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A Mathematical Model for the Dynamics of Zika Virus via Homotopy Perturbation Method

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ABSTRACT: Zika virus is a member of the *Flavivirus genus* of the *Flaviviridae* family, which includes other globally relevant human's pathogens such as dengue virus, yellow fever virus, West Nile virus and tick-borne encephalitis virus. In this paper, a deterministic mathematical model of Zika virus was formulated using ordinary differential equations with two control strategies: treatment for humans and insecticide spray for mosquitoes. Homotopy Perturbation Method was used to obtain the approximate solution of the model. From the result obtained, 59% effective administration of insecticide spray proved effective which showed a great reduction in the infected humans as well as infected vector population. Numerical results were offered in the form of Graphs. This research work contributes to new field of knowledge included to the dynamics of Zika virus in population's dynamics with the application of Homotopy Perturbation Method and can be further extended to study the pattern of Zika associated diseases that pose a significant public health risk. © JASEM

https://dx.doi.org/10.4314/jasem.v21i4.1

Keywords: Homotopy Perturbation method, Zika virus, Modelling, Numerical Simulations

Zika virus is a mosquito-borne *flavivirus* diseases that was first identified in Uganda in 1947 in monkeys through a network that monitored yellow fever. It was later identified in humans in 1952 in Uganda and Tanzania. Outbreaks of Zika virus disease have been recorded in Africa. America. Asia and the Pacific. From the 1960s to 1980s, human infections were found across Africa and Asia, typically accompanied by mild illness. The first large outbreak of disease caused by Zika infection was reported from the Island of Yap (Federated States of Micronesia) in 2007. In July 2015 Brazil reported an association between Zika virus infection and Guillain-Barré syndrome. In October 2015 Brazil reported an association between Zika virus infection and microcephaly (WHO, 2016). Zika virus (ZIKV) is primarily transmitted to people through the bite of an infected mosquito from the Aedes genus, mainly Aedes aegypti in tropical regions. Aedes mosquitoes usually bite during the day, peaking during early morning and late afternoon/evening. This is the same mosquito that transmits dengue, chikungunya and yellow fever. Sexual transmission of Zika virus is also possible. Other modes of transmission such as blood transfusion are being investigated (Musso et al., 2015). The incubation period of Zika virus disease is not clear, but is likely to be a few days. The

symptoms are similar to other infections such as dengue, fever, skin rashes, conjunctivitis, muscle and joint pain, malaise, and headache. These symptoms are usually mild and last for 2-7 days. The recent ZIKV incidence in Brazil and French Polynesia possibly compels World Health Organization (WHO) to declare a Public Health Emergency of International Concern in response to the clusters of microcephaly and other neurological disorders. Apart from the major outbreak in French Polynesia, ZIKV incident shows 42 Guillain-Barre syndrome cases between March 2014 and May 2015 in same region and severe brain lesions were reported (Ebenezer and Kazeem, 2016). Consequently, at this time there were no recognized vaccines or drugs to cure the ZIKV infection.

The Homotopy Analysis Method is one of the wellknown methods to solve the linear and non-linear equations. In the last decade, the idea of Homotopy was combined with perturbation. The fundamental work was done by (He, 2000). This method involves a free parameter, whose suitable choice results into fast convergence. He introduced Homotopy Perturbation Method and its application (He, 2006, 2008, 2009). These methods are independent of the assumption of small parameter as well as they cover all the advantages of the perturbation method. Muhammad *et al.* (2014) developed a model of SIR by Homotopy Perturbation Method (HPM).The models were solved by NSFD, RK4 and compared the HPM solution with NSFD and RK4, it found that the HPM solution have a good agreement with other standard method of NSFD and RK4. Abubakar *et al.* (2013) built a model of SIR for infectious virus dynamics, using Homotopy Perturbation Method (HPM). The analytical solution was obtained to present the model graphically, which gives the better understanding of the infectious virus dynamics. Their result shows that the reductions of disease frequency match or even supersede the infection rate.

A lot of mathematical models have been built up as a tool for investigating the dynamics transmission of infectious diseases. Ebenezer and Kazeem, (2016) formulated a mathematical model for optimal control strategies of Zika virus to examine the effect of mass treatment and insecticide, in other to minimize the number of Zika infected hosts and vectors with optimal cost of mass treatment and insecticide. Adam *et al.* (2014) developed a mathematical model to simulate vector-borne transmission for people and mosquitoes, the model incorporated delays as a result of the intrinsic (human) and extrinsic (vector) incubation periods. Onuorah *et al.* (2016) developed

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a deterministic model for the dynamics of Zika virus. The model incorporates vital dynamics for both the human and vector compartments. Bawa *et al.* (2016) developed a deterministic model of tuberculosis disease using HPM to analyze the rate of TB infections on human populations. Their result revealed that TB disease can be drastically reduced with 65% effectiveness in vaccine administration. The objective of this paper is to develop a mathematical model for the dynamics of zika virus using HPM with two control strategies that is, treatment for humans and insecticide spray for mosquitoes. The impact of these two strategies is to be further examined.

MATERIALS AND METHODS

The clinical symptoms of Zika virus was used to determine the model of Zika virus represented by ordinary differential equations (1) to (5). The deterministic model of Zika virus with five compartments and control strategies of both humans and vectors were formulated. The positions of ordinary differential equations of the non-linear system are as follow: The data used to test the model is on table 1 and the systems of non-linear equations are as follow:

$$\frac{dS_h}{dt} = \wedge_h - (\mu_h + \beta_h I_v) S_h \tag{1}$$

$$\frac{dI_h}{dt} = \beta_h S_h I_v - (\mu_h + \mu_0 + \lambda + \delta_h) I_h$$
⁽²⁾

$$\frac{dR_h}{dt} = (\lambda + \delta_h)I_h - \mu_h R_h \tag{3}$$

$$\frac{dS_{\nu}}{dt} = \wedge_{\nu} - \beta_{\nu}S_{\nu}I_{h} - (\mu_{\nu} + \delta_{\nu})S_{\nu}$$
⁽⁴⁾

$$\frac{dI_{\nu}}{dt} = \beta_{\nu} S_{\nu} I_{h} - (\mu_{\nu} + \delta_{\nu}) I_{\nu}$$
⁽⁵⁾

where; S_h represents susceptible humans, S_v is susceptible vectors, I_h infectious humans, I_v infectious vectors, R_h is recovered humans, μ_h natural death rate of humans, μ_v natural death of vectors, δ_v represent death rate from insecticide, δ_h represent recovery rate from treatment, β_h the spread rate from humans to mosquitoes, β_v spread rate from mosquitoes to humans, \wedge_h represents recruitment number of susceptible

humans, \wedge_{ν} recruitment number of susceptible vectors, μ_0 death due to infection, λ probability of infectious humans being recovered

Solution of the Model Using Homotopy Perturbation Method (HPM): Homotopy Perturbation Method (HPM) was developed by (He, 2000). The HPM provides solution to various linear and non-linear differential equations (Jiya, 2010)

The basic ideas of the method are by considering the following non-linear differential equation:

$$A(U) - f(r) = 0, r \in \Omega$$
⁽⁶⁾

with the boundary condition

$$B\left(U,\frac{\partial U}{\partial n}\right) = 0, r \in \Gamma$$
⁽⁷⁾

Where A is the general differential operator, B the boundary operator, f(r) is the analytical function and Γ is the boundary of the domain Ω . The A operator can be divided into two major parts L and N been the linear and nonlinear component respectively. Equation (6) can be written as follows:

$$L(U) + N(U) - f(r) = 0, r \in \Omega$$
⁽⁸⁾

The HPM is composed as follows:

$$H(V,h) = (1-h) \left[L(V) - L(U_0) \right] + h \left[A(V) - f(r) \right] = 0$$

$$(9)$$

Where $V(r, P) : \Omega \in [0, 1] \rightarrow R$

From equation (8), $P \in [0,1]$ is an embedded parameter and U_0 is the approximation that satisfies the boundary condition. The solution to equation (10) can be assumed as power series in *h* as follows:

$$V = V_0 + hV_1 + h^2 V_2 + \dots (10)$$

With approximation best obtain when:

$$U = \lim_{n \to \infty} = v_0 + hv_1 + h^2 v_2 + \dots$$

$$h \to 1$$
(11)

The rate of convergence majorly depends on the nonlinear operator A(V)

Solution of the Model Equations

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$$\frac{dS_h}{dt} + (\mu_h + \beta_h I_v)S_h - \gamma_h = 0 \tag{12}$$

$$\frac{dI_h}{dt} + (\mu_h + \mu_0 + \lambda + \delta_h)I_h - \beta_h S_h I_v = 0$$
⁽¹³⁾

$$\frac{dR_h}{dt} + \mu_h R_h - (\lambda + \delta_h) I_h = 0 \tag{14}$$

$$\frac{dS_{\nu}}{dt} + \beta_{\nu}S_{\nu}I_{h} + (\mu_{\nu} + \delta_{\nu})S_{\nu} - \gamma_{\nu} = 0$$
⁽¹⁵⁾

$$\frac{dI_{\nu}}{dt} + (\mu_{\nu} + \delta_{\nu})I_{\nu} - \beta_{\nu}S_{\nu}I_{h} = 0$$
⁽¹⁶⁾

With the following initial conditions $S_h(0) = S_0$, $I_h(0) = I_0$, $R_h(0) = R_0$, $S_v(0) = \phi_0$ and $I_v(0) = \tau_0$. Let

$$S_h = a_0 + ha_1 + h^2 a_2 + \dots$$
(17)

$$I_h = b_0 + hb_1 + h^2b_2 + \dots$$
(18)

$$R_h = c_0 + hc_1 + h^2 c_2 + \dots$$
(19)

$$S_{\nu} = d_0 + hd_1 + h^2d_2 + \dots$$
(20)

$$I_{v} = e_{0} + he_{1} + h^{2}e_{2} + \dots$$
⁽²¹⁾

Applying Homotopy Perturbation Method (HPM) to equations (12) - (16) using equations (17) - (21), we obtained the orders of h as follows:

$$h^{0}: a_{0}' = 0, b_{0}' = 0, c_{0}' = 0, d_{0}' = 0 \text{ and } e_{0}' = 0$$
 (22)

$$h^{1}: a_{1}' + \mu_{h}a_{0} + \beta_{h}a_{0}e_{0} - \wedge_{h} = 0$$

$$b_{1}' + \alpha b_{0} + \beta_{h}a_{0}e_{0} = 0$$

$$c_{1}' + \mu_{h}c_{0} - (\lambda + \delta_{h})b_{0} = 0$$

$$d_{1}' + \beta_{\nu}b_{0}d_{0} + (\lambda + \delta_{\nu})d_{0} - \wedge_{\nu} = 0$$

$$e_{1}' + (\mu_{\nu} + \delta_{\nu})e_{0} - \beta_{\nu}(b_{0}d_{0}) = 0$$
(23)

$$h^{2}: a'_{2} + \mu_{h}a_{1} + \beta_{h}(a_{0}e_{1} + a_{1}e_{0}) = 0$$

$$b'_{2} + \alpha b_{1} + \beta_{h}(a_{0}e_{1} + a_{1}e_{0}) = 0$$

$$c'_{2} + \mu_{h}c_{1} - (\lambda + \delta_{h})b_{1} = 0$$

$$d'_{2} + \beta_{\nu}(b_{0}d_{1} + b_{1}d_{0}) + (\mu_{\nu} + \delta_{\nu})d_{1} = 0$$

$$e'_{2} + (\mu_{\nu} + \delta_{\nu})e_{1} - \beta_{\nu}(b_{0}d_{1} + b_{1}d_{0}) = 0$$
(24)

where α is denoted as $\mu_h + \mu_0 + \lambda + \delta_h$

Solving equations (22) - (24) using the initial condition and collecting the coefficients of power of 'h' by expansion, we obtained:

$$h^0: a_0 = S_0, b_0 = I_0, c_0 = R_0, d_0 = \phi_0 \text{ and } e_0 = \tau_0$$
 (25)

$$h^{1}: a_{1}(t) = (-S_{0}\tau_{0}\beta_{h} - S_{0}\mu_{h} + \wedge_{h})t$$

$$b_{1}(t) = (-S_{0}\tau_{0}\beta_{h} - \alpha I_{0})t$$

$$c_{1}(t) = (\lambda I_{0} + I_{0}\delta_{h} - R_{0}\mu_{0})t$$

$$d_{1}(t) = (I_{0}\phi_{0}\beta_{v} - \mu_{v}\phi_{0} - \phi_{0}\delta_{v} + \wedge_{v})t$$

$$e_{1}(t) = (I_{0}\phi_{0}\beta_{v} - \mu_{v}\tau_{0} - \tau_{0}\delta_{v})t$$
(26)

$$h^{2}: a_{2} = -\frac{1}{2}t^{2} \begin{cases} S_{0}\mu_{h}^{2} + \wedge_{h}\mu_{h} + \beta_{h}I_{0}S_{0}\phi_{0}\beta_{v} - \beta_{h}S_{0}\mu_{0}\tau_{v} \\ -\beta_{h}S_{0}\tau_{v}\delta_{v} - S_{0}\tau_{0}^{2}\beta_{h}^{2} + \beta_{h}\tau_{0}\wedge_{h} \end{cases}$$

$$b_{2} = -\frac{1}{2}t^{2} \begin{cases} -\alpha S_{0}\tau_{0}\beta_{h} + \alpha S_{0}\mu_{h} + \wedge_{h}\alpha + \beta_{h}I_{0}S_{0}\phi_{0}\beta_{v} \\ -\beta_{h}S_{0}\mu_{v}\tau_{0} - \beta_{h}S_{0}\tau_{0}\delta_{v} - S_{0}\tau_{0}^{2}\beta_{h}^{2} + \beta_{h}S_{0}\mu_{h}\tau_{0} \\ +\beta_{h}\tau_{0}\wedge_{h} \end{cases}$$

$$c_{2} = -\frac{1}{2}t^{2} \begin{cases} \lambda S_{0}\sigma_{0}\beta_{h} + \beta_{h}S_{0}\tau_{0}\delta_{h} + \lambda I_{0}\mu_{h} - \lambda S_{0}\mu_{h} + I_{0}\mu_{h}\delta_{h} \\ +R_{0}\mu_{h}^{2} - S_{0}\mu_{h}\delta_{h} - \lambda \wedge_{h} - \wedge_{h}\delta_{h} \end{cases}$$

$$d_{2} = \frac{1}{2}t^{2} \begin{cases} I_{0}^{2}\phi_{0}\beta_{v}^{2} + 2\beta_{v}I_{0}\mu_{v}\phi_{0} + 2\beta_{v}I_{0}\phi_{0}\delta_{v} - \beta_{v}I_{0}\wedge_{v} \\ +\beta_{v}S_{0}\phi_{0}\tau_{0}\beta_{h} - \beta_{h}S_{0}\mu_{h}\phi_{0} - \beta_{v}\phi_{0}\wedge_{h} + \mu_{v}^{2}\phi_{0} \\ +2\mu_{v}\phi_{0}\delta_{v} + \phi_{0}\delta_{v}^{2} - \mu_{v}\wedge_{v} - \wedge_{v}\delta_{v} \end{cases}$$

$$e_{2} = -\frac{1}{2}t^{2} \begin{cases} I_{0}^{2}\phi_{0}\beta_{v}^{2} + 2\beta_{v}I_{0}\mu_{v}\phi_{0} + 2\beta_{v}I_{0}\phi_{0}\delta_{v} - \beta_{v}I_{0}\wedge_{v} \\ +\beta_{v}S_{0}\phi_{0}\tau_{0}\beta_{h} - \beta_{v}S_{0}\mu_{h}\tau_{0}\phi_{0} - \beta_{v}\phi_{0}\wedge_{h} - \mu_{v}^{2}\tau_{0} \\ -2\mu_{v}\tau_{0}\delta_{v} - \tau_{0}\delta_{v}^{2} \end{cases}$$

$$(27)$$

Substituting the values into the series in equations (17) – (21) taking the lim as $h \rightarrow 1$, we obtained the result as follows:

$$S_{h}(t) = S_{0} + (-S_{0}\tau_{0}\beta_{h} + S_{0}\mu_{h} + \wedge_{h})t$$

$$-\frac{1}{2}t^{2} \begin{cases} S_{0}\mu_{h}^{2} + \wedge_{h}\mu_{h} + \beta_{h}I_{0}S_{0}\phi_{0}\beta_{v} - \beta_{v}S_{0}\mu_{v}\tau_{0} \\ -\beta_{h}S_{0}\tau_{0}\delta_{v} - S_{0}\tau_{0}^{2}\beta_{h}^{2} + \beta_{h}\tau_{0}\wedge_{h} \end{cases}$$
(28)

$$I_{h}(t) = I_{0} + (-S_{0}\tau_{0}\beta_{h} + S_{0}\mu_{h} + \wedge_{h})t$$

$$-\frac{1}{2}t^{2} \begin{cases} -\alpha S_{0}\tau_{0}\beta_{h} + \alpha S_{0}\mu_{h} + \wedge_{h}\alpha + \beta_{h}I_{0}S_{0}\phi_{0}\beta_{v} \\ -\beta_{h}S_{0}\mu_{v}\tau_{0} - \beta_{h}S_{0}\tau_{0}\delta_{v} - S_{0}\tau_{0}^{2}\beta_{h}^{2} + \beta_{h}S_{0}\mu_{h}\tau_{0} \\ +\beta_{h}\tau_{0}\wedge_{h} \end{cases}$$
(29)

$$R_{0}(t) = R_{0} + (\lambda I_{0} + I_{0}\delta_{h} + R_{0}\mu_{h})t$$

$$-\frac{1}{2}t^{2} \begin{cases} \lambda S_{0}\tau_{0}\beta_{h} + S_{0}\tau_{0}\beta_{h}\delta_{h} + \lambda I_{0}\mu_{h} - \lambda S_{0}\mu_{h} \\ +I_{0}\mu_{h}\delta_{h} + R_{0}\mu_{h}^{2} - S_{0}\mu_{h}\delta_{h} - \lambda \wedge_{h} \wedge_{h}\delta_{h} \end{cases}$$
(30)

$$S_{\nu}(t) = \phi_{0} + (-I_{0}\phi_{0}\beta_{\nu} - \mu_{\nu}\phi_{0} - \phi_{0}\delta_{\nu} + \wedge_{\nu})t + \frac{1}{2}t^{2} \begin{cases} I_{0}^{2}\phi_{0}\beta_{\nu}^{2} + 2\beta_{\nu}I_{0}\mu_{\nu}\phi_{0} + 2\beta_{\nu}I_{0}\phi_{0}\delta_{\nu} - \beta_{\nu}I_{0}\wedge_{\nu} \\+ \beta_{\nu}S_{0}\phi_{0}\tau_{0}\beta_{h} - \beta_{\nu}S_{0}\mu_{h}\phi_{0} - \beta_{\nu}\phi_{0}\wedge_{h} + \mu_{\nu}^{2}\phi_{0} \\+ 2\mu_{\nu}\phi_{0}\delta_{\nu} + \phi_{0}\delta_{\nu}^{2} - \mu_{\nu}\wedge_{\nu} - \wedge_{\nu}\delta_{\nu} \end{cases}$$
(31)

$$I_{\nu}(t) = \tau_{0} + (I_{0}\phi_{0}\beta_{\nu} - \mu_{\nu}\tau_{0} - \tau_{0}\delta_{\nu})t - \frac{1}{2}t^{2} \begin{cases} I_{0}^{2}\phi_{0}\beta_{\nu}^{2} + 2\beta_{\nu}I_{0}\mu_{\nu}\phi_{0} + 2\beta_{\nu}I_{0}\phi_{0}\delta_{\nu} \\ -\beta_{\nu}I_{0}\wedge_{\nu} + \beta_{\nu}S_{0}\phi_{0}\tau_{0}\beta_{h} - \beta_{\nu}S_{0}\mu_{h}\phi_{h} \\ -\beta_{\nu}\phi_{0}\wedge_{h} - \mu_{h}^{2}\tau_{0} - 2\mu_{\nu}\tau_{0}\delta_{\nu} - \tau_{0}\delta_{\nu}^{2} \end{cases}$$
(32)

Numerical Simulations: The simulations were carried out using the following variables and parameters for initial conditions at time (t). Computations were run in maple17 software for investigation.

Para	meter Description	Value	Source
μ_{h}	Natural death rate of human	0.0008	Assumed
μ_{v}	Natural death rate of vector	0.0714	Mojumder, et al.(2016)
δ_{v}	Death rate from insecticide	0.59	Assumed
$\delta_{\scriptscriptstyle h}$	Recovery rate from treatment	0.05	Assumed
$eta_{\scriptscriptstyle h}$	Probability of human to mosquitoes getting infected	0.5	Ebenezer & Kazeem (2016)
β_{v}	Probability of mosquitoes to human getting infected	0.005	Assumed
\wedge_h	Recruitment number of susceptible human	100	Ebenezer & Kazeem (2016)
\wedge_{v}	Recruitment number of susceptible vector	1000	Ebenezer & Kazeem (2016)
μ_0	Death rate due to infectious	0.5	Assumed
λ	Probability of human recovered from infection	0.03	Assumed
S_h	Susceptible human	11000	"
I_h	Infectious human	500	,,
R_h	Recovered human	200	"
S_{v}	Susceptible vector	3000	,,
I_{v}	Infectious vector	300	Assumed

Table1. Parameter Descriptions, Values and Source used in simulations

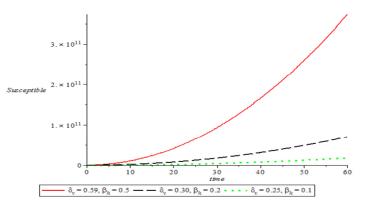


Fig 1: Graph of Susceptible humans against time with different rate of death from insecticide δ_v and humans to mosquitoes getting infected β_h .

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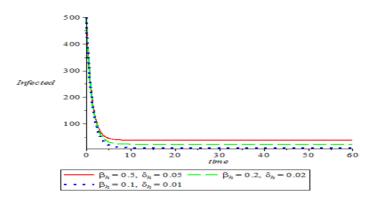


Fig 2: Graph of Infected humans against time with different rate of humans to mosquitoes getting infected β_h .

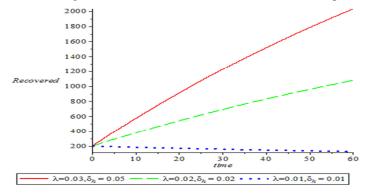


Fig 3: Graph of Recovered humans from infection against time with different rate.

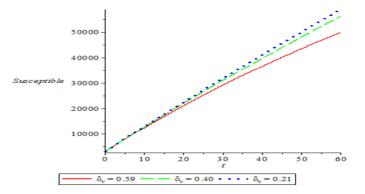


Fig 4: Graph of Susceptible vectors against time with different rate of death from insecticide δ_v

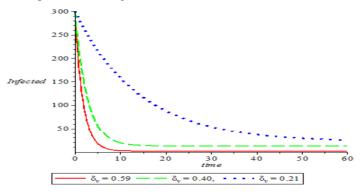


Fig 5: Graph of Infected vectors against time with different rate.

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RESULTS AND DISCUSSION

We used Homotopy Perturbation Method to determine the approximate solution of our model formulated. Mathematical software Maple17 specially designed for this purpose was used for the simulations which always give the best output. The graph in figure 1 shows the various trends for the susceptible population as the parameter values were varied. For $\delta_v = 0.59$, $\beta_h = 0.5$, the susceptible population experienced significant increase over time, this could be attributed to 59% effective use of insecticide. This can be observed in figure 5, where the infected vector population diminishes very fast particularly for $\delta_v = 0.59$. For $\delta_v = 0.30$, $\beta_h = 0.2$ gave a fall on the susceptible population signalling eminent danger. The graph for, $\delta_v = 0.25$, $\beta_h = 0.1$, signifies that the susceptible population is most likely to go extinct if no measure is taken.

The corresponding infected population is in figure 2. The three profiles show significant decline in the infected class with the graph for $\beta_h = 0.1$ and $\delta_h = 0.01$ giving the best fit. This could be further explained from figure 4, the graph of susceptible vector population which leads to decline in the infected human compartment. Consequently, the recovered rate increased as depicted in figure 3. The graph for $\delta_h = 0.05$ gives the best fit of this case.

Conclusion: In this paper, the research work contributes to new field of knowledge inclined to the dynamics of Zika virus in population dynamics with the applications of Homotopy Perturbation Method. The result of this work can further be extended to other field of knowledge to study the pattern of Zika associated diseases in many part of the world which pose a significant public health risk. It can stimulate expedited research required to address open questions and to better inform countermeasure development and clinical management.

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