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Mathematical Modelling of Compressive Strength of Recycled Ceramic Tile Aggregate Concrete Using Modified Regression Theory

[1]ONYIA, ME; *[2]AMBROSE, EE; [1]OKAFOR, FO; [2]UDO, JJ

*[1]Department of Civil Engineering, University of Nigeria, Nsukka, Enugu State, Nigeria.
[2]Department of Civil Engineering, Akwa Ibom State University, Ikot Akpaden, Akwa Ibom State, Nigeria.

*Corresponding Author Email: edidiongambrose@aksu.edu.ng
Co-Authors Email: michael.onyia@unn.edu.ng; fidelis.okafor@unn.edu.ng; johnnyudo@aksu.edu.ng

ABSTRACT: At present, the large quantity of wastes generated by the ceramic industry is not reused in any significant quantity. Research has shown the feasibility of incorporating these wastes into concrete production. This will benefit both the ceramic and concrete industries. However, not much research data is available on the use of ceramic wastes as fine aggregate material compared to their use as coarse aggregate material. Moreover, there are presently no models for predicting the properties of ceramic waste aggregate concretes. In this study, a modified regression theory based on Taylor's series was adopted to formulate mathematical model for predicting compressive strength of concrete into which Recycled Ceramic Tile (RCT) is incorporated as fine aggregate. Preliminary tests on RCT indicate that it is a suitable fine aggregate material for concrete production. It has also been established that addition of RCT improves compressive strength of concrete and reduces concrete's workability. The formulated model is a function of the mix proportions of its constituents and its predicted responses are in good agreement with experimentally observed data. The model has been tested using student's t-test and analysis of variance and has been confirmed to be adequate and hence is validated.

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Recycling of industrial wastes and by-products is a sustainable way of preserving our environment while still meeting our needs. A good number of these wastes like fly ash, silica fumes, ground granulated blast-furnace slag, metakaolin, have been incorporated into concrete production with immense benefits (Ogirigbo and Black, 2017; Ambrose and Forth, 2018). Research has also confirmed the feasibility of incorporating even more others in concrete production. These materials include: recycled concrete aggregate (Tahar *et al.*, 2020; Paewchompo *et al.*, 2020), polystyrene aggregate (Tang *et al.*, 2008), periwinkle and palm kernel shell (Egamana and Sule, 2017), quarry sand (Ambrose *et al.*, 2018; Kaish *et al.*, 2021) and of recent, ceramic wastes (Bartosz *et al.*, 2016; Halicka *et al.*, 2013; Awoyera *et al.*, 2018; Ambrose *et al.*, 2021; 2021b; Elci, 2016). Unlike organic wastes like sawdust which are biodegradable (Etim *et al.*, 2017), ceramic wastes are non-biodegradable (Halicka, *et al.*, 2013; Zimbili *et al.*, 2015). They are generated by industries producing sanitary wares, electrical insulators, porcelain, ceramic tiles, bricks, etc. due to production error, cracks, off-standard products, size discrepancy, glazing fault among others. Some are generated during transportation and distribution and as construction and demolition wastes. These wastes are

*Corresponding Author Email: edidiongambrose@aksu.edu.ng

presently not recycled in any significant quantity (Awoyera *et al*., 2018; Elci, 2016; Zimbili, *et al*, 2014), rather, millions of tonnes are disposed of in landfills all over the world. Therefore, recycling of ceramic wastes into concrete production will benefit both the ceramics and construction industries. From literature, the use of ceramic wastes as aggregate in concrete production has produced concrete with comparable and even improved strength and durability properties compared to concrete with conventional aggregate. This has been reported for both when used as coarse aggregate (Daniyal and Akmad, 2015; Awoyera *et al*., 2016) and when used as fine aggregate (Elci, 2016; Alves *et al*., 2014; Aliabdo, 2014). However, literature also shows that the use of ceramic wastes aggregate reduces concrete workability (Awoyera *et al*., 2016, Alves *et al*., 2014, Halicka *et al*., 2013). The improved strength and durability properties and reduced workability of ceramic waste aggregate concrete is due to the intrinsic characteristics of ceramic waste aggregates. They are usually rough textured and irregularly shaped (Bartosz *et al*., 2016) and although these properties increase friction during mixing and placement of fresh concrete, they enhance aggregate/cement paste bonding and refine the pore structure of the resulting concrete (Medina *et al*., 2012).

Properties of concrete are determined by the proportions of mixed constituents. For this reason, concrete mix design is an important aspect of concrete production and concrete mix optimization is of even more importance. Concrete mix optimization requires careful selection and proportioning of concrete constituents with the aim of achieving the desired properties at optimum level. The traditional method of achieving this, which is based on trial-and-error, is no longer efficient and could require too many trial mixes, especially when dealing with concrete with many constituents (Simon, 2003).

A far more efficient and economical way of achieving mix optimization is the use of statistical experimental design methodology. The process of optimization of concrete mix using statistical and mathematical procedures for model building is generally referred to as response surface methodology (RSM). RSM majorly involves formulation of model equations for responses which are usually concrete properties, through well-designed experiments and optimization of these properties using the formulated model equations (Anya, 2015).

Design of experiment selects points for evaluation of a desired response, thereby relating the response with some independent variables. Model formulation in statistical methods usually requires fitting empirical models to experimental data for each response. Once these equations are established, concrete mix optimization can easily be carried out. Presently, there are no such model equations for concretes incorporating recycled ceramic as aggregate. The need for such is imminent. In this study, mathematical models were formulated using Osadebe's regression theory for predicting the compressive strength of concrete with partial or full replacement of river sand with recycled ceramic tile aggregates.

Osadebe's Regression Theory: Osadebe's regression model is a modified regression theory and a form of mixture experiment which is a general technique for modelling relationships between responses and components of a mixture. Mixture experiment techniques are mainly for cases where responses depend on the mass or volume proportions of individual components and not on their total mass or volume. This is typical of concrete properties.

Let us consider an arbitrary amount, S of a given mixture with q components. Let the proportion of the ith component of the mixture be S_i. Then from the principle of absolute volume (or mass).

$$S_1 + S_2 + \ldots + S_q = S \quad \text{or}$$

$$\frac{S_1}{S} + \frac{S_2}{S} + \ldots + \frac{S_q}{S} = 1 \qquad (1)$$

Where S_i/S is the proportion of the ith constituent of the mixture.

$$\text{Let} \quad \frac{S_i}{S} = Z_i \qquad (2)$$

Therefore, substituting Equation 2 into Equation 1 yields:

$$Z_1 + Z_2 + \ldots + Z_q = \sum_{i=1}^{q} Z_i = 1 \qquad (3)$$

Regression model equation: In Osadebe's regression model, a response, $\hat{y}$ is expressed as a function of the mixture proportions, Z_i. Using Taylor's series with the assumption that a response function, $\hat{y}$ = F(Z) is continuous and differentiable with respect to its predictors, Z_i; Osadebe expanded the response in the neighbourhood of a chosen point $Z^{(0)} = (Z_1^{(0)}, Z_2^{(0)}, \ldots, Z_q^{(0)})^T$ as follows (Anya, 2015; Mama and Osadebe, 2011; Okere et al., 2011; Onwuka et al., 2011):

$$\hat{y} = F(Z) = F(Z^{(0)}) + \sum_{i=1}^{q} \frac{\partial F(Z^{(0)})}{\partial Z_i}(Z_i - Z^{(0)}) + \frac{1}{2!}\sum_{i=1}^{q-1}\sum_{j=1}^{q} \frac{\partial^2 F(Z^{(0)})}{\partial Z_i \partial Z_j}(Z_i - Z_i^{(0)})(Z_j - Z_j^{(0)}) + \frac{1}{2!}\sum_{i=1}^{q} \frac{\partial^2 F(Z^{(0)})}{\partial^2 Z_i^2}(Z_i - Z^{(0)}) + \ldots \quad (4)$$

For the purpose of convenience and without loss of generality of the formulation, the origin can be taken as $Z^{(0)} = 0$, which implies that:

$$Z_1^{(0)} = 0;\ Z_2^{(0)} = 0;\ \ldots, Z_q^{(0)} = 0 \quad (5)$$

Let: $b_0 = F(0);\ b_i = \frac{\partial F(0)}{\partial Z_i};\ \ b_{ij} = \frac{\partial^2 F(0)}{\partial Z_i \partial Z_j};$

and $b_{ii} = \frac{\partial^2 F(0)}{\partial Z_i^2}$ (6)

Substituting Equation 6 into Equation 4 gives:

$$\hat{y} = b_0 + \sum_{i=1}^{q} b_i Z_i + \sum_{1 \le i \le j \le q}^{q} b_{ij} Z_i Z_j + \sum_{i=1}^{q} b_{ii} Z_i^2 \quad (7)$$

The number of constant coefficients, N in the polynomial in Equation 7 is given as:

$$N = C_n^{q+n} = \frac{(q+n)!}{(q)!(n)!} \quad (8)$$

Where n is the degree of the polynomial of the response function. However, taking advantage of Equation 1, the number of constant coefficients in Equation 7 can be reduced to that in Equation 9.

$$N = C_n^{q+n-1} = \frac{(q+n-1)!}{(q-1)!(n)!} \quad (9)$$

The reduction is as follows. Multiplying Equation 3 by b_0 gives the expression in Equation 10

$$b_0 Z_1 + b_0 Z_2 + \ldots + b_0 Z_q = b_0 \quad (10)$$

Multiplying Equation (3) successively by $Z_1, Z_2, \ldots, Z_q$ and rearranging the terms gives:

Substituting Equations (10) and (11) into Equation (7) and simplifying gives equation 12:

$$\begin{aligned} Z_1^2 &= Z_1 - Z_1 Z_2 - \ldots\ldots\ldots - Z_1 Z_q \\ Z_2^2 &= Z_2 - Z_1 Z_2 - \ldots\ldots\ldots - Z_2 Z_q \\ &\ldots\ldots\ldots\ \ldots\ldots\ldots\ \ldots\ldots\ldots\ \ldots\ldots\ldots\ \ldots\ldots \\ Z_q^2 &= Z_q - Z_1 Z_q - \ldots\ldots\ldots - Z_{q-1} Z_q \end{aligned} \quad (11)$$

$$\hat{y} = \sum_{i=1}^{q} \beta_i Z_i + \sum_{i \le i \le j \le q}^{q} \beta_{ij} Z_i Z_j \quad (12)$$

Where: $\beta_i = b_0 + b_i + b_{ii}$ and

$$\beta_{ij} = b_{ij} - b_{ii} - b_{ij} \quad (13)$$

Equation (12) is Osadebe's regression model equation. $\hat{y}$ is the response function while Z is the predictor which are the proportions of the mixture components and β_i is the model coefficient.

For a 5-component mixture ($q = 5$) adopted in this study, Osadebe's regression model equation is given as:

$$\begin{aligned} \hat{y} = {} & \beta_1 Z_1 + \beta_2 Z_2 + \beta_3 Z_3 \\ & + \beta_4 Z_4 + \beta_5 Z_5 + \beta_{12} Z_1 Z_2 \\ & + \beta_{13} Z_1 Z_3 + \beta_{14} Z_1 Z_4 + \beta_{15} Z_1 Z_5 \\ & + \beta_{23} Z_2 Z_3 + \beta_{24} Z_2 Z_4 + \beta_{25} Z_2 Z_5 \\ & + \beta_{34} Z_3 Z + \beta_{35} Z_5 Z_5 + \beta_{45} Z_4 Z_5 \end{aligned} \quad (14)$$

Regression model coefficients: For Osadebe's model equation, the minimum number of experiments (design points) to determine the coefficients of the model is given by N as in Equation 9. Let the k^{th} response be $\hat{y}^k$ and the vector corresponding to the set of predictors at point k is given as:

$$Z^{(k)} = \left\{ Z_1^{(k)},\ Z_2^{(k)},\ \ldots \ldots,\ Z_q^{(k)} \right\} \quad (15)$$

Substituting Equation 15 into Equation 12, yields:

$$\hat{y}^{(k)} = \sum_{i=1}^{q} \beta_i Z_i^{(k)} + \sum_{i \le i \le j \le q}^{q} \beta_{ij} Z_i^{(k)} Z_j^{(k)} \qquad k = 1, 2,\ \ldots, N \quad (16)$$

By substituting the predictor vector at each of the N design points, Equation 16 can be generalized in matrix form as:

$$[Z][\beta]=[\hat{y}] \quad (17)$$

Where: $[Z]$ is an $N \times N$ matrix whose elements are the mixture component proportions; $[\beta]$ is a column matrix whose elements are estimates of the model coefficients; and $[\hat{y}]$ is a column matrix whose elements are experimental responses at the various design points. Since $[Z]$ can easily be determined and $[\hat{y}]$ can be determined through experiments, Equation 17 can be rearranged to give Equation 18. By solving Equation 18, the model coefficients can be determined.

$$[\beta]=[Z]^{-1}[\hat{y}] \quad (18)$$

MATERIALS AND METHODS

Materials: Concrete samples produced for laboratory experiments in this study were made of five constituents, namely: cement, water, river sand (RS), recycled ceramic tiles (RCT) and granite chippings. RS and RCT were used as fine aggregates, while granite chippings were used as coarse aggregate (CA). Cement used was Portland Limestone cement (strength class 32.5R) manufactured by United Cement Company of Nigeria (Unicem).

The cement conformed to NIS 444-1 (2008) and was acquired in 50kg bags from a dealer at Ikot Akpaden, Akwa Ibom State. RS was obtained from a mining site at Ikot Ekong, Akwa Ibom State while granite chippings were from a quarry in Akamkpa, Cross River State all in Nigeria.

RCT used were derived from recycled floor and wall tiles that had passed through complete manufacturing process. These tiles were either broken or cracked during transportation and distribution process and were considered as wastes (Fig. 1(a)).

They were obtained from a tile dealer in Uyo, broken into smaller pieces and crushed into the required size (Fig. 1(b)) using a hammer mill.

Particle size distribution test, specific gravity test and bulk density test were carried out on the aggregates used while X-ray fluorescence test was carried out on cement and RCT.

Design of experiment: 15 different mixes corresponding to 15 design points were required to formulate Osadebe's regression models based on Equations 9 and 14. These mix ratios were selected based on authors' experience on concrete mix design and were further transformed into component proportions.

Methods: The methodology for achieving the aim of this study involved preparation and characterization of materials, design of experiment, production and test of samples, formulation of regression models and validation of models.

Fig. 1 Recycled ceramic tiles before (a) and after (b) crushing

This is presented in Table 1, while Table 2 presents additional mix ratios that would be used as control points to validate the models. From Table 1, elements of *Z* matrix and inverse *Z* matrix (Z^{-1}) were generated as presented in Table 3 and Table 4 respectively. These would be used to generate coefficients of the model using Equation 18.

Sample preparation and testing: Batching of concrete components for preparation of test samples was carried out by weight using the real component ratios in Tables 1 and 2. Mixing, compaction and curing of concrete samples were carried out in accordance with BS EN 12390-2 (2009).

For each fresh mix workability was measured in duplicate using slump test and in accordance with BS EN 12350-2 (2009). Three concrete cubes of *100mm* x *100mm* x *100 mm* were prepared for each of the 21 mixes. Specimens were left in the mold for about 24 hours after casting before being demolded and cured by immersion in water till test date (see Fig. 2). Concrete cube samples were tested in triplicate for compressive strength on the 28th day after casting using a compression testing machine conforming to BS EN 12390-4 (2009) and having a test range of 0 – 2000kN. Maximum load at failure was recorded for each test and compressive strength was computed by dividing failure load by cross-sectional area of sample.

Table 1 Components in real ratios and proportions for model calibration

N	*Components in Real Ratios*					*Components in Proportions*				
	Water	Cement	RS	RCT	CA	Water	Cement	RS	RCT	CA
	S_1	S_2	S_3	S_4	S_5	Z_1	Z_2	Z_3	Z_4	Z_5
1	0.550	1	0.75	0.75	3.00	0.0909	0.1653	0.1240	0.1240	0.4959
2	0.500	1	0.50	0.75	2.50	0.0952	0.1905	0.0952	0.1429	0.4762
3	0.500	1	1.50	0.00	3.00	0.0833	0.1667	0.2500	0.0000	0.5000
4	0.625	1	1.25	0.75	3.75	0.0847	0.1356	0.1695	0.1017	0.5085
5	0.400	1	1.00	0.00	2.00	0.0909	0.2273	0.2273	0.0000	0.4545
6	0.450	1	1.25	0.00	2.50	0.0865	0.1923	0.2404	0.0000	0.4808
7	0.450	1	0.00	1.00	2.00	0.1011	0.2247	0.0000	0.2247	0.4494
8	0.600	1	0.00	1.50	3.00	0.0984	0.1639	0.0000	0.2459	0.4918
9	0.650	1	2.50	0.00	4.50	0.0751	0.1156	0.2890	0.0000	0.5202
10	0.525	1	0.00	1.25	2.50	0.0995	0.1896	0.0000	0.2370	0.4739
11	0.550	1	1.25	0.50	3.25	0.0840	0.1527	0.1908	0.0763	0.4962
12	0.415	1	0.50	0.50	2.00	0.0940	0.2265	0.1133	0.1133	0.4530
13	0.575	1	2.00	0.00	3.75	0.0785	0.1365	0.2730	0.0000	0.5119
14	0.475	1	0.75	0.50	2.50	0.0909	0.1914	0.1435	0.0957	0.4785
15	0.525	1	1.75	0.00	3.25	0.0805	0.1533	0.2682	0.0000	0.4981

Fig. 2 Concrete samples (a) before demolding (b) during curing

Table 2 Components in real ratios and proportions at control point for model validation

C	*Components in Real Ratios*					*Components in Proportions*				
	Water	Cement	RS	RCT	CA	Water	Cement	RS	RCT	CA
	S_1	S_2	S_3	S_4	S_5	Z_1	Z_2	Z_3	Z_4	Z_5
C_1	0.510	1	1.25	0.25	2.95	0.0856	0.1678	0.2097	0.0419	0.4950
C_2	0.585	1	1.75	0.25	3.70	0.0803	0.1373	0.2402	0.0343	0.5079
C_3	0.485	1	0.50	0.75	2.45	0.0935	0.1929	0.0964	0.1446	0.4725
C_4	0.520	1	1.00	0.50	2.90	0.0878	0.1689	0.1689	0.0845	0.4899
C_5	0.560	1	0.50	1.00	2.95	0.0932	0.1664	0.0832	0.1664	0.4908
C_6	0.460	1	1.00	0.25	2.45	0.0891	0.1938	0.1938	0.0484	0.4748

RESULTS AND DISCUSSION

Materials characterization: Fig. 3 presents particle size distribution curves of the two fine aggregate materials used in laboratory experiments while that of coarse aggregate is presented in Fig. 4. Test results of physical properties of the three aggregate materials are presented in Table 5 while chemical composition of cement and RCT are presented in Table 6. From Table 5, it could be seen that specific gravity and bulk density of RCT are lower than those of RS. These values are within the range found in literature (Higashiyama *et al.*, 2012; Binci, 2007; Awoyera *et al.*, 2016) and is an indication that the former is a lighter fine aggregate compared to the latter. Fig. 3 shows that both RS and RCT are suitable fine aggregate materials for concrete production as both materials satisfy the general grading requirement of BS 882: 1992 (Neville, 2011). Moreover, RS falls within the limits of medium grading while RCT falls within coarse grading.

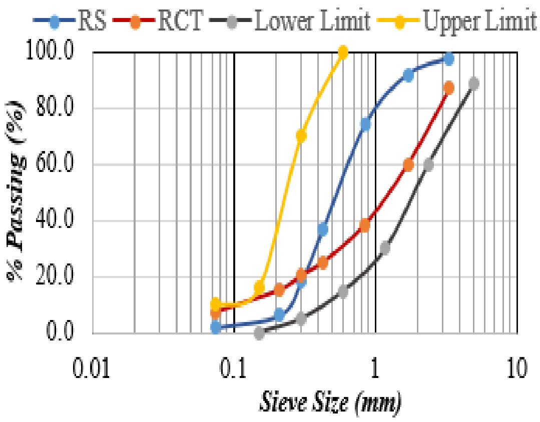

Fig. 3: Particle size distribution curve for fine Aggregates

Table 3 Elements of Z matrix for Osadebe's model

Z_1	Z_2	Z_3	Z_4	Z_5	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_1Z_5	Z_2Z_3	Z_2Z_4	Z_2Z_5	Z_3Z_4	Z_3Z_5	Z_4Z_5
0.0909	0.1653	0.1240	0.1240	0.4959	0.0150	0.0113	0.0113	0.0451	0.0205	0.0205	0.0820	0.0154	0.0615	0.0615
0.0952	0.1905	0.0952	0.1429	0.4762	0.0181	0.0091	0.0136	0.0454	0.0181	0.0272	0.0907	0.0136	0.0454	0.0680
0.0833	0.1667	0.2500	0.0000	0.5000	0.0139	0.0208	0.0000	0.0417	0.0417	0.0000	0.0833	0.0000	0.1250	0.0000
0.0847	0.1356	0.1695	0.1017	0.5085	0.0115	0.0144	0.0086	0.0431	0.0230	0.0138	0.0689	0.0172	0.0862	0.0517
0.0909	0.2273	0.2273	0.0000	0.4545	0.0207	0.0207	0.0000	0.0413	0.0517	0.0000	0.1033	0.0000	0.1033	0.0000
0.0865	0.1923	0.2404	0.0000	0.4808	0.0166	0.0208	0.0000	0.0416	0.0462	0.0000	0.0925	0.0000	0.1156	0.0000
0.1011	0.2247	0.0000	0.2247	0.4494	0.0227	0.0000	0.0227	0.0454	0.0000	0.0505	0.1010	0.0000	0.0000	0.1010
0.0984	0.1639	0.0000	0.2459	0.4918	0.0161	0.0000	0.0242	0.0484	0.0000	0.0403	0.0806	0.0000	0.0000	0.1209
0.0751	0.1156	0.2890	0.0000	0.5202	0.0087	0.0217	0.0000	0.0391	0.0334	0.0000	0.0601	0.0000	0.1504	0.0000
0.0995	0.1896	0.0000	0.2370	0.4739	0.0189	0.0000	0.0236	0.0472	0.0000	0.0449	0.0898	0.0000	0.0000	0.1123
0.0840	0.1527	0.1908	0.0763	0.4962	0.0128	0.0160	0.0064	0.0417	0.0291	0.0117	0.0758	0.0146	0.0947	0.0379
0.0940	0.2265	0.1133	0.1133	0.4530	0.0213	0.0106	0.0106	0.0426	0.0257	0.0257	0.1026	0.0128	0.0513	0.0513
0.0785	0.1365	0.2730	0.0000	0.5119	0.0107	0.0214	0.0000	0.0402	0.0373	0.0000	0.0699	0.0000	0.1398	0.0000
0.0909	0.1914	0.1435	0.0957	0.4785	0.0174	0.0130	0.0087	0.0435	0.0275	0.0183	0.0916	0.0137	0.0687	0.0458
0.0805	0.1533	0.2682	0.0000	0.4981	0.0123	0.0216	0.0000	0.0401	0.0411	0.0000	0.0763	0.0000	0.1336	0.0000

Table 4 Elements of inverse Z matrix (Z^{-1})

532400	120273	209455	0	63360	-117993	-712890	-757731	0	1335630	0	637927	0	-1310430	0
15972	3608	5204	0	1030	-1593	-20199	-21244	0	37398	0	19138	0	-39313	0
0	0	7200	0	1394	-6490	0	0	2394	0	0	0	-8585	0	4087
639	-120	4399	-1740	324	-2045	2535	2246	2394	-4452	2746	-1418	-6868	0	1362
6921	2005	8771	1741	2605	-8456	-12832	-13423	599	24041	-2059	10632	-4292	-19656	3406
-734712	-165976	-280767	0	-80758	147904	976659	1036738	0	-1827142	0	-880340	0	1808393	0
-514298	-108245	-149629	34810	-44479	47787	627343	670051	-2394	-1175354	-41186	-574135	-25755	1218700	36785
-593732	-119752	-170915	36550	-51857	54749	719860	772804	-2394	-1353438	-43932	-657774	-27472	1397792	39510
-661773	-154350	-300960	-19146	-92154	196851	925331	981153	-599	-1733648	22653	-818673	21462	1657694	-23842
-19592	-6014	-22569	-6962	-5201	18879	37308	38780	-2694	-69453	8237	-31896	13736	57659	-10218
-6442	-3748	-15912	-5222	-3040	13196	18535	18531	-2694	-33836	5492	-15594	12019	26209	-7493
-1065	200	-1139	1740	-146	669	-2733	-2801	-449	5343	-2059	1063	1717	0	-341
-2822	-241	-707	0	-282	236	3168	3599	0	-6233	0	-2835	0	6115	0
-8731	-3207	-31313	-5222	-7885	30835	21387	22191	-5387	-40069	6178	-17011	25755	28829	-16349
160	-1684	-23956	-2611	-5069	22419	5545	5548	-5387	-10240	2059	-4253	23179	6552	-12262

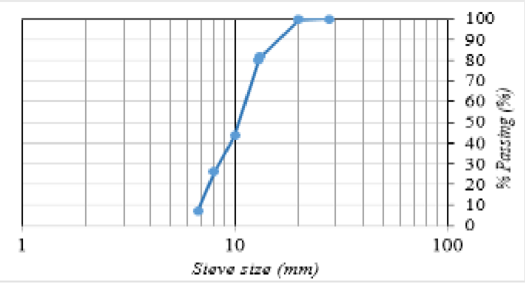

Fig. 4: Particle size distribution curve for CA

Fig. 4, shows that CA is also suitable for concrete production because it satisfies the grading requirement of BS 882: 1992 for coarse aggregate of 20 to 5mm nominal size (Neville, 2011). Again, from Table 5, uniformity coefficient (C_u) and gradation coefficient (C_c) values indicate that RCT has a larger range of particle sizes than RS and CA and can be classified as well graded because its C_u value is greater than 4 and its C_c is within the range of 1 and 3 (Ambrose *et al*, 2019)

Table 5: Physical properties of aggregates

Property	Sand	RCT	CA
Specific gravity	2.61	2.40	2.39
Bulk density (kg/m^3)	1635	1373	1386
Uniformity coefficient (C_u)	2.85	17	1.84
Gradation coefficient (C_c)	0.73	1.78	0.87

Table 6: Chemical composition of cement and RCT

Compound	% Composition by mass	
	Cement	RCT
Iron Oxide (Fe_2O_3)	2.25	3.07
Aluminum Oxide (Al_2O_3)	4.73	17.50
Silicon dioxide (SiO_2)	19.84	66.13
Calcium Oxide (CaO)	70.32	5.70
Manganese Oxide (MnO)	0.01	0.58
Magnesium Oxide (MgO)	1.47	2.14
Zinc Oxide (ZnO)		0.42
Sulfur trioxide (SO_3)	0.03	-
Sodium Oxide (Na_2O)	0.08	0.09
Potassium Oxide (K_2O)	0.72	1.02
LOI (Loss of Ignition)	1.01	3.30

Experimental responses

Workability: Results of workability of fresh concrete measured in terms of slump height is presented in Table 7 Slump heights for the 21 mixes range from 5mm to 82.5mm representing very low to very high workability according to Neville (2011). The results show that the level of replacement of RS with RCT affects concrete workability. For instance, in comparing mix No 5 with mix No 7, the two mix compositions are similar in terms of cement, fine aggregate and coarse aggregate content; except that the latter uses 100% RCT as fine aggregate while the former uses 100% RS. Surprisingly, although mix No 7 has a higher water-cement ratio, it still has a far lower slump (13.75mm) compared to mix No 5 (78.75mm). Several literatures have also reported this trend (Halicka *et al.*, 2013; Awoyera *et al.*, 2016; Alves *et al.*, 2014) and link it to the intrinsic properties of ceramic waste aggregates.

Compressive strength: Table 7 also presents average characteristic compressive strength results for the 15 design points and 6 control points. The results show that replacement of RS with RCT improves compressive strength of resulting concrete. This can be demonstrated by again comparing mix No 5 with mix No 7. Both use the same mix ratio of 1:1:2 (cement: fine aggregate: Coarse aggregate), but although mix No 7 uses a water/cement ratio of 0.45 while mix No 5 uses a water/cement ratio of 0.4, its compressive strength is far higher than that of the latter because in the former mix, RCT was used as 100% fine aggregate. It has been reported that introduction of ceramic waste aggregate refines concrete pore structure and strengthen aggregate/cement paste bonding (Medina *et al.*, 2012) and this is obviously the explanation for improved strength of RCT concrete. Moreover, there is a relationship between concrete strength and aggregate's shape, size and surface (Mkpaidem *et. al*, 2022). Therefore, the irregular and angular shape of RCT combined with its rough texture improves aggregate-cement paste bonding and hence, strength.

Model formulation: Equation 14 was adopted to formulate Osadebe's regression model for predicting characteristic compressive strength of RCT concrete. Equation 18 was used to obtain model coefficients, using the inverse ***Z*** matrix ($\boldsymbol{Z}^{-1}$) in Table 4 and compressive strength experimental responses in Table 7. Solving Equation 18 simultaneously gave the following model coefficients:

β_1 = 3077601.12, β_2 = 84843.05, β_3 = 21913.44, β_4 = -17.57, β_5 = 70959.33, β_{12} = -4192299.31, β_{13} = -2606714.28, β_{14} = -3022129.62, β_{15} = -4079379.81, β_{23} = -189975.34, β_{24} = -94674, β_{25} = 1273.44, β_{34} = -15349.95, β_{35} = -169764.03, β_{45} = -80504.12.

Therefore, the resulting model according to Equation 14 is given as:

$$\begin{aligned}\hat{y} = {} & 3077601.12Z_1 + 84843.05Z_2 + 21913.44Z_3 \\ & - 17.57Z_4 + 70959.33Z_5 - 4192299.31Z_1Z_2 \\ & - 2606714.28Z_1Z_3 - 3022129.62Z_1Z_4 \\ & - 4079379.81Z_1Z_5 - 189975.34Z_2Z_3 \\ & - 94674Z_2Z_4 + 1273.44Z_2Z_5 \\ & - 15349.95Z_3Z_4 - 169764Z_3Z_5 \\ & - 80504.12Z_4Z_5 \end{aligned} \quad (19)$$

Model validation and test of adequacy: Tests of adequacy for the proposed model was evaluated using student's t-test and analysis of variance using responses at the six control points. Table 8 shows experimental results at the control points with their corresponding model predicted responses and percentage differences. The small values of percentage differences already show that model predicted values For the student's *t*-test and analysis of variance, the null hypothesis was that there is no significant difference between the experimental and model predicted values. On the other side, the alternative hypothesis was that there is a significant difference between the experimental values and model predicted values.

Table 7 Compressive strength and slump tests results including percentage replacement of RS with RCT for each mix

N	Component in Real Ratios					% Replacement Of sand with RCT	Slump (mm)	Compressive Strength (N/mm^2)
	Water	Cement	RS	RCT	CA			
	S_1	S_2	S_3	S_4	S_5			
1	0.550	1	0.75	0.75	3.00	50.00	40.00	33.844
2	0.500	1	0.50	0.75	2.50	60.00	45.00	31.061
3	0.500	1	1.00	0.00	3.00	0.000	60.00	28.186
4	0.625	1	1.25	0.75	3.75	37.50	47.50	23.766
5	0.400	1	1.00	0.00	2.00	0.000	78.75	35.271
6	0.450	1	1.25	0.00	2.50	0.000	47.50	28.374
7	0.450	1	0.00	1.00	2.00	100.0	13.75	42.291
8	0.600	1	0.00	1.50	3.00	100.0	5.000	35.436
9	0.650	1	2.50	0.00	4.50	0.000	17.50	20.454
10	0.525	1	0.00	1.25	2.50	100.0	10.00	39.138
11	0.550	1	1.25	0.50	3.25	28.60	15.00	25.197
12	0.425	1	0.50	0.50	2.00	50.00	5.000	40.417
13	0.575	1	2.00	0.00	3.75	0.000	10.00	23.872
14	0.475	1	0.75	0.50	2.50	40.00	40.00	33.977
15	0.525	1	1.75	0.00	3.25	0.000	70.00	26.891
Control Points for Model Validation								
C_1	0.510	1	1.25	0.25	2.95	16.7	47.5	27.294
C_2	0.585	1	1.75	0.25	3.70	12.5	22.5	25.427
C_3	0.485	1	0.50	0.75	2.45	60.0	10.0	34.644
C_4	0.520	1	1.00	0.50	2.90	33.3	30.0	28.397
C_5	0.560	1	0.50	1.00	2.95	66.7	15.0	30.511
C_6	0.460	1	1.00	0.25	2.45	20.0	52.5	31.286

Student t-test: A two-tail student *t*-test was carried out, using an alpha level of 0.05 to compare experimental and model predicted results. From the test result in Table 9, *t* stat value was 1.281114628 and is less than the *t* critical (two-tail) value which was 2.570581836. Thus, the null hypothesis cannot be rejected and this implies that there is no significant difference between the experimental results and model predicted results at all the control points.

Table 8: Compressive strength tests results at control points including percentage replacement of RS

Control Points	Experimental Response (N/mm^2)	Model Response (N/mm^2)	Difference (%)
C_1	27.294	28.740	5.30
C_2	25.427	23.384	8.03
C_3	34.644	31.813	8.17
C_4	28.397	27.877	1.83
C_5	30.511	31.397	2.91
C_6	31.286	28.486	8.95

Analysis of variance: Table 10 presents analysis of variance results carried out at 0.05 alpha level. This was to further test for adequacy of the proposed model. From the test result, value of *F* was 0.28867 and is less than *Fcrit* which was 4.96460.

Table 9: T-Test: paired two samples for mean

	Experimental	Model
Mean	29.59316667	28.61616667
Variance	10.65858937	9.181398167
Observations	6.000000000	6.000000000
Pearson Correlation	0.826411172	
Hypothesized Mean Difference	0.000000000	
df	5.000000000	
t Stat	1.281114628	
P(T<=t) one-tail	0.128176321	
t Critical one-tail	2.015048373	
P(T<=t) two-tail	0.256352643	
t Critical two-tail	2.570581836	

Thus, the null hypothesis cannot be rejected. Moreover, *P*-value for variation between groups was 0.60282 which is greater than 0.05 and is an indication that there is an insignificant variation between the experimental and model predicted results.

With the results of student *t*-test and analysis of variance, it has been established that the proposed model is adequate for predicting characteristic compressive strength of concrete incorporating RCT as full or partial replacement for RS as fine aggregate.

Table 10: Analysis of variance (single factor)

Groups	*Count*	*Sum*	*Average*	*Variance*		
Experimental	6	177.559	29.59317	10.6585		
Model	6	171.697	28.61617	9.18139		
Source of Variation	*Sum of Square*	Degree of Freedom	*Mean Square*	*F-value*	*P-value*	*F crit*
Between Groups	2.86359	1.0	2.863587	0.28867	0.60282	4.96460
Within Groups	99.19994	10	9.91999			
Total	102.0635	11				

Conclusion: In this study, a modified regression model based on Taylor's series has been applied to formulate mathematical model for predicting and optimization of compressive strength of concrete incorporating RCT as fine aggregate. This model can predict characteristic compressive strength of RCT concretes using their mix proportions. Addition of RCT has been found to improve compressive strength of concrete although it's incorporation also reduces concrete workability at fresh state.

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