INVESTIGATING THE EFFECTS OF HALL AND ION-SLIP CURRENTS ON CONVECTIVE FLOW IN ROTATING FLUID WITH WALL TEMPERATURE OSCILLATIONS

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ABSTRACT
The hydro magnetic free convective flow in a rotating fluid with wall temperature oscillations in the presence of a strong magnetic fluid is presented. The governing equations for the problem are solved by perturbation method to obtain analytical expressions for the velocity and temperature distribution. Effects of the Hall parameter β_e and the ion-slip parameter β_i on the velocity and temperature are discussed with the aid of graphs.

Nomenclature

Symbol	Quantity
$\vec{B}$	Magnetic induction vector
C_p	Specific heat, J/kgK
E	Electric field, $volts/m$
e	Electric charge, $coul/m^3$
Er	Rotational parameter,
g	Acceleration due to gravity, m/s^2
Gr	Grashof number
Ec	Eckert number
$\vec{J}$	Current density, A/m^2
$J_{x^+}, J_{y^+}, J_{z^+}$	Components of current density, A/m^2
k	Thermal conductivity, w/mk
M	Magnetic field parameter
Pr	Prandtl number
q	Complex velocity of the fluid *m/s*
T_∞^+	Temperature of the fluid in free stream, K
T^+	Dimensional temperature of the fluid, K

T_w^+	Temperature of the fluid at the plate, K
β^+	Coefficient of volumetric expansion, $1/K$
ω_e	Electron cyclotron frequency, Hz
ω_i	Ion-cyclotron frequency, Hz
τ_e	Collision time of electrons, s
τ_i	Collision time of ions, s
σ	Electrical conductivity, Ω/m
β_e	Hall current parameter
β_i	Ion-slip current parameter
θ	Dimensionless temperature of fluid
μ_e	Magnetic permeability H/m
ρ	Fluid density, Kg/m^3
ν	Kinematics viscosity of the fluid, m^2/s
Ω	Angular velocity of the fluid, $1/s$
t^+	Dimensional time, s
t	Dimensionless time
U^+	Free stream velocity m/s
u^+, v^+, w^+	Velocity components, m/s
w_0	Suction velocity, m/s
w	Dimensionless suction velocity
x^+, y^+, z^+	Cartesian coordinate

Abbreviation

MHD	Magnetohydrodynamic

1.0 INTRODUCTION

In an ionised gas where the density is low and the magnetic field is very strong, conductivity normal to the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions. When electric field **E** is applied, there will be an electrical current in the direction of **E**. If the magnetic field **H** is perpendicular to **E**, there will be an electromagnetic force **J X B,** which is perpendicular to both **E** and **H**. We thus have a new component of electric current density in the direction perpendicular to both **E** and **H**, which is known as Hall current. For the same electromagnetic force, the motion of ions is different from that of electrons, when the electromagnetic force is very large (such as in a very strong magnetic field) the diffusion velocity of ions cannot be neglected. If we consider the diffusion velocity of ions as well as that of electrons, we have the phenomenon known as ion-slip current.

The study of magnetohydrodynamics (MHD) viscous flows with Hall and Ion-slip currents has important application that varies from the study of the universe to engineering. Engineers employ MHD principles in the design of heat exchangers, electromagnetic pumps and space propulsion. The effect of Hall current is very important in the study of coronal plasma flows in the configuration of plasma sheet formation in the active region of the sun in the magnetic tail region.

Emad *et al* (2005) studied the effects of viscous dissipation and joule heating on MHD free convection flow past a semi- infinite vertical flat plate in the presence of the combined effect of Hall and ion-slip currents for the case of power-law variation of the wall temperature .They found that the magnetic field acts as a retarding force on the tangential flow but has a propelling effect on the induced lateral flow. The skin –friction factor for the tangential flow and the Nusselt number decreases but the skin-friction factor for the lateral flow increases as the magnetic field increases . Jordon (2007) analysed the effects of thermal radiation and viscous dissipation on MHD free-convection flow over a semi–infinite vertical porous plate. The network simulation method is used to solve the boundary –layer equations based on the finite difference formulation. It was found that an increase in viscous dissipation leads to an increase of both velocity and temperature profiles, an increase in the magnetic parameter leads to an increase in the temperature profiles and a decrease in the velocity profiles finally an increase in the suction parameter leads to an increase in the local skin-friction and Nusselt number. Cookey *et al.* (2003) investigated the influence of viscous dissipation and radiation on the problem of unsteady magneto hydrodynamics free-convection flow past an infinite heated vertical plate in an optically thin environment with time dependent suction. The results shows that increased cooling (Gr >0)of the plate and Eckert number leads to a rise in the velocity profiles ,while increases in magnetic field, radiation and Darcy's parameters are associated with a decrease in the velocity .Kinyanjui *et al.* (2001) investigated the MHD stokes problem for a vertical infinite plate in a dissipating rotating fluid with Hall current. Ram *et al.* (1991) used the finite difference method to solve the MHD stokes problem for a vertical plate with Hall and ion-slip currents. Emad *et al.* (2001) studied Hall current effects on magneto hydrodynamic free convection flow past a semi-infinite vertical plate with mass transfer. Adel *et al.* (2003) used similarity analysis in magnetohydrodynamics and investigated Hall current effects on free convection flow and mass transfer past a semi-infinite vertical flat plate.

In spite of all these studies, the effects of Hall and ion-slip currents on convective flow in a rotating fluid with wall temperature oscillations in the presence of a strong magnetic field has received little attention. Hence, the main objective of the present investigation is to study the effects of Hall parameters β_e and Ion-slip parameter β_i on velocity and temperature fields.

2.0 MATHEMATICAL ANALYSIS

In this study, unsteady free convection magneto hydrodynamic flow past a semi-infinite vertical porous plate subjected to a strong magnetic field inclined at an angle α to the plate and constant suction is studied. The x^+-axis is taken along the plate in vertically upward direction, which is the direction of flow.

The z^+-axis taken normal to the plate, since the plate is semi-infinite in length and for a two dimensional free convective fluid flow the physical variables are functions of $x^+, z^+ and\, t^+$.

The fluid is permeated by a strong uniform magnetic field H $=(H_0\sqrt{1-\psi^2},\ 0,\ H_0\psi)$ where $H_0 = |H|$ is the magnitude of the magnetic field and $\psi = \cos\alpha$. The temperature of the fluid and the plate are assumed to be the same initially. At time $t^+ > 0$ the porous plate starts moving impulsively in its own plane with a constant velocity U and its temperature is instantaneously raised or lowered to T_w^+, which is maintained constant thereafter.

The electric current density $\vec{J}$ represents the relative motion of charged particles in a fluid and its equation may be derived from the diffusion velocities of charged particles i.e. electrons and ions.

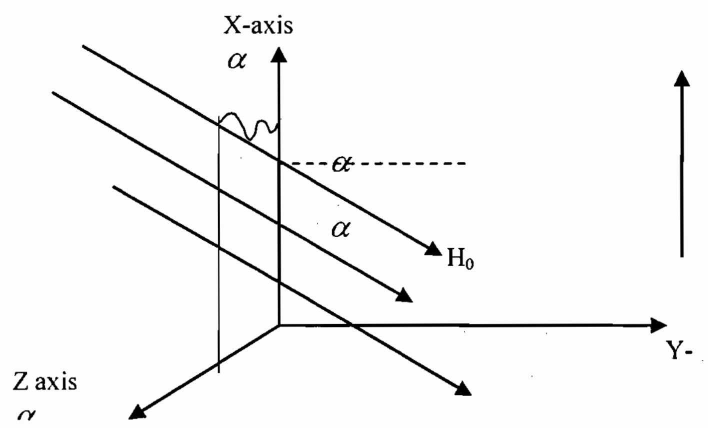


Figure 1: Flow configuration

Taking Hall current, the Ion-slip current and collisions between electrons and the neutral particles into account, we obtain Ohm's law of the form.

$$\vec{J} = \sigma\left[\vec{E} + \vec{q} \times \vec{B}\right] - \frac{\omega_e \tau_e}{\beta_0} \vec{J} \times \vec{B} + \frac{\omega_e \tau_e \omega_i \tau_i}{B_o^2}\left(\vec{J} \times \vec{B}\right) \times \vec{B}] \qquad (1)$$

In our study of effects of Hall and ion-slip currents on convective flow in a rotating fluid with wall temperature oscillations, we make the following assumptions:

(i) A strong magnetic field of uniform strength is applied transversely to the direction of flow (and to the wall).

(ii) The fluid and wall are in a state of rigid rotation with uniform angular velocity about the Z^+ axis.

(iii) The temperature of the wall fluctuates with time about a constant non-zero mean.

(iv) The free stream velocity oscillates about a constant, non-zero mean.

(v) The suction velocity (w_0) perpendicular to the surface of the wall is constant.

With the usual boundary layer equations and the Boussinesq approximation, the problem is governed by the following set of equations

$$\frac{\delta u^+}{\delta t^+}=\frac{1}{\rho}B_0\vec{J}_{y^+}+\beta^+g\left(T^+-T^+_\infty\right)+\nu\frac{\delta^2u^+}{\delta z^{+2}}+w_0\frac{\delta u^+}{\delta z^+}-2\Omega v^++\frac{dU^+}{dt^+}\quad\text{...................}(2)$$

$$\frac{\partial v^+}{\partial t^+}=-\frac{1}{\rho}B_0\vec{J}_{x^+}+\nu\frac{\partial^2v^+}{\partial z^{+2}}+w_0\frac{\partial v^+}{\partial z^+}+2\Omega\left(U^+-u^+\right)\quad\text{...}(3)$$

$$\frac{\partial T^+}{\partial t^+}=\frac{k}{\rho C_\rho}\frac{\partial^2T^+}{\partial Z^{+2}}+\frac{\nu}{C_p}\left[\left(\frac{\partial u^+}{\partial z^+}\right)^2+\left(\frac{\partial v^+}{\partial z^+}\right)^2\right]+w_0\frac{\partial^2T^+}{\partial Z^{+2}}\quad\text{...}(4)$$

The equation of conservation of electric charge gives constant. This constant is assumed to be zero since 0 at the plate which is assumed to be electrically non-conducting.

The magnetic induction, electric current density, complex velocity, and electric field are given in component forms as,

$$\left.\begin{aligned}\vec{B}&=\left(0,\ \ 0,\ \ B_0\right)\\ \vec{J}&=\left(\vec{J}_{x^+},\vec{J}_{y^+},\ 0\right)\\ \vec{q}&=\left(u^+,\ \ v^+,\ \ 0\right)\\ \vec{E}&=\left(E_{x^+},E_{y^+},0\right)\end{aligned}\right\}\quad\text{..}(5)$$

Expanding equation (1) and comparing coefficients of i, j on both sides, equation

(1) can now be written in component form as

$$J_{x^+}=\sigma\left[E_{x^+}+B_0V^+\right]-\omega_e\tau_e\vec{J}_{y^+}-\omega_e\tau_e\omega_i\tau_i\vec{J}_{x^+}$$

.

But $\omega_e\tau_e=\beta_e$ (the Hall parameter) and $\omega_i\tau_i=\beta_i$ (the Ion-slip parameter).

$$\left(1+\beta_e\beta_i\right)\vec{J}_{x^+}+\beta_e\vec{J}_{y^+}=\sigma\left[E_{x^+}+B_0V^+\right]\quad\text{..}(6)$$

Similarly,

$$(1+\beta_e\beta_i)J_{y^+}-\beta_e J_{x^+}=\sigma\left[E_{y^+}-B_0u^+\right] \quad (7)$$

Solving equations (6) and (7) simultaneously, we get

$$J_{x^+}=\sigma\left[\alpha(E_{x^+}+B_0V^+)-\beta\left(E_{y^+}+\left(U^+-u^+\right)\right)B_0\right] \quad (8)$$

$$J_{y^+}=\sigma\left[\alpha(E_{y^+}+B_0(U^+-u^+)+\beta\left(E_{x^+}+v^+B_0\right)\right] \quad (9)$$

w $^+$-the free stream velocity

$$\left.\alpha=\frac{1+\beta_e\beta_i}{(1+\beta_e\beta_i)^2+\beta_e^2} \qquad \beta=\frac{\beta_e}{(1+\beta_e\beta_i)^2+\beta_e^2}\right\} \quad (10)$$

On introducing non –dimensional quantities,

$$z=w_0\frac{z^+}{v}, \quad u=\frac{u^+}{u_0}, \quad v=\frac{v^+}{u_0}, \quad t=\frac{t^+w_0^2}{v}$$

$$w=\frac{vw^+}{w_0^2}, \quad \theta=\frac{T^+-T_\infty^+}{T_w^+-T_\infty^+}, \quad \Pr=\frac{\rho v c_\rho}{k}, \quad Er=\frac{\Omega v}{w_0^2}-$$

$$Gr=\frac{vg\beta^+\left(T_w^+-T_\infty^+\right)}{U_0^3}, \quad Ec=\frac{U_0^2}{c_p\left(T_w^+-T_\infty^+\right)}, \quad E_x=\frac{E_{x^+}}{B_0u_0} \quad (11)$$

$$E_y=\frac{E_{y^+}}{B_0u_0}, \quad J_x=\frac{J_{x^+}}{\sigma B_0u_0}, \quad J_y=\frac{J_{y^+}}{\sigma B_0\, u_0}$$

$$M^2=\frac{\sigma B_0^2 v}{\rho w_0^2}, \quad B_0^2=M^2\alpha_0, \qquad \alpha_0=\alpha+i\beta$$

Substituting equation (11) into (2) we get, $\frac{\partial u}{dt}-\frac{\partial u}{\partial z}=2M_1v+\frac{dU}{dt}+\frac{\partial^2u}{\partial z^2}+Gr\theta-M_1u$, where $M_1=2\left(Er+B_0^2\right)$

Since $q=u+iv$ and $\overline{q}=u-iv$ so that $\frac{dq}{\partial t}=\frac{\partial u}{\partial t}+i\frac{\partial v}{\partial t}$ and $\frac{dq}{\partial z}=\frac{\partial u}{\partial z}+i\frac{\partial v}{\partial z}$

Hence

$$\frac{\partial q}{\partial t}-\frac{\partial q}{\partial z}+M_1(q-U)=\frac{\partial U}{\partial t}+\frac{\partial^2q}{\partial z^2}+Gr\theta \quad (12)$$

Non-dimensionlising equation (4) we have

$$\frac{\partial\theta}{\partial t}-\frac{\partial\theta}{\partial z}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial z^2}+Ec\left(q'.\bar{q}'\right)\quad(13)$$

The bar denotes the complex conjugate of the corresponding quantity. Now, we have to solve equations (12) and (13) with the boundary conditions.

$$\left.\begin{array}{llll} q=0, & \theta=1+\varepsilon e^{i\omega t} & at & z=0 \\ q\to U(t), & \theta\to 0 & as & z\to\infty \end{array}\right\}\quad(14)$$

2.1 Solution of the Problem by Perturbation Method

To solve the equations (12) and (13) we represent the velocity and temperature fields in the neighborhood of the wall as

$$\left.\begin{array}{l} q=(1-q_0)+\varepsilon(1-q_1)e^{i\omega t}+0(\varepsilon^2) \\ \theta=\theta_0+\varepsilon\theta_1 e^{i\omega t}+0(\delta^2) \\ u(t)=1+\varepsilon.e^{i\omega t}+0(\varepsilon^2) \end{array}\right\}\quad(15)$$

where $\varepsilon<<1$

Differentiating (14), we get

$$\left.\begin{array}{l} \dfrac{\partial q}{\partial t}=0+\varepsilon i\omega(1-q_1)e^{i\omega t} \\ \dfrac{\partial q}{\partial z}=0-\dfrac{\partial q_0}{\partial z}+\varepsilon\left(0-\dfrac{\partial q_1}{\partial z}\right)e^{i\omega t} \\ \dfrac{\partial^2 q}{\partial z^2}=-\dfrac{\partial^2 q_0}{\partial z^2}+\varepsilon\left(-\dfrac{\partial^2 q_1}{\partial z^2}\right)e^{i\omega t} \\ \dfrac{\partial u}{\partial t}=\varepsilon i\omega t e^{i\omega t} \end{array}\right\}\quad(16)$$

Substituting equations (15) and (16) in equation (12) and comparing the coefficients of ε^0 and $\varepsilon^{'}$ we obtain

$$\frac{\partial^2 q_0}{\partial z^2}+\frac{\partial q_0}{\partial z}-M_1 q_0=Gr\theta_0.\quad(17)$$

$$\frac{\partial^2 q_1}{\partial z^2}+\frac{\partial q_1}{\partial z}-M_2 q_1=Gr\theta_1\quad(18)$$

where $M_2 = (M_1 + i\omega)$

Equations (17) and (18) can be rewritten as

$$q_0'' + q_0' - M_1 q_0 = Gr\theta_0 \qquad (19)$$

$$q_1'' + q_1' - M_2 q_1 = Gr\theta_1 \qquad (20)$$

Similarly from equation (13), we get

$$\theta_0'' + \Pr\theta_0' = -\Pr Ec(q'.\bar{q}') \qquad (21)$$

$$\theta_1'' + \Pr\theta_1' - M_3\theta_1 = \Pr Ec\left(q_0'\bar{q}_1' + \bar{q}_0' q_1^1\right) \qquad (22)$$

where $M_3 = \Pr i\omega$

The above equations are still non-linear. Applying perturbation method using

$$\left.\begin{aligned} \theta_0 &= \theta_{01} + Ec\theta_{02} + 0(Ec)^2 \\ \theta_1 &= \theta_{11} + Ec\theta_{12} + 0(Ec)^2 \\ q_0 &= q_{01} + Ecq_{02} + 0(Ec)^2 \\ q_1 &= q_{11} + Ecq_{12} + 0(Ec)^2 \end{aligned}\right\} \qquad (23)$$

where $Ec << 1$

Substituting the equation (23) together with their differentials in equations (19) to (22) and equating the coefficients of $(Ec)^0$ and $(Ec)^1$ on both sides we get the following set of eight equations

$$q_{01}'' + q_{01}' - M_1 q_{01} = Gr\theta_{01} \qquad (24)$$

$$q_{02}'' + q_{02}' - M_1 q_{02} = Gr\theta_{01} \qquad (25)$$

$$q_{11}'' + q_{11}' - M_2 q_{11} = Gr\theta_{11} \qquad (26)$$

$$q_{12}'' + q_{12}' - M_2 q_{12} = Gr\theta_{12} \qquad (27)$$

$$\theta_{01}'' + \Pr\theta_{01}' = 0 \qquad (28)$$

$$.\theta_{02}'' + \Pr\theta_{02}' = -Ec\Pr\left(q_{01}'\,\bar{q}_{01}'\right) \qquad (29)$$

$$\theta_{11}'' + \Pr\theta_{11}' - M_3\theta_{11} = 0 \;\; \theta_{02}'' + \Pr\theta_{02}' = -Ec\Pr\left(q_{01}'\,\bar{q}_{01}'\right) \qquad (30)$$

$$\theta_{12}'' + \Pr\theta_{12}' - M_3\theta_{12} = -Ec\Pr\left(q_{01}'\bar{q}_{11}' + \bar{q}_{01}' q_{11}'\right) \qquad (31)$$

$D(D+\Pr)=0$, $D=0$, $D=-\Pr$

$\Rightarrow \theta_{01}=C_1e^{01}+C_2e^{-\Pr z}$ And $\theta_{01}=C_1+C_2e^{\Pr z}$

Using modified boundary conditions given by equation (32) i.e. $\theta_{01}=1$ at $z=0$, $\theta_{01}=0$ at $z=\infty$, we obtain $1=c_1+c_2$ and $0=c_1+0 \Rightarrow c_1=0$, $c_2=1$

Hence

$$\theta_{01}=e^{-\Pr z}$$

From equation (24)

$$\left(D^2+D-M_1\right)q_{01}=Gr\theta_{01}$$

Its auxiliary equation is $\left(D^2+D-M_1\right)=0$

$$D=\frac{-1\pm\sqrt{1+4M_1}}{2}$$

$$Complementary\ \ function=C_3e^{\left(\frac{-1+\sqrt{1+4M_1}}{2}\right)z}+C_4e^{\left(\frac{-1-\sqrt{1+4M_1}}{2}\right)z}$$

$$Particular\ \ Integral=\frac{Gr}{\left(D^2+D-M_1\right)}e^{-\Pr z}=\frac{Gr}{(\Pr)^2-\Pr-M_1}.e^{-\Pr z}$$

Hence, the complete solution becomes

$$q_{01}=C_3e^{\left(\frac{-1+\sqrt{1+4M_1}}{2}\right)z}+C_4e^{\left(\frac{-1-\sqrt{1+4M_1}}{2}\right)z}+\frac{Gr}{(\Pr)^2-\Pr-M_1}.e^{-\Pr z}$$

Applying boundary conditions given by equation (32)

$$C_3=0$$

$$C_4=1-\frac{Gr}{(\Pr)^2-\Pr-M_1}=A_4 \qquad \text{.........(34)}$$

And $A_3 = \dfrac{Gr}{(\Pr)^2 - \Pr - M_1}$ then the complete solution becomes

$$q_{01} = A_3 e^{-\Pr z} + A_4 e^{-\alpha_2 \Pr z} \qquad (35)$$

where $\alpha_2 = \dfrac{1+\sqrt{1+4M_1}}{2}$

Similarly the solutions to the remaining six equations (25), (26), (27), (28), (30) and (31)

$$q_{02} = A_{15} e^{-\alpha_1 z} + A_{10} e^{-\Pr z} - A_{11} e^{-(\alpha_2+\bar{\alpha}_2)z} - A_{12} e^{-(\alpha_2+\Pr)z} - A_{13} e^{-(\bar{\alpha}_2+\Pr)z} - A_{14} e^{-2\Pr z} \qquad (36)$$

$$q_{11} = A_1 e^{-\alpha z} + A_2 e^{-\alpha_1 z} \qquad 37$$

$$q_{12} = A_{34} e^{-\alpha_1 z} + A_{25} e^{-\alpha z} - A_{26} e^{-(\alpha_2+\bar{\alpha}_1)z} - A_{27} e^{-(\alpha_2+\bar{\alpha})z} - A_{28} e^{-(\Pr+\bar{\alpha}_1)z} - A_{29} e^{-(\Pr+\bar{\alpha})z}$$
$$-A_{30} e^{-(\alpha_1+\bar{\alpha}_2)z}$$

$$-A_{31} e^{-(\alpha_1+\Pr)z} - A_{32} e^{-(\alpha+\alpha_2)z} - A_{33} e^{-(\alpha+\Pr)z} \qquad (38)$$

$$\theta_{02} = A_9 e^{-\Pr z} - A_5 e^{-(\alpha_2+\bar{\alpha}_2)z} - A_6 e^{-(\bar{\alpha}_2+\Pr)z} - A_7 e^{-(\bar{\alpha}_2+\Pr)z} - A_8 e^{-2\Pr z} \qquad (39)$$

$$\theta_{12} = A_{24} e^{-\alpha z} - A_{16} e^{-(\alpha_2+\alpha_1)z} - A_{17} e^{-(\alpha_2+\alpha)z} - A_{18} e^{-(\Pr+\bar{\alpha}_1)z} - A_{19} e^{-(\Pr+\bar{\alpha})z}$$
$$-A_{20} e^{-(\alpha_1+\bar{\alpha}_2)} - A_{21} e^{-(\alpha_1+\Pr)z} - A_{22} e^{-(\alpha+\bar{\alpha}_2)z} - A_{23} e^{-(\alpha+\Pr)z} \qquad (40)$$

where the constants appearing in the above are listed below

$$A_1 = \frac{Gr}{(\alpha^2-\alpha-M_2)}, \quad \alpha = \frac{\sqrt{(\Pr)^2+4M_3}}{2},$$

$$A_2 = 1 - A_1, \ \alpha_1 = \frac{1}{2}\left[1+(1+4M_2)^{1/2}\right]$$

$$A_5 = \frac{\Pr Ec \alpha_2 \bar{\alpha}_2 A_4 \bar{A}_4}{(\alpha_2+\bar{\alpha}_2)(\alpha_2+\bar{\alpha}_2-p_r)},$$

$$A_6 = \frac{(\Pr)^2 Ec A_4 \bar{A}_3}{(\bar{\alpha}_2+\Pr)},$$

$$A_8 = \frac{1}{2}\Pr Ec A_3 \bar{A}_3$$

$$A_9 = \sum_{i=5}^{8} A_i ,$$

$$A_{10} = \frac{GrA_9}{\left((\Pr)^2 - \Pr - M_1\right)},$$

$$A_{11} = \frac{GrA_5}{\left[(\alpha_2 + \bar{\alpha}_2)^2 - (\alpha_2 + \bar{\alpha}_2) - M_1\right]} \quad A_{12} = \frac{GrA_6}{\left[(\alpha_2 + \Pr)^2 - (\alpha_2 + \Pr) - M_1\right]},$$

$$A_{13} = \frac{GrA_7}{\left[(\bar{\alpha}_2 + \Pr)^2 - (\bar{\alpha}_2 + \Pr) - M_1\right]} \quad A_{14} = \frac{GrA_8}{\left(4(\Pr)^2 - 2\Pr - M_1\right)}$$

$$A_{15} = \sum_{i=11}^{14} A_i - A_{10}$$

$$A_{17} = \frac{\Pr Ec\alpha_2 \bar{\alpha} A_4 \bar{A}_1}{\left[(\alpha_2 + \bar{\alpha})^2 - \Pr(\alpha_2 + \bar{\alpha}) - M_3\right]} \quad A_{16} = \frac{\Pr Ec\alpha_1 \alpha_2 A_4 \bar{A}_2}{\left[(\alpha_2 + \bar{\alpha}_1)^2 - \Pr(\alpha_2 + \bar{\alpha}_1) - M_3\right]}$$

$$A_{18} = \frac{(\Pr)^2 Ec\bar{\alpha}_1 A_3 \bar{A}_2}{\left[(\Pr + \bar{\alpha}_1)^2 - \Pr(\Pr + \bar{\alpha}_1) - M_3\right]}$$

$$A_{19} = \frac{(\Pr)^2 Ec\bar{\alpha} A_3 \bar{A}_1}{\left[(\Pr + \bar{\alpha})^2 - \Pr(\Pr + \bar{\alpha}) - M_3\right]}$$

$$A_{20} = \frac{\Pr Ec\alpha_1 \bar{\alpha}_2 A_2 \bar{A}_4}{\left[(\alpha_1 + \bar{\alpha}_2)^2 - \Pr(\alpha_1 + \bar{\alpha}_2) - M_3\right]}$$

$$A_{21} = \frac{(\Pr)^2 Ec\alpha_1 A_2 \bar{A}_3}{\left[(\alpha_1 + \Pr)^2 - \Pr(\alpha_1 + P_r) - M_3\right]}$$

$$A_{22} = \frac{\Pr Ec\alpha \bar{\alpha}_2 A_1 \bar{A}_4}{\left[(\alpha + \bar{\alpha}_2)^2 - \Pr(\alpha + \bar{\alpha}_2) - M_3\right]}$$

$$A_{23} = \frac{(\Pr)^2 Ec\alpha A_1 \bar{A}_3}{\left[(\alpha + \Pr)^2 - \Pr(\alpha + \Pr) - M_3\right]}$$

$$A_{24} = \sum_{i=16}^{23} A_i$$

$$A_{25} = \frac{GrA_{24}}{\left(\alpha^2 - \alpha - M_2\right)}$$

$$A_{26} = \frac{GrA_{16}}{\left[\left(\alpha_2 + \bar{\alpha}_2\right)^2 - \left(\alpha_2 + \bar{\alpha}\right) - M_2\right]}$$

$$A_{27} = \frac{GrA_{17}}{\left[\left(\alpha_2 + \bar{\alpha}\right)^2 - \left(\alpha_2 + \bar{\alpha}\right) - M_2\right]}$$

$$A_{28} = \frac{GrA_{18}}{\left[\left(\Pr + \bar{\alpha}_1\right)^2 - \left(\Pr + \bar{\alpha}_1\right) - M_2\right]}$$

$$A_{29} = \frac{GrA_{19}}{\left[\left(\Pr + \bar{\alpha}\right)^2 - \left(\Pr + \bar{\alpha}\right) - M_2\right]}$$

$$A_{30} = \frac{GrA_{20}}{\left[\left(\alpha_1 + \bar{\alpha}_2\right)^2 - \left(\alpha_1 + \bar{\alpha}_2\right) - M_2\right]}$$

$$A_{31} = \frac{GrA_{21}}{\left[\left(\alpha_1 + \Pr\right)^2 - \left(\alpha_1 + \Pr\right) - M_2\right]}$$

$$A_{32} = \frac{GrA_{22}}{\left[\left(\alpha + \bar{\alpha}_2\right)^2 - \left(\alpha + \bar{\alpha}_2\right) - M_2\right]}$$

$$A_{33} = \frac{GrA_{23}}{\left[\left(\alpha + \Pr\right)^2 - \left(\alpha + \Pr\right) - M_2\right]}$$

$$A_{34} = \sum_{i=26}^{33} A_i - A_{25}$$

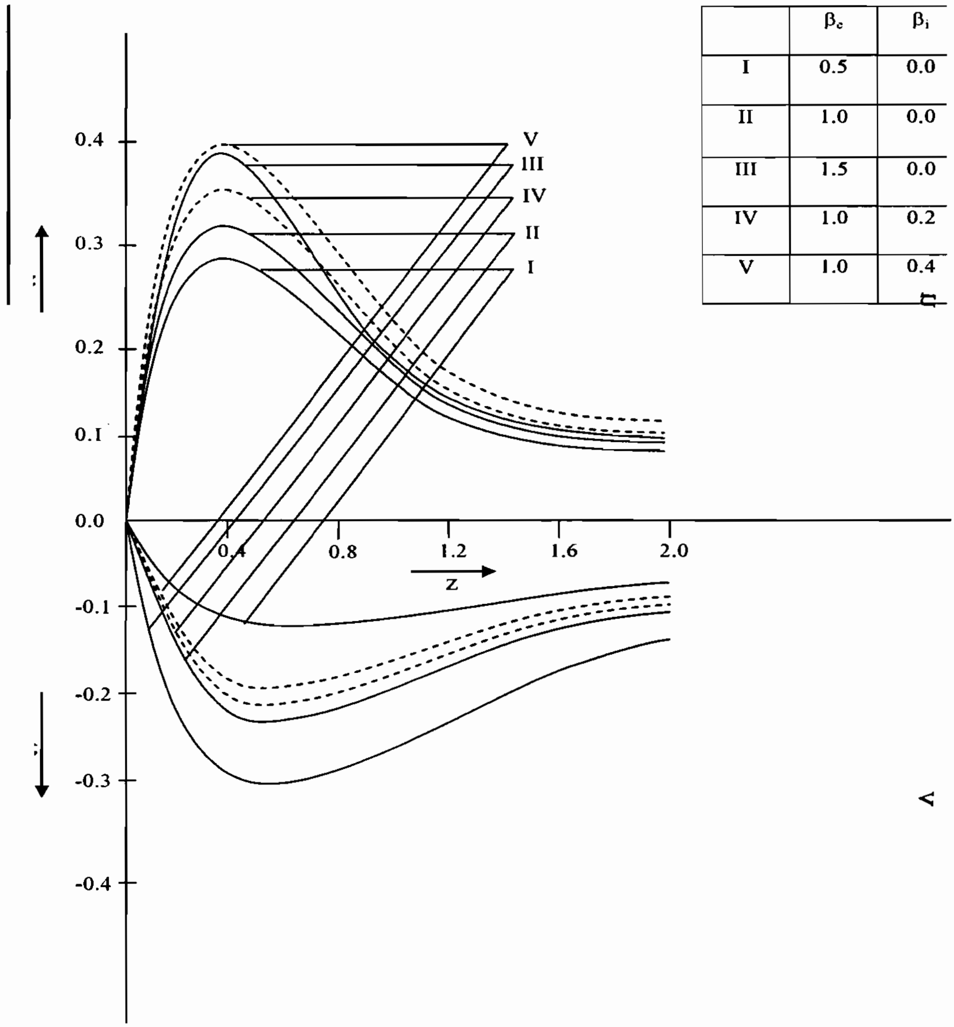
A computer programme was written to solve the reduced equations with specified values of the parameters, the initial and boundary conditions.

The computations are performed using small values of Δt, in our research we set =0.00125 and, the Prandtl number is taken as 0.71 which corresponds to air, magnetic parameter which signifies a strong magnetic field. Two cases are considered:

(i) When the Grashof number, Gr>0(10) corresponding to convective cooling of the plate.

(ii) When the Grashof number, Gr<0(-10) corresponding to convective heating of the plate.

Graphs on our calculations are presented in Figures 2 - 4.



	β_e	β_i
I	0.5	0.0
II	1.0	0.0
III	1.5	0.0
IV	1.0	0.2
V	1.0	0.4

Figure 2: Transient primary and secondary velocity profiles at Gr = 10.0

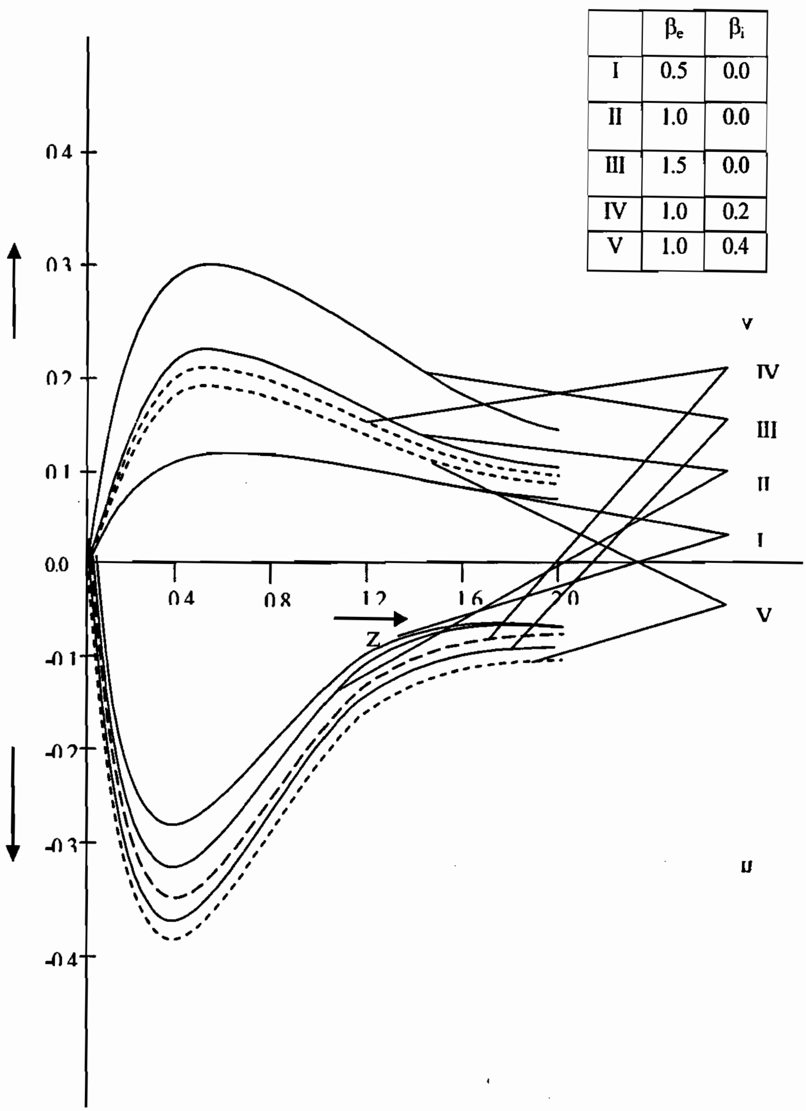


Figure 3: Transient primary and secondary velocity profiles at Gr = -10.0

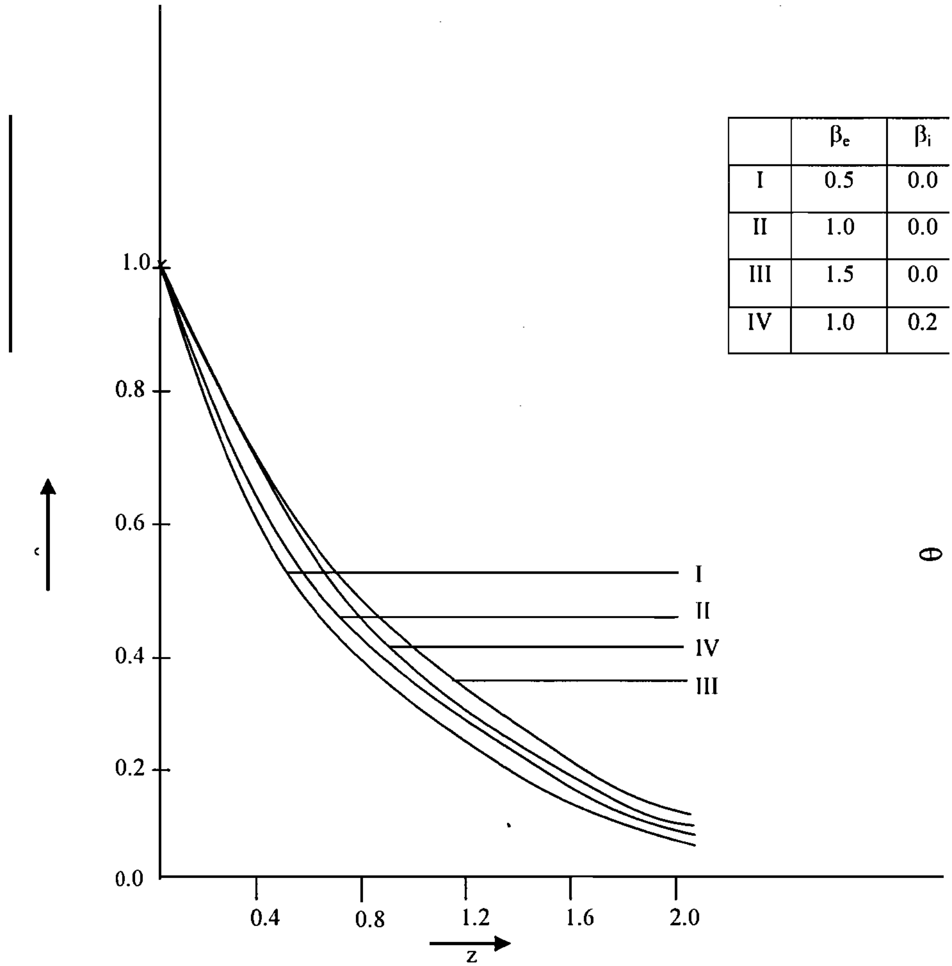


Figure 4: Transient temperature profile at Gr > 0 and Gr < 0

3.0 RESULTS AND CONCLUSIONS

In order to study the effects of β_e and β_i on velocity and temperature fields, computations (numerical calculations) are carried out using the above figures. Primary and secondary velocities fields and temperature fields are shown graphically.

From Figure 2, we note that for positive value of Gr,

(a) As β_e increases, u increases as v decreases.

(b) As β_i increases, both u and v increase.

From Figure 3, we note that for negative value of Gr,

(a) As increases, u decreases but v increases.
(b) As increases, both u and v decrease.

From Figure 4, we note that for both cases, (Gr<0) or (Gr>0),

(a) As increases, also increases
(b) As increases, also increases

We conclude that with the cooling of the system as the Hall parameter increases primary velocity increases but secondary velocity decreases whereas the increase in ion-slip parameter increases both primary and secondary velocities.

On the other hand with heating of the system as Hall parameter increases vice versa increases in velocities takes place. The same happens to the velocities with the increase in the ion-slip parameter. Finally, it is interesting to note that with the increase of Hall parameter orion-slip parameter the rise in temperature takes place in each case.

In power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. This class of flow has many applications in the design of MHD generators, pumps and flow meters. In many cases, the flow in these devices will be accompanied by heat either dissipated internally through viscous heating, joule heating or that produced by electric currents in the walls. We strongly recommend that the designers of these devices should take into consideration the effects of the parameters discussed in this paper. The problem was first solved without subjecting the flow to a strong magnetic field inclined at angle to the plate and the results were found to coincide with those of Chamkha (2004) who considered unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption.

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