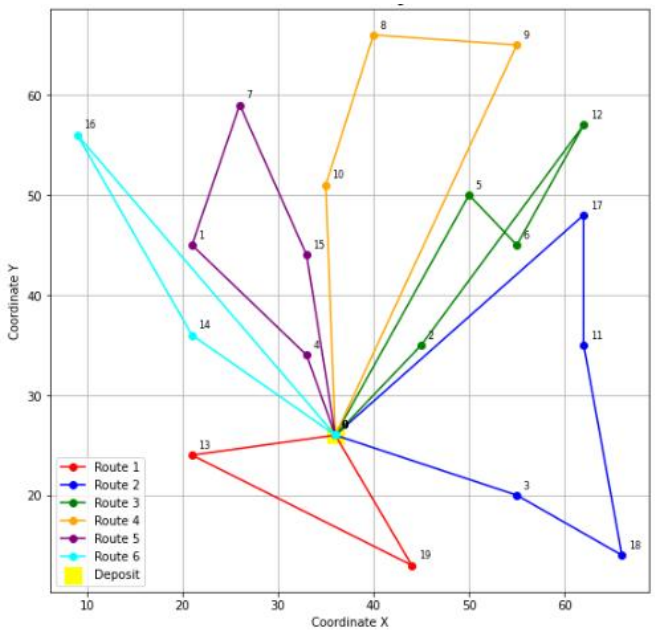

Figure 3. *Routes of optimization of (P2): Average scenario*

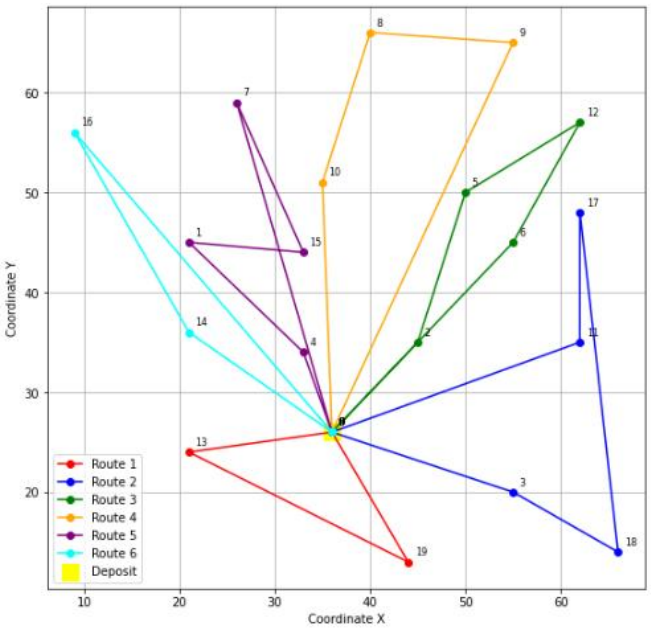

Figure 4. *Routes of optimization of (P3): Optimistic scenario*

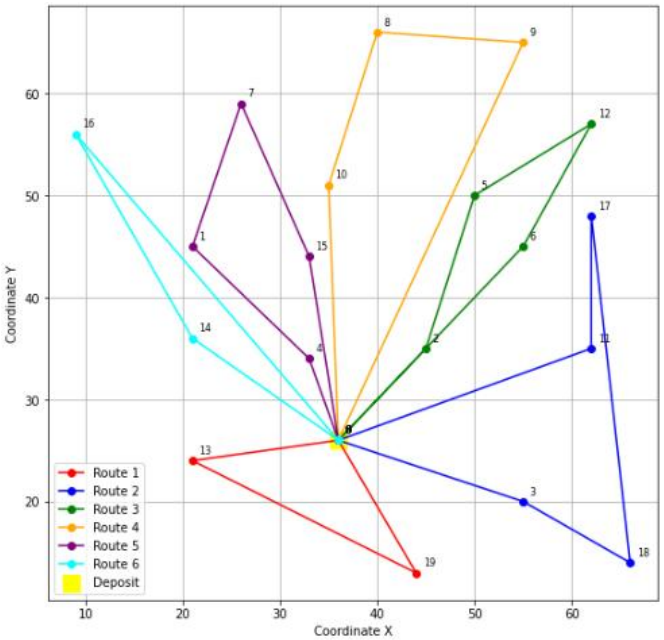

Figure 5. *Routes of optimization of (P4): Pessimistic scenario*

The obtained Pareto front presents a non-uniform distribution characterized by a relatively high spacing metric, indicating a limited density of intermediate trade-off solutions. From a managerial perspective, the front provides three strategic alternatives corresponding to time-oriented, environmentally-oriented, and balanced operational policies. Therefore, the final selection
strongly depends on the decision-maker's preference structure and sustainability priorities. In the following lines, we will attempt to carry out a comparative analysis of f_2 between our fuzzy and classic MEET models when the vehicle speed is fixed for all routes by using relative gaps (%) given by $\frac{fixed\ speed}{fuzzy\ speed} \times 100$, summarized in Table 6 below.

Table 6. Comparative analysis of fuzzy and fixed vehicle speeds on f_2

	Routes	Fuzzy average speed	Speed (20 km/h)	Speed (40 km/h)	Speed (50 km/h)	Fuzzy speed vs speed 20 km/h)	Fuzzy speed vs speed 40 km/h)	Fuzzy speed vs speed 50 km/h)
Scenario 1	1	5685,346652	22470,409	5830,809961	1265,096378	25%	98%	449%
	2	13811,24419	40561,40684	10525,21363	2283,629502	34%	131%	605%
	3	8290,95789	35893,03706	9313,82593	2020,797717	23%	89%	410%
	4	10080,79608	39900,28503	10353,6602	2246,40798	25%	97%	449%
	5	13597,75167	29947,36593	7770,99338	1686,053164	45%	175%	806%
	6	8215,42935	32850,961	8524,442551	1849,527162	25%	96%	444%
Scenario 2	**Routes**	**Fuzzy optimistic speed**						
	1	8643,198388	22470,409	5830,809961	1265,096378	38%	148%	683%
	2	23260,6164	40561,40684	10525,21363	2283,629502	57%	221%	1019%
	3	19196,38387	35893,03706	9313,82593	2020,797717	53%	206%	950%
	4	27100,84063	39900,28503	10353,6602	2246,40798	68%	262%	1206%
	5	20822,24371	29947,36593	7770,99338	1686,053164	70%	268%	1235%
	6	12535,94028	32850,961	8524,442551	1849,527162	38%	147%	147%
Scenario 3	**Routes**	**Fuzzy pessimistic speed**						
	1	6123,720753	22470,409	5830,809961	1265,096378	27%	105%	484%
	2	11700,12721	40561,40684	10525,21363	2283,629502	29%	111%	512%
	3	8062,085013	35893,03706	9313,82593	2020,797717	22%	87%	399%
	4	10666,70253	39900,28503	10353,6602	2246,40798	27%	103%	475%
	5	9261,553059	29947,36593	7770,99338	1686,053164	31%	119%	549%
	6	8526,702964	32850,961	8524,442551	1849,527162	26%	100%	461%

5. Discussions

In Scenario 1, the fuzzy speed corresponds to an average traffic situation, representing normal traffic conditions. The observed deviations range between 23% and 45% compared to 20 km/h, between 89% and 175% compared to 40 km/h, and between 410% and 605% compared to 50 km/h. These results indicate that the average fuzzy speed lies between low and high deterministic speeds, which corresponds to a realistic representation of urban traffic conditions. Route 5 shows the largest deviation with respect to 40 km/h (175%), while Route 2 presents the largest difference with respect to 50 km/h (605%). Scenario 2 corresponds to optimistic traffic conditions, characterized by smoother traffic flow and relatively higher speeds. The observed deviations range between 38% and 70% compared to 20 km/h, between 147% and 268% compared to 40 km/h, and between 683% and 1235% compared to 50 km/h. These differences are the largest among the three scenarios, which can be explained by the fact that optimistic conditions amplify the discrepancies between fuzzy speeds and certain deterministic speeds. Route 5 shows the highest deviations with respect to 20 km/h and 40 km/h, while Route 4 presents the largest difference with respect to 50 km/h (1206%). Scenario 3 represents unfavorable traffic conditions, characterized by higher congestion and lower speeds. The observed deviations range between 17% and 31% compared to 20 km/h, between 87% and 119% compared to 40 km/h, and between 399% and 549% compared to 50 km/h. Overall, the differences are smaller than those observed in the optimistic scenario, which is consistent since pessimistic speeds are closer to the lower speeds used in the deterministic model. Route 5 presents the smallest deviation compared to 20 km/h (17%), while Route 2 shows the largest difference compared to 50 km/h (549%).

6. Conclusion

In this paper, we presented a modeling approach for a multi-objective green vehicle routing problem, integrating both the environmental and economic impacts of the routes. The inherent variability of vehicle speed was taken into account by adapting the MEET model, allowing for a more realistic representation of actual traffic conditions and their influence on CO_2 emissions. We exploited the fundamental dependence between speed and travel time by representing both variables in a fuzzy form to capture the uncertainty associated with speed. Defuzzification was performed using an interval-based approach known as the Nearest Interval Approximation (NIA), thereby preserving partial information about uncertainty at each confidence level. This approach highlighted three speed management policies: average speed, pessimistic speed, and optimistic speed. Each of these policies provides a different perspective on route performance in terms of travel time and emissions, offering decision-makers tools to balance environmental and economic objectives under uncertain conditions. Comparing the third scenarios reveals the importance of incorporating speed uncertainty into the Green Vehicle Routing Problem. Together, they provide a broader representation of possible operational environments. The analysis shows that deterministic speed assumptions can either overestimate or underestimate system performance, whereas fuzzy speed modeling allows the model to capture the variability of real-world traffic conditions. This improves the robustness and realism of the routing solutions, particularly in the context of multi-objective optimization involving travel time and environmental impact. Overall, this study illustrates the importance of incorporating speed variability into green vehicle routing models and demonstrates the relevance of fuzzy modeling combined with interval-based defuzzification to obtain robust and realistic solutions. The FMOGVRP based on the Fuzzy MEET opens several promising avenues for future research, such as: Extension to heterogeneous fleets: Study the integration of vehicles from different categories (weight, engine type, energy capacity) into the fuzzy model to assess the impact on emissions and travel times. Developing a fully fuzzy

multi-objective optimization framework, in which Pareto dominance is handled directly in the fuzzy domain without relying on an initial defuzzification step.

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