Modelling the Transmission Dynamics of COVID-19 Incorporating public Enlightenment Campaign

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ABSTRACT: A mathematical model to study the transmission dynamics of COVID-19 incorporating public enlightenment campaign as control is presented. The effective reproduction number \( R_e \) was computed using the next generation method. Using the Lyapunov method, the global stability of the disease-free equilibrium was found to be globally asymptotically stable whenever \( R_e \leq 1 \). Sensitivity analysis was conducted on the effective reproduction number in order to determine parameters of the model that are most sensitive and targeted by way of intervention strategies. Numerical simulations of the COVID-19 model shows that if 90% of both treatment and public enlightenment campaign is achieved, the pandemic will be greatly controlled and subsequently eradicated in the population.

DOI: https://dx.doi.org/10.4314/jasem.v26i10.16

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Cite this paper as: ASHEZUA, T, T; AONDONA, L, C; AMAONYEIRO, A. U. (2022), Modelling the transmission dynamics of COVID-19 incorporating public enlightenment campaign. J. Appl. Sci. Environ. Manage. 26 (10) 1721-1726

Keywords: COVID-19, Public enlightenment campaign, Global stability, Pandemic

Corona virus disease COVID-19 is an infectious disease which was first identified amidst an outbreak of respiratory illness caused in Wuhan City, China (Cennimo, 2021). The corona virus has been responsible for 270,327,277 cases and 5,316,017 deaths globally, European Center for Disease Prevention and Control (ECDC), (2021). In Nigeria, it is on record that 217, 481 cases have been confirmed while 2,981 deaths recorded so far (Nigeria Centre for Disease Control (NCDC), (2021) and World Health Organization, (WHO), (2021). The COVID-19 virus spreads primarily through droplets of saliva or discharge from the nose when an infected person coughs or sneezes (WHO, 2020). Thus, the disease spreads from one person to another as a result of close contact with an infected person. Symptoms of COVID-19 may range from mild symptoms to severe illness. Symptoms may appear within 2-14 days after exposure to the virus and these include: fever, chills, cough, shortness of breath or difficulty in breathing, sore throat, congestion or runny nose, fatigue, headache, muscle pain, loss of taste or smell, nausea, vomiting, and diarrhea (Felman, 2021). A test can detect the infection, even if there are no symptoms (CDC, 2021). Over the last two years, numerous models have been developed in order to understand the spread and control of COVID-19, see for example, Gweryina et al., (2021), William et al., (2021) and Iboi et al., (2021) just to mention a few. Mathematical models can help play a key role in educating the public about COVID-19 pandemic stressing the available control/prevention strategies among other steps that could help curb the spread of the pandemic. This could be achieved through the organization of workshops, television adverts and radio jingles to help enlighten the public about COVID-19. In this paper, a mathematical model to study the transmission dynamics of COVID-19 incorporating public

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enlightenment campaign as a control strategy is presented. This research is a modification of the work by Olaniyi et al., (2020) and based on the following assumptions: all successfully treated individuals become susceptible after treatment from the isolation centre, proper COVID-19 protocols are fully observed during burial rites of the dead, hence no need for the dead compartment, the present study incorporates the parameters of birth, death, and control strategies like the public enlightenment campaign, which was not captured in the works of Olaniyi et al., (2020), we neglect immigration of individuals into the country in order to contain the spread of the pandemic, this study did not incorporate recovery class since individuals who are properly treated from the hospital moved to their various houses (become susceptible). Individuals who are either symptomatic (I) or Isolated (H) can die due to COVID-19 infections.

MATERIALS AND METHOD
The total population at time \( t \), denoted by \( N \) is subdivided into five (5) compartments of the susceptible individuals \( S \), exposed individuals \( E \), symptomatic individuals \( I \), asymptomatic individuals \( A \) and isolated individuals \( H \).

Let \( \Lambda \) be the recruitment number into the susceptible into the population, \( \gamma \) be the recovery rate and \( \beta \) be the rate of transmission of the COVID-19 pandemic. \( \mu \) is the natural death rate associated to all the compartments. \( \alpha \) represents the rate at which humans exposed to COVID-19 progress from the exposed state to either the symptomatic or asymptomatic compartments after the incubation period of 14 days. The parameter \( l_1 \) represents the rate of individuals with symptoms after the incubation period. As a result, the rate of the individuals that do not show symptoms after incubation period is \((1 - l_1)\).

The parameters \( h_1 \) and \( h_2 \) represent the rates at which individuals are isolated in both the symptomatic and asymptomatic compartments respectively. COVID-19 is known to have caused several deaths; the parameter \( \delta \) represents the rate at which people die due to COVID-19 pandemic. The above description of variables and parameters are summarized on Tables 1 and 2 respectively.

While the flow diagram used in the formulation of the COVID-19 model is presented in figure 1. With the descriptions of the variables and parameters on Tables 1 and 2, assumptions and the flow diagram in Figure 1, the following set of non-linear ordinary differential equations were derived:

\[
\begin{align*}
\frac{dS}{dt} &= \Lambda - \lambda S + \gamma H - \mu S \quad (1) \\
\frac{dE}{dt} &= \lambda S - (\alpha + \mu)E \quad (2) \\
\frac{dI}{dt} &= l_1 \alpha E - (h_1 + \mu + \delta)I \quad (3) \\
\frac{dA}{dt} &= (1 - l_1)\alpha E - (h_2 + \mu)A \quad (4) \\
\frac{dH}{dt} &= h_1 I + h_2 A - (\mu + \gamma + \delta)H \quad (5)
\end{align*}
\]

where,

\[
\lambda = \frac{(1-\psi)\beta(t+A+H)}{N} \quad (6)
\]

\[
N(t) = S(t) + E(t) + I(t) + A(t) + H(t) \quad (7)
\]

Basic Properties of the Model: For Model (1) – (5) to be epidemiologically meaningful, it is important to prove that all its state variables are positive for all time \( t \).

Positivity and boundedness of solutions: Consider the biologically feasible region

\[
\mathcal{M} = \left\{ (S, E, I, A, H) \in \mathbb{R}_+^5 : N \leq \frac{\Lambda}{\mu} \right\}
\]

It can be shown that the set \(\mathcal{M}\) is a positive invariant set.

Lemma 1. The region \(\mathcal{M}\) is positively invariant for the system (1) – (5)

Proof. The total human population is given by (7) and its rate of change is

\[
\frac{dN}{dt} = \Lambda - \mu N - \delta (1 + H) \quad (8)
\]

Since the right-hand side of (2.8) is bounded by \(\Lambda - \mu N\), a standard comparison theorem as outlined in the works of Iboi and Okoungahae (2016) was applied here.

\[
N(t) \leq N(0)e^{-\mu t} + \frac{\Lambda}{\mu}(1 - e^{-\mu t})
\]

\[
N(t) \leq \frac{\Lambda}{\mu} + \left[ N(0) - \frac{\Lambda}{\mu} \right] e^{-\mu t}
\]

As \( t \to \infty \), the population size \( N(t) \) approaches

\[
0 \leq N(t) \leq \frac{\Lambda}{\mu} \Rightarrow N(t) \leq \frac{\Lambda}{\mu} \quad (9)
\]

Thus, \(\mathcal{M}\) is positively invariant. In this region, the model (1) – (5) can be considered as being
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epidemiologically meaningful and mathematically well posed.

Table 1. Description of Variables in the COVID-19 Model

<table>
<thead>
<tr>
<th>Variables</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S(t) )</td>
<td>Number of susceptible individuals at time, ( t ).</td>
</tr>
<tr>
<td>( E(t) )</td>
<td>Number of exposed individuals (though infectious) at time, ( t ).</td>
</tr>
<tr>
<td>( I(t) )</td>
<td>Number of symptomatic individuals at time, ( t ).</td>
</tr>
<tr>
<td>( A(t) )</td>
<td>Number asymptomatic individuals at time, ( t )</td>
</tr>
<tr>
<td>( H(t) )</td>
<td>Number of isolated individuals undergoing treatment at time, ( t ).</td>
</tr>
</tbody>
</table>

Table 2. Description of Parameters in the COVID-19 Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda )</td>
<td>Recruitment number into the susceptible class</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Public enlightenment campaign</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Effective transmission coefficient</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Rate of disease progression from exposed class</td>
</tr>
<tr>
<td>( l_1 )</td>
<td>Proportion of exposed with symptoms after the incubation period</td>
</tr>
<tr>
<td>( 1-l_1 )</td>
<td>Proportion of exposed without symptoms after the incubation period</td>
</tr>
<tr>
<td>( h_1 )</td>
<td>Isolation rate for symptomatic class</td>
</tr>
<tr>
<td>( h_2 )</td>
<td>Isolation rate for asymptomatic class after confirmation</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Recovery rate from isolation class</td>
</tr>
<tr>
<td>( \delta )</td>
<td>COVID-19 induced mortality rate</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Natural death rate</td>
</tr>
</tbody>
</table>

Fig 1. Flow diagram for the model (1)-(5) where

\[
\lambda = \frac{(1-\psi)\beta(1+A+\mu)}{N} \]

\[
F = \begin{pmatrix}
0 & \beta(1-\psi) & \beta \varepsilon_1(1-\psi) & \beta \varepsilon_2(1-\psi) \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
V = \begin{pmatrix}
k_1 & 0 & 0 & 0 \\
-\alpha & k_2 & 0 & 0 \\
-(1-\varepsilon_2) & 0 & k_3 & 0 \\
0 & -h_1 & -h_2 & k_4
\end{pmatrix}
\]

\[
k_1 = \alpha + \mu, k_2 = h_1 + \mu + \delta, k_3 = h_2 + \mu, k_4 = \mu + \gamma + \delta
\]

RESULTS AND DISCUSSION

The disease-free equilibrium (DFE) of the model (1) – (5) is given by

\[
E_0 = (S_0^0, E_0^0, A_0^0, I_0^0, H_0^0) = \left( \frac{\Lambda}{\mu}, 0, 0, 0, 0 \right) \quad (10)
\]

Effective Reproduction Number \( R_c \): Local Stability of Disease-Free Equilibrium: The linear stability of \( E_0 \) can be established using the next generation method on the model (1) – (5). Thus, it follows that the matrices \( F \) and \( V \) which represents new infection and rate of transfer of individuals respectively are given by

It follows that the effective reproduction number \( R_c \) of the model (1)-(5) is given by

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$$R_c = \rho(FV^{-1}) = \frac{\beta(1-\psi)l_1\alpha}{k_1k_2} + \frac{\beta(1-\psi)(1-l_1)\alpha}{k_1k_3} + \frac{\beta(1-\psi)[l_1h_1k_3+(1-l_1)h_2k_2]I}{k_1k_3k_4}$$ (13)

Where \(\rho(FV^{-1})\) is the spectral radius of the matrix \(FV^{-1}\). Equation (13) is the effective reproduction number \(R_c\) of the COVID-19 model (1)-(5). By definition, it is the average number of secondary COVID-19 cases generated by an infected COVID-19 individual in a completely susceptible population.

The results below follow from Theorem 2 in van den Driessche and Watmough (2002).

**Lemma 2.** The disease-free equilibrium \(E_0\) is locally asymptotically stable when \(R_c < 1\) and unstable when \(R_c > 1\).

**Global Stability of the Disease-Free Equilibrium State Lemma 3.** If \(R_c \leq 1\), the disease-free equilibrium \(E_0\) of the model (2.1) - (2.5) is globally asymptotically stable.

**Proof:** Here, we follow a similar approach as outlined in the works of Ibio and Okoungahe (2016) to establish the global stability of the disease-free equilibrium.

From the model (1) – (5), we construct the linear Lyapunov function as follows:

\[ V = B_1E + B_2I + B_3A + B_4H \]

With Lyapunov derivative

\[
\frac{dV}{dt} = B_1E \frac{dE}{dt} + B_2I \frac{dI}{dt} + B_3A \frac{dA}{dt} + B_4H \frac{dH}{dt}
\]

\[ = \frac{\beta(1-\psi)(1+A+H)S}{k_1k_2} [l_1\alpha k_3 k_4 + (1 - l_1)\alpha k_2 k_4 + l_1 h_1 k_3 + (1 - l_1) h_2 k_2] - I[k_1 k_2 k_3 (k_4 + h_3)] - A[k_1 k_2 k_3 (k_4 + h_2)] - k_1 k_2 k_3 h_4 - H[k_1 k_2 k_3 k_4]
\]

\[ = k_1 k_2 k_3 k_4 \left[ \frac{\beta(1-\psi)}{k_1k_2} l_1\alpha k_3 k_4 + (1 - l_1)\alpha k_2 k_4 + l_1 h_1 k_3 + (1 - l_1) h_2 k_2 \right] - 1 \right] (I + A + H) \]

(14)

It follows from (14) that since \(S(t) \leq N(t)\) and \(N(t) \leq \frac{\Lambda}{\mu}\) for all \(t > 0\)

\[
\frac{dV}{dt} \leq k_1 k_2 k_3 k_4 (R_c - 1) (I + A + H)
\]

Hence, \(\frac{dV}{dt} \leq 0\) if \(R_c \leq 1\) with \(\frac{dV}{dt} = 0\) if and only \(I = A = H = 0\).

Therefore, \(\frac{dV}{dt}\) is a Lyapunov function in \(\mathbb{M}\) and it follows from Lasalle’s invariance principle (Lasalle and Lefschete, 1976) that every solution of the equations in (1) - (5) with initial conditions in \(\mathbb{M}\) converges to \(E_0\) as \(t \to \infty\), that is \((E(t), I(t), A(t), H(t)) \to (0,0,0,0)\) as \(t \to \infty\).

Substituting \(E = I = A = H = 0\) into (1) gives \(S(t) \to \frac{\Lambda}{\mu}\) as \(t \to \infty\).

Thus, \((S, E, I, A, H) \to (\frac{\Lambda}{\mu}, 0, 0, 0, 0)\) as \(t \to \infty\) for \(R_c \leq 1\), so the DFE, \(E_0\), is globally asymptotically stable in \(\mathbb{M}\) if \(R_c \leq 1\). Hence the proof.

Having established the proof for local and global stability of the disease-free equilibrium state, it is extremely important to establish the bounds for both the COVID-19 transmission rate and the public enlightenment campaign.

From (13), we obtain,

\[ \beta < \frac{1}{C + D + E} \]

(15)

Where \(C = \frac{(1-\psi)l_1\alpha}{k_1k_2}\), \(D = \frac{(1-\psi)(1-l_1)\alpha}{k_1k_3}\) and \(E = \frac{(1-\psi)l_1h_1k_3+(1-l_1)h_2k_2}{k_1k_2k_3k_4}\)

and \(\psi < \frac{F + G + J + 1}{F + G + J}\)

(16)

Where \(F = \frac{l_1\alpha}{k_1k_2}\), \(G = \frac{(1-l_1)\alpha}{k_1k_3}\)

and \(J = \frac{l_1h_1k_3+(1-l_1)h_2k_2}{k_1k_2k_3k_4}\)

From (15) an upper bound for the infection rate \(\beta\) was obtained.

This suggests that for COVID-19 to be put under control in the population, the infection rate \(\beta\) should not exceed the value given by the right-hand side of (15) while equation (16) suggests that for public enlightenment campaign about COVID-19 to be effective, the campaign rate should target a value greater than the right hand of (16).

**Sensitivity Analysis:** Sensitivity analysis is conducted on the effective reproduction number using the normalized forward sensitivity index in order to

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determine the parameters that are most sensitive and that should be targeted by way of intervention strategies.

Mathematically, the normalized forward sensitivity index of a parameter \( (\alpha) \) that depends on differentiable \( (R_c) \) (Williams et al, 2020) is expressed as:

\[
X_{R_c}^\alpha = \frac{\partial R_c}{\partial x} \times \frac{\alpha}{R_c} \quad (17)
\]

The results obtained from the analysis are presented in Table 3. The table shows that parameters with positive index will increase the endemicity of the pandemic while those with negative index will reduce the endemicity of the pandemic.

Table 3. Numerical Value of the Sensitivity Index

<table>
<thead>
<tr>
<th>S/NO</th>
<th>Parameter</th>
<th>Sensitivity index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \beta )</td>
<td>+1.00000</td>
</tr>
<tr>
<td>2</td>
<td>( \alpha )</td>
<td>+0.02534</td>
</tr>
<tr>
<td>3</td>
<td>( \beta_1 )</td>
<td>-0.01074</td>
</tr>
<tr>
<td>4</td>
<td>( \gamma )</td>
<td>-0.06352</td>
</tr>
<tr>
<td>5</td>
<td>( \lambda_1 )</td>
<td>-0.34187</td>
</tr>
<tr>
<td>6</td>
<td>( \lambda_2 )</td>
<td>-0.35016</td>
</tr>
<tr>
<td>7</td>
<td>( \delta )</td>
<td>-0.54515</td>
</tr>
<tr>
<td>8</td>
<td>( \psi )</td>
<td>-1.00000</td>
</tr>
</tbody>
</table>

Numerical Simulations: In this section, numerical simulation of the COVID-19 model (1)-(5) is performed using a set of reasonable estimated parameter values and initial conditions for the variables presented on Table 1. Some parameter values were assumed while others were gotten from other published articles as acknowledged in this paper. The simulation of the model (1)-(5) was done using Maple 17 mathematical software.

Table 4. Initial Conditions for the Variables

<table>
<thead>
<tr>
<th>S/NO</th>
<th>Variable at Initial Condition</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( S(0) )</td>
<td>1000</td>
<td>Assumed</td>
</tr>
<tr>
<td>2</td>
<td>( E(0) )</td>
<td>500</td>
<td>Assumed</td>
</tr>
<tr>
<td>3</td>
<td>( I(0) )</td>
<td>250</td>
<td>Assumed</td>
</tr>
<tr>
<td>4</td>
<td>( A(0) )</td>
<td>200</td>
<td>Assumed</td>
</tr>
<tr>
<td>5</td>
<td>( H(0) )</td>
<td>159</td>
<td>Assumed</td>
</tr>
</tbody>
</table>

Table 5. Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda )</td>
<td>20</td>
<td>Assumed</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.001-0.9</td>
<td>Assumed</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.5</td>
<td>Assumed</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.2</td>
<td>Williams et al (2020)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.1923</td>
<td>Olaniyi et al (2020)</td>
</tr>
<tr>
<td>( \lambda_1 )</td>
<td>0.5</td>
<td>Olaniyi et al (2020)</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>0.33604</td>
<td>Olaniyi et al (2020)</td>
</tr>
<tr>
<td>( \delta_2 )</td>
<td>0.19466</td>
<td>Olaniyi et al (2020)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.03</td>
<td>Assumed</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.005</td>
<td>Williams et al (2020)</td>
</tr>
</tbody>
</table>

Figure 2 shows that strengthening public enlightenment campaign at 90% level among the susceptible population will greatly reduce the number of persons who will be exposed or infected with COVID-19 pandemic. Figure 3 reveals that, administering proper treatment at 90% level will lead to the prompt recovery of individuals in the isolation centres.

Conclusion: A mathematical model to study the transmission dynamics of COVID-19 incorporating public enlightenment campaign is presented. The model was found to be locally asymptotically stable when \( R_c < 1 \) and globally asymptotically stable whenever \( R_c \leq 1 \). A bound for COVID-19 transmission rate and that of public enlightenment campaign was obtained. Finally, numerical simulations conducted on the model reveal that, 90% of treatment and public enlightenment campaign must be attained for COVID-19 to be put under control in the population.

REFERENCES

Modelling the transmission dynamics of COVID-19 using mathematical models...


