Magnetopolaron in a cylindrical nanocrystal

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ABSTRACT

Magnetopolaron states in a cylindrical nanocrystal (nano-size crystal) with a parabolic confinement potential are investigated applying the Feynman variational principle. Similar effects of cyclotron and confinement frequencies respectively on electron and polaron energy levels are obtained for the case of a cylindrical nanocrystal. Splitting of electron energy level is observed. Energy degeneracy for confinement and cyclotron frequencies is found. Cyclotron and confinement frequencies reduce considerably regions of weak and intermediate polaron and regions of strong coupling are shifted to ones of weak and intermediate polaron. It is observed that the polaron mass and energy increase with increase of Fröhlich electron-phonon coupling constant, confinement and cyclotron frequencies.

Key words: polaron, cylindrical nanocrystal, parabolic, confinement, cyclotron frequency.

RESUME

Dans ce travail, les états du magnétopolaron dans un nano-cristal de forme cylindrique avec un potentiel de confinement parabolique ont été étudiés à l'aide du principe variationel de Feynman. Dans ce cas, les effets similaires des fréquences du cyclotron et de confinement respectivement sur les niveaux d'énergie de l'électron et du polaron ont été obtenus. La séparation du niveau d'énergie de l'électron a été observée. La dégénérescence de l'énergie pour les fréquences du cyclotron et de confinement a été déterminée. Il a été en outre remarqué que les fréquences du cyclotron et de confinement réduisaient considérablement les domaines des polarons de liaisons faible et intermédiaire, alors que le domaine du polaron de liaison forte passait pour ceux des polarons de liaisons faible et intermédiaire. Par ailleurs, lors d'une augmentation aussi bien de la constante a de liaison électron - phonon de Fröhlich, que de la fréquence de confinement ou du cyclotron, la masse et l'énergie du polaron croissent.

Mots clés: : polaron, nano-cristal cylindrique, parabolique,confinement,frequence de cyclotron

1. INTRODUCTION

Recent developments in 2D, 1D and 0D nanocrystal fabrication have stimulated theoreticians' interest in formulating models describing physical phenomena associated with quantum wires Hudgins et al (1997), Lucsak et al (1995), Nagamune et al (1992), Osorio et al (1988;1995), Tsukamoto et al (1993). The present paper investigates confinement, cyclotron frequency, and Fröhlich electron-phonon coupling constant effects on polaron energy and effective mass. Polaron energy is evaluated in Pokatilov et al (1995), using perturbation theory. The polaron and bipolaron concept has a renewed attention due to the assumption of their possible participation in high-temperature superconductors. The carrier here is a condensed bipolaron gas (that is, charged hard-core bosons). Landau introduced the polaron concept Landau (1965), while Pekar originally suggested the possibility of large bipolaron formation Osorio et al (1988).

A polaron is a quasi particle that arises as a result of a conduction electron (or hole) together with its self-induced polarization in an ionic crystal or in a polar semiconductor Devreese (1998). Polarons may be classified using the Fröhlich electron-phonon coupling constant value α that is weak-coupling if $\alpha < 1$, strong-coupling if $\alpha \geq 7$ and intermediate-coupling between these ranges. The majority of crystals are weak or intermediate-coupling polarons. Strong-coupling is not attained even in strong ionic crystals such as alkaline halides. The polaron character is well pronounced only for strong-coupling Pekar (1963). Strong-coupling polarons play an important role in bipolaron states formation that, in general, do not exist for weak-coupling polarons. For nanocrystals it is possible to reduce the lower bound of the electron-phonon coupling constant's threshold value to within weak or intermediate-coupling range. Some review on the polaron theory is found in Devreese (1986). When investigating the polaron problem in nanocrystals, it is necessary to consider both the electron and the phonon confinement. The electron confinement is described in Hudgins et al (1997), Wendler et al (1993),Miyake (1975),Zhu (1992), by means of a parabolic potential. In Wendler et al (1993),Miyake (1975), magnetopolaron phenomena are examined and in Pokatilov et al (1995),Tsukamoto et al (1993),Haupt and Wendler (1994), Wendler and Kugler (1994), magnetopolarons in layers, wires and dots are investigated. In Devreese et al (2000) the high magnetic field cyclotron resonance in CdS is examined: Explanation of the non-monotonic temperature dependence of the cyclotron mass is discussed.

This work also arises from recent advances in fabrication of nanocrystals with strong ionic coupling Hudgins et al (1997). In our investigation of polaron states in a cylindrical nanocrystal using Feynman variational principle we resolve to upper bound polaron ground state energy for arbitrary Fröhlich electron-phonon coupling constant. In this paper electron confinement is selected in form of transverse parabolic potential because rigid interface boundaries are absent. Then we examine electron interaction only with 3D longitudinal polar optical phonons (3D-phonon approximation). For rigid interface boundaries, interface-like phonon modes are localized at the neighbourhood of a sharp boundary. There is also quantisation of bulk phonons. For the parabolic potential, interface-like phonon modes are rather smoothly distributed in space. For this we do the 3D-phonon approximation. Consequently, interface phonons may not be considered. This approach seems to be adequate as integral polaron effects results from summation over all phonon spectra. In phenomena with integral phonon effects we resort to numerical results. It is expected that to confine an electron without using a magnetic field one needs to affect the crystal structure. It is also expected that the phonon spectrum have to change. The present investigation shows that this is not so in the absence of the magnetic field when the confinement is present.

The model with parabolic confinement is preferable as it examines polaron states covering all values of Fröhlich electron-phonon coupling constant. It is of principal importance in this investigation. In the problem, the magnetic field is considered to be in the direction of the axis of the cylindrical nanocrystal.

2. FEYNMAN VARIATIONAL PRINCIPLE.

The Feynman variational principle is one of the most effective methods when investigating the polaron problem for arbitrary values of the electron-phonon coupling constant α. For the Feynman variational principle, the exact and the trial (model) systems are considered. The action functional of the exact system is defined as:

$$S[\vec{r}] = \int L dt \tag{2.1}$$

and that of the trial system as:

$$S_0[\vec{r}] = \int L_0 dt \tag{2.2}$$

where $\vec{r}$ is the radius vector, t the time, L and L_0 are the Langrangians of the exact and trial systems respectively.

The statistical sum of state of the exact system is defined by

$$Z = Sp \int D\vec{r} \exp\{S[\vec{r}]\} \tag{2.3}$$

and that of the trial system by.

$$Z_0 = Sp \int D\vec{r} \exp\{S_0[\vec{r}]\} \tag{2.4}$$

Here $D\vec{r}$ denotes path integration and Sp the spur.

In our evaluations, the statistical sum for the exact system is defined as $\tilde{Z}$:

$$\tilde{Z} = \frac{Z}{Z_L} = Sp \int D\vec{r} \exp\{S[\vec{r}]\} \tag{2.5}$$

where Z_L is the statistical sum due to the lattice vibrations. The expression in 2.5 may also be written in the form:

$$\tilde{Z} = Sp\int D\vec{r}\exp\{S_0[\vec{r}]\}\frac{Sp\int D\vec{r}\exp\{S[\vec{r}]-S_0[\vec{r}]\}\exp\{S_0[\vec{r}]\}}{Sp\int D\vec{r}\exp\{S_0[\vec{r}]\}}$$

or

$$\tilde{Z} = Z_0\langle exp\{S[\vec{r}]-S_0[\vec{r}]\}\rangle \qquad (2.5')$$

The angle brackets in 2.5' denote averaging over electron paths and are defined as follows

$$\langle F[\vec{r}]\rangle = \frac{Sp\int D\vec{r}F[\vec{r}]exp\{S_0\}}{Sp\int D\vec{r}\, exp\{S_0\}}$$

The basis of Feynman variational method is the Jensen-Feynman inequality Devreese et al (2000):

$$\langle exp(S[\vec{r}]-S_o[\vec{r}])\rangle \geq exp\langle S[\vec{r}]-S_o[\vec{r}]\rangle \qquad (2.6)$$

The Feynman's statistical sum Z_F is evaluated with the help of 2.5' and 2.6:

$$\ln\tilde{Z} \geq \ln Z_F \equiv \ln Z_O - \langle S[\vec{r}]-S_O[\vec{r}]\rangle \qquad (2.7)$$

The total momentum $\vec{p}$ of the polaron is considered to be the only continuous quantum number. The dependence of the energy on the momentum (for the case of an isotropic crystal) has the form:

$$E = E_0(\nu)+p^2E_2(\nu)+p^4E_4(\nu)+\cdots, \qquad (2.8)$$

Here the quantities E_0 , E_2 , E_4 ,$\cdots$, are coefficients of expansion of 2.8 and ν is the totality of discrete quantum numbers of the system.

To evaluate the sum of state for the system using 2.3 is limited to the first two terms of the expansion as higher order terms for low temperatures are exponentially small (in the sum of state expression) compared to the first two terms. If the polaron effective mass is defined by $M = \frac{1}{2E_2}$ then from the sum of state Z the expression follows

$$ln Z = ln\left(\frac{V}{(2\pi\hbar)^3}\left(\frac{2\pi m_e}{\lambda}\right)^{\frac{3}{2}}\right) - \lambda E_0 + \frac{3}{2}ln\left(\frac{M}{m_e}\right),\ \lambda \equiv \frac{1}{T} \qquad (2.9)$$

Comparing 2.7 and 2.9 it is possible to obtain the expressions for the Feynman variational polaron energy E and effective mass M (in units of the electron's mass m_e) at low temperatures T (absolute temperature) i.e., $T \to 0$($\lambda \to \infty$). The polaron ground state energy is obtained as the coefficient of λ in 2.4 and the polaron effective mass from the term independent of λ.

3. ELECTRON WAVE FUNCTION AND ENERGY IN A QUASI 1D CYLINDRICAL NANOCRYSTAL.

Consider an electron in a quasi-1D nanocrystal with a transverse parabolic confinement potential. The electron's motion (in the nanocrystal) in the presence of a magnetic field is characterized by a cyclotron frequency ω. The electron's motion in the direction of the axis of the nanocrystal is free. The magnetic field is positioned parallel to the cylindrical axis of the nanocrystal. The electron's Hamiltonian in cylindrical coordinates is:

$$\hat{\mathrm{H}}_e = -\frac{\hbar^2}{2m_e}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}+\frac{\partial^2}{\partial z^2}\right)-\hat{\ell}_z\frac{\hbar\omega}{2}+\frac{m_e\tilde{\omega}^2\rho^2}{2} \tag{3.1}$$

Here $\rho^2 = x^2 + y^2$, m_e- electron's band mass, Ω- frequency characterizing the parabolic confinement potential, $\omega = \frac{eH}{cm_e}$- cyclotron frequency, e- electron's charge, c- light speed, $\tilde{\omega}^2 = \Omega^2 + \frac{\omega^2}{4}$. The operators $\hat{\ell}_z = -i\frac{\partial}{\partial\varphi}$ (the z- component of angular momentum operator) and $\hat{p}_z$ commute with each other and with the Hamiltonian H_e of the electron's system. Thus they have a common system of eigenfunctions. The eigenfunctions $\Psi_{nmp_z}(\varphi, z, \rho)$ of the Hamiltonian 3.1 are found to be:

$$\Psi_{nmp_z}(\varphi,z,\rho)= A_{|m|}\xi^{\frac{|m|}{2}} exp\left\{im\varphi+\frac{ip_z z}{\hbar}-\frac{\xi}{2}\right\}F(-n,|m|+1,\xi) \tag{3.2}$$

where, $F(\nu,\gamma,x)$ is a confluent hypergeometric function Gradstein (1971), $A_{|m|}$ the normalization constant and $n,|m| = 0,1,2,\cdots$, $\quad \xi = \frac{\rho^2}{2a^2}$, $\quad a = \left(\frac{m_e^2\omega^2}{\hbar^2}+\frac{4m_e^2\Omega^2}{\hbar^2}\right)^{-\frac{1}{4}}$

The eigenvalues of 3.1 are

$$E_{nmp_z} - \frac{p_z^2}{2m_e} = E_{nm} = -\frac{\hbar m\omega}{2}+\hbar\sqrt{\Omega^2+\frac{\omega^2}{4}}(2n+|m|+1)$$

Here m is the angular momentum quantum number (along the z-axis) and n that of the two-dimensional oscillator respectively. The ground state corresponds to $n = m = 0$.

The limiting expressions for the ground state energy for weak $\omega << \Omega$ and strong magnetic fields $\omega >> \Omega$ are respectively

$$E_{00} \approx \hbar\Omega + \frac{\hbar\omega^2}{8\Omega}$$

and

$$E_{00} \approx \frac{\hbar\Omega^2}{\omega} + \frac{\hbar\omega}{2}$$

The dependence of the ground state energy spectrum on confinement and cyclotron frequencies for weak and strong magnetic fields show similar effects of confinement and cyclotron frequency on electron's energy spectrum. Effects differ only in values of energy levels. This is seen in figures 1 and 2. In these figures the splitting of the energy levels is observed for the case of the confinement and cyclotron frequency. Degeneracies are observed for the two cases. Thus the subjection of an electron to a confinement gives similar effects as for the case of the magnetic field.

4. FEYNMAN POLARON IN A QUASI 1D CYLINDRICAL NANOCRYSTAL.

Consider the motion of an electron in a magnetic field (positioned parallel to the cylindrical axis). The Hamiltonian that describes electron interacting with lattice vibrations is:

$$\hat{H} = \hat{H}_e + \hat{H}_{ph} + \hat{H}_{e-ph}$$

$$\hat{H}_{ph} = \sum_{\vec{q}} \hbar\omega_{\vec{q}} \hat{b}^+_{\vec{q}} \hat{b}_{\vec{q}}, \quad \hat{H}_{e-ph} = \sum_{\vec{q}} \left[\gamma_{\vec{q}} \hat{b}_{\vec{q}} \exp\{i\vec{q}\vec{r}\} + \gamma^*_{\vec{q}} \hat{b}^+_{\vec{q}} \exp\{-i\vec{q}\vec{r}\} \right]$$

$\hat{H}_{ph}$ - phonon contribution Hamiltonian, $\hat{H}_{e-ph}$ -electron-phonon interaction Hamiltonian, $\omega_{\vec{q}}$ - phonon frequency numbered by wave vector $\vec{q}$.

Consider the magnetic cyclotron motion as fast and the translatory motion along the z-axis as slow. The ground state wave function from 3.2 for the fast motion may be selected:

$$\psi(\rho) = \frac{a}{\sqrt{\pi}} exp\left\{ -\frac{1}{2}(\rho\, a)^2 \right\}$$

Averaging $\hat{H}$ by $\psi(\rho)$ eliminates the fast motion. In fact, this is an adiabatic separation of the slow translatory motion along the z-axis and of the fast motion in the xy-plane. This approximation is valid only in the strong-confinement limit or in the limit of strong magnetic field. This procedure yields the Lagrangian of the slow subsystem:

$$L = -\frac{m_e \dot{z}^2}{2\hbar^2} - \frac{1}{2}\sum_{\vec{q}} \left[\frac{\dot{Q}^2_{\vec{q}}}{\hbar^2} + \omega^2_{\vec{q}} Q^2_{\vec{q}} \right] - \sum_{\vec{q}} \tilde{\gamma}_{\vec{q}}(z) Q_{\vec{q}} \qquad (4.1)$$

where

$$\tilde{\gamma}_q(z) = \gamma_{\bar{q}} \chi_{\bar{q}}(z) exp\left\{-\frac{q_\perp^2}{4a^2}\right\}$$

$$\gamma_{\bar{q}} = \left[\frac{\alpha\pi}{V}\left(\frac{\hbar\omega_o}{q}\right)^2 R_p\right]^{\frac{1}{2}}, R_p = \left(\frac{\hbar}{2m_e\omega_o}\right)^{\frac{1}{2}}, \quad q^2 = q_\perp^2 + q_{//}^2, \chi_{\bar{q}}(z) = \begin{cases} cos\, q_{//} z & q_{//} < 0 \\ sin\, q_{//} z & q_{//} \geq 0 \end{cases}$$

Here $Q_{\bar{q}}$ are the vibrational normal coordinates, ω_o - non-dispersional polar optical frequency. The slow subsystem trial Lagrangian L_o is selected in one-oscillatory approximation:

$$L_0 = -\frac{m_e \dot{z}^2}{2\hbar^2} - \frac{M_f \dot{Z}_f^2}{2\hbar^2} - \frac{M_f \omega_f (Z_f - z)^2}{2} \tag{4.2}$$

Where Z_f is the coordinate of fictitious particle. The quantities M_f and ω_f are the mass and the elastic coupling frequency of fictitious particle respectively. Both serve as variational parameters. The quantities z, m_e are the electron coordinate and mass respectively. In the expressions 4.1 and 4.2, $t = -i\hbar\tau$ and $\dot{f} = \frac{df}{d\tau}$.

In the evaluation of the magnetopolaron states the ground state for which in particular $m = 0$, is considered. Thus the inequality in 2.6 can now be valid, as the action cannot be complex.
In 4.2 changing of variables

$$\rho_1 = Z_f - z\,, \rho_2 = \frac{MZ_f + m_e z}{M + m_e} \tag{4.3}$$

and then applying it to 2.4 gives

$$Z_0 = \frac{l\sqrt{m_e}u}{\sqrt{2\pi\hbar^2\lambda}} \frac{1}{2\,sinh\left(\frac{\lambda\hbar v}{2}\right)} \tag{4.4}$$

Here l is a parameter that characterizes the length of the wire and

$$v = u\omega_f\,, u^2 = \frac{M_f + m_e}{m_e}, a_1 = \frac{1}{u^2}, a_1 + a_2 = 1$$

The exact action functional $S[z]$ may be obtained from the expression 2.5 after integration over phonon variables $Q_{\bar{q}}$. Thus from 2.3, 2.5 and 4.1 is obtained the result:

$$\tilde{Z} = \prod_{\vec{q}} Sp \int Dz\, exp\left\{-\frac{m_e}{2\hbar^2}\int_0^\lambda \dot{z}^2(\tau)d\tau + \Phi_{\omega_{\vec{q}}}[z]\right\} \tag{4.5}$$

where

$$\Phi_{\omega_{\vec{q}}}[z] = -\frac{\hbar}{4m_e\omega_{\vec{q}}}\int_0^\lambda\int_0^\tau \tilde{\gamma}_{\vec{q}}(\tau)\tilde{\gamma}_{\vec{q}}(\tau')F_{\omega_{\vec{q}}}(|\tau-\tau'|)d\tau d\tau'$$

is the electron-phonon interaction influence phase and

$$F_{\omega_{\vec{q}}}[z] = \frac{cosh\,\hbar\omega_{\vec{q}}\left(|\tau-\tau'|-\frac{\lambda}{2}\right)}{sinh\left(\frac{\lambda\hbar\omega_{\vec{q}}}{2}\right)}.$$

Thus from 4.5:

$$S[z] = -\frac{m_e}{2\hbar^2}\int_0^\lambda \dot{z}^2(\tau)d\tau + \Phi_L[z] \tag{4.6}$$

where $\Phi_L[z] = \sum_{\vec{q}}\Phi_{\omega_{\vec{q}}}[z]$.

Similarly from 2.4 and 4.2 after integration over the coordinates of the fictitious particle is obtained:

$$Z_0 = Sp\int Dz\, exp\left\{-\frac{m_e}{2\hbar^2}\int_0^\lambda \dot{z}^2(\tau)d\tau + \frac{M_f\omega_f^2}{2}\int_0^\lambda z^2(\tau)d\tau + \Phi_{\omega_f}[z] - ln\left[2\,sinh\left(\frac{\lambda\hbar\omega_f}{2}\right)\right]\right\} \tag{4.7}$$

where

$$\Phi_{\omega_f}[z] = \frac{\hbar M_f\omega_f^3}{4}\int_0^\lambda\int_0^\lambda z(\tau)z(\tau')F_{\omega_f}(|\tau-\tau'|)d\tau d\tau'$$

From 4.7 the action functional $S_0[z]$ of the trial system is obtained:

$$S_0[z] = -\frac{m_e}{2\hbar^2}\int_0^\lambda \dot{z}^2(\tau)d\tau + \frac{M_f\omega_f^2}{2}\int_0^\lambda z^2(\tau)d\tau + \Phi_{\omega_f}[z] - ln\left[2\,sinh\left(\frac{\lambda\hbar\omega_f}{2}\right)\right] \tag{4.8}$$

Considering 4.4, 4.6 and 4.8 the expression for 2.7 may now be obtained. The resultant expression should be in an explicit form when we evaluate the following:

$$\langle z^2(\tau)\rangle,\ \langle z(\tau)z(\tau')\rangle,\ \langle cos(q_{||}, z(\tau)-z(\tau'))\rangle \tag{4.9}$$

These are obtained with the help of the productive function

$$\Psi_{\vec{q}}(\xi,\eta) \equiv \langle exp\{iq_{||}(\xi\, z(\tau)-\eta\, z(\tau'))\}\rangle$$

that after path integration yields

$$\Psi_{\vec{q}}(\xi,\eta) \equiv exp\{\Gamma_{\vec{q}} + \Gamma'_{\vec{q}}\} \tag{4.10}$$

where

$$\Gamma_{\vec{q}} = -\frac{\hbar q_{||}^2 a_2}{4mv}\left[(\xi^2+\eta^2)F_v(0) - 2\xi\eta\, F_v(|\tau-\tau'|)\right]$$

and

$$\Gamma'_{\vec{q}} = -\frac{\hbar^2 q_{||}^2 a_1}{4m}\lambda\left[\frac{1}{6}(\xi^2+\eta^2)F_v(0) - \frac{2\xi\eta}{\lambda^2}\left(|\tau-\tau'| - \frac{\lambda}{2}\right)^2 + \frac{\xi\eta}{6}\right]$$

From 4.10 the expressions for 4.9 is obtained:

$$\frac{1}{q_{||}^2}\frac{\partial^2\Psi_{\vec{q}}}{\partial\xi\,\partial\eta}\bigg|_{\xi=\eta=0} = \langle z(\tau)z(\dot{\tau}')\rangle, \langle z(\tau)z(\tau')\rangle|_{\tau=\tau'} = \langle z^2(\tau)\rangle$$
$$\langle\cos(q_{||}, z(\tau) - z(\tau'))\rangle = \Psi_{\vec{q}}(1,1) = \Psi_{\vec{q}}(-1,-1) \tag{4.11}$$

In further evaluations of the polaron energy and mass, the Feynman's units Feynman (1955), are used: $\hbar\omega_0$ for energy and R_p for length. Considering equations 4.4 to 4.11 then we now compare 2.7 with 2.9 to obtain the polaron energy and mass. The Feynman polaron dimensionless variational energy is found to be:

$$E = \frac{v}{4}\left(1-\frac{1}{u}\right)^2 - \frac{\alpha\sqrt{2}}{v\,a\pi\sqrt{\pi}}\int_0^1 (1-\tau)^{\frac{1-v}{v}}\frac{ln\left[\sqrt{A-1}+\sqrt{A}\right]}{\sqrt{A-1}}d\tau \tag{4.12}$$

and the dimensionless variational polaron mass as:

$$M = exp\left\{-\frac{u^2-1}{u^2} - \frac{\alpha}{a^3v^3\sqrt{2\pi}}\int_0^1 \frac{(1-\tau)^{\frac{1-v}{v}}ln^2(1-\tau)}{\sqrt{A}}\left[1 - \frac{\sqrt{\pi}\,ln(2A-1)}{4\sqrt{(A-1)}}\right]d\tau\right\} \tag{4.13}$$

Here

$$A = \frac{2}{v\,a^2}\left[a_2\tau - a_1\,ln(1-\tau)\right]$$

The polaron energy and mass are found by minimizing the polaron variational energy and mass. The numerical results are shown on the figures 1-8 below. In the figures 3, 5 and 7 the absolute value of the polaron energy is considered for convenience.

From expressions 4.12 and 4.13 the analytic dimensionless expressions for the polaron energy and effective mass for the strong-coupling polarons are obtained respectively as:

$$E = -\frac{\alpha^2}{\pi}ln^2\left(\frac{2\pi}{e\alpha^2 a}\right) \tag{4.14}$$

and

$$M = \frac{4\alpha^4}{\pi^2} ln^2\left(\frac{2\pi}{\alpha^2 a^2}\right) \qquad (4.15)$$

Rigorous weak-coupling expansion for the polaron ground state energy and effective mass yield Devreese (1987):

$$E = -\alpha - 0.01592\,\alpha^2 - \cdots, \quad M = 1 + \frac{\alpha}{6} + 0.02363\,\alpha^2 + \cdots$$

The expansion for the strong-coupling polaron yields [13]:

$$E = -0.108513\alpha^2 - 2.836, \; M = 1 + 0.0227019\alpha^4$$

Polaron excitations in Haupt (1994) and Zhu (1992) for the strong-coupling polaron in a 3D structure yield the energy:

$$E = -\frac{\alpha^2}{9\pi}$$

Since $a^{-1} > 1$ then from the above results, polaron ground state energy and effective mass in a cylindrical nanocrystal are greater than those in 3D structures. Also effect of the confinement or the cyclotron frequency leads to increase in polaron ground state energy and effective mass.

5. CONLUSION

Figures 1 and 2 show the effects of cyclotron and confinement frequencies respectively on electron's energy levels. They have similar effects. The splitting of energy levels is observed. There is similar energy degeneracy for confinement and cyclotron frequencies. Figure 3 is a plot of the polaron energy versus Fröhlich electron-phonon coupling constant. The plot deviates slightly from a linear relation. The regions of strong-coupling polarons are shifted to ones of weak and intermediate polarons. Thus introducing a confinement or a magnetic field shifts the regions of strong-coupling polarons to ones of weak and intermediate polarons. This is in agreement with Pokatilov et al (2000;1999;1998) on the effect of the confinement. It can be seen from the results that the effect of the confinement is similar to that of the magnetic field. Thus instead of introducing an electron confinement to shift the regions of strong-coupling polarons to ones of weak and intermediate polarons, it is sufficient to introduce only the magnetic field. In figure 4, the polaron mass increases with increase Fröhlich electron-phonon coupling constant. Intensifying the confinement or the magnetic field leads to increase in polaron mass. Figure 5 is the plot of the polaron energy versus the cyclotron frequency. Here it is observed that the polaron energy increases with increase in cyclotron frequency. Figure 6 is a plot of polaron mass versus cyclotron. It shows that the polaron mass increases with an increase in cyclotron frequency. Figures 5 and 7 show similar behaviors for the confinement frequency as for the cyclotron frequency on the polaron energy levels. The confinement and cyclotron frequencies also have similar effects on the polaron mass as seen in figures 6 and 8. These results confirm the fact that instead of providing an electronic confinement it is sufficient to apply a magnetic field.

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Figure-1: Plot of electron energy versus cyclotron frequency

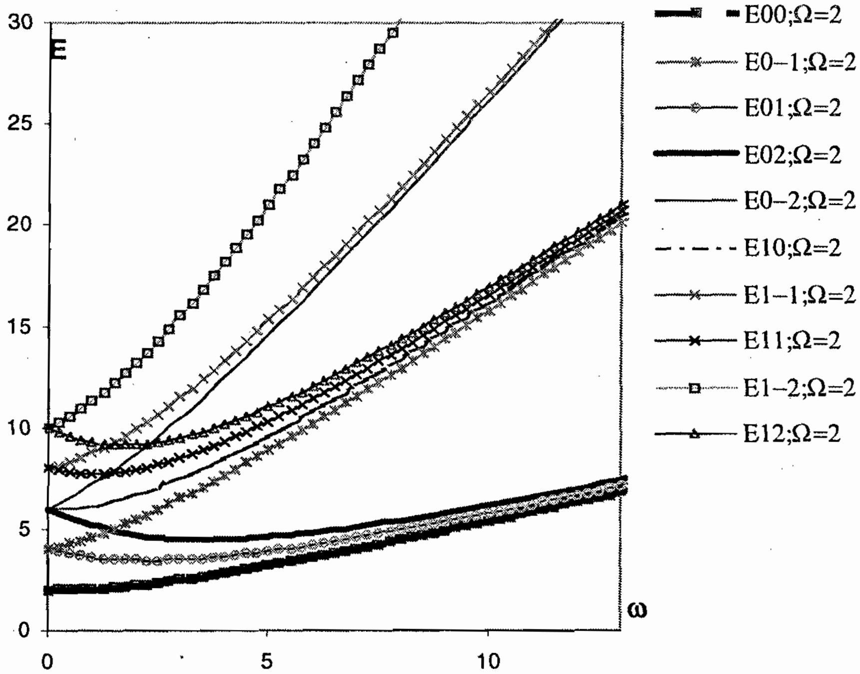


Figure-2: Plot of electron energy versus confinement frequency

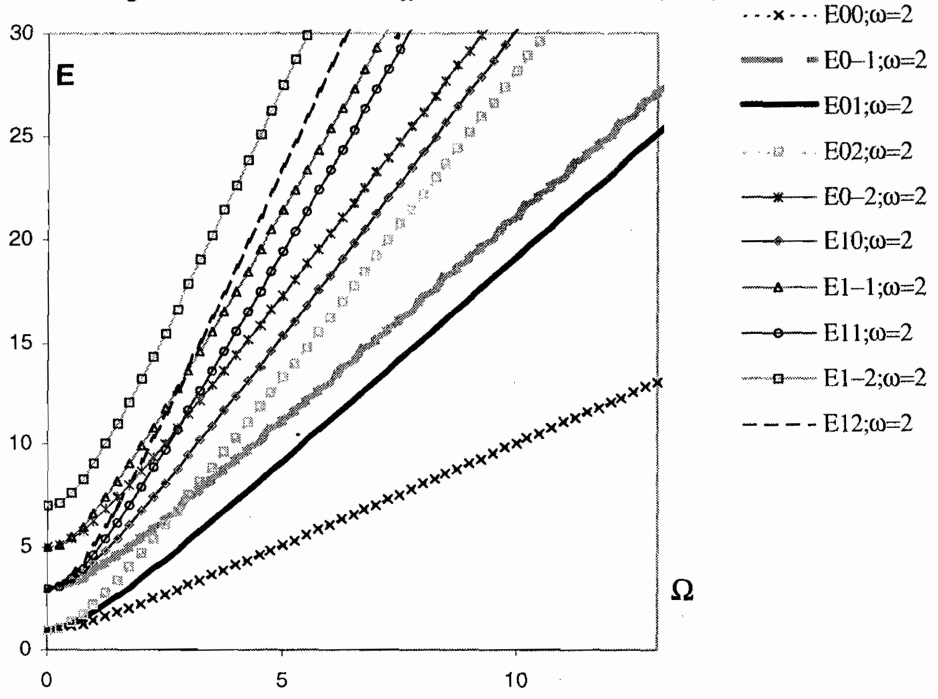


Figure - 3: Plot of polaron energy versus Fröhlich electron -phonon coupling constant

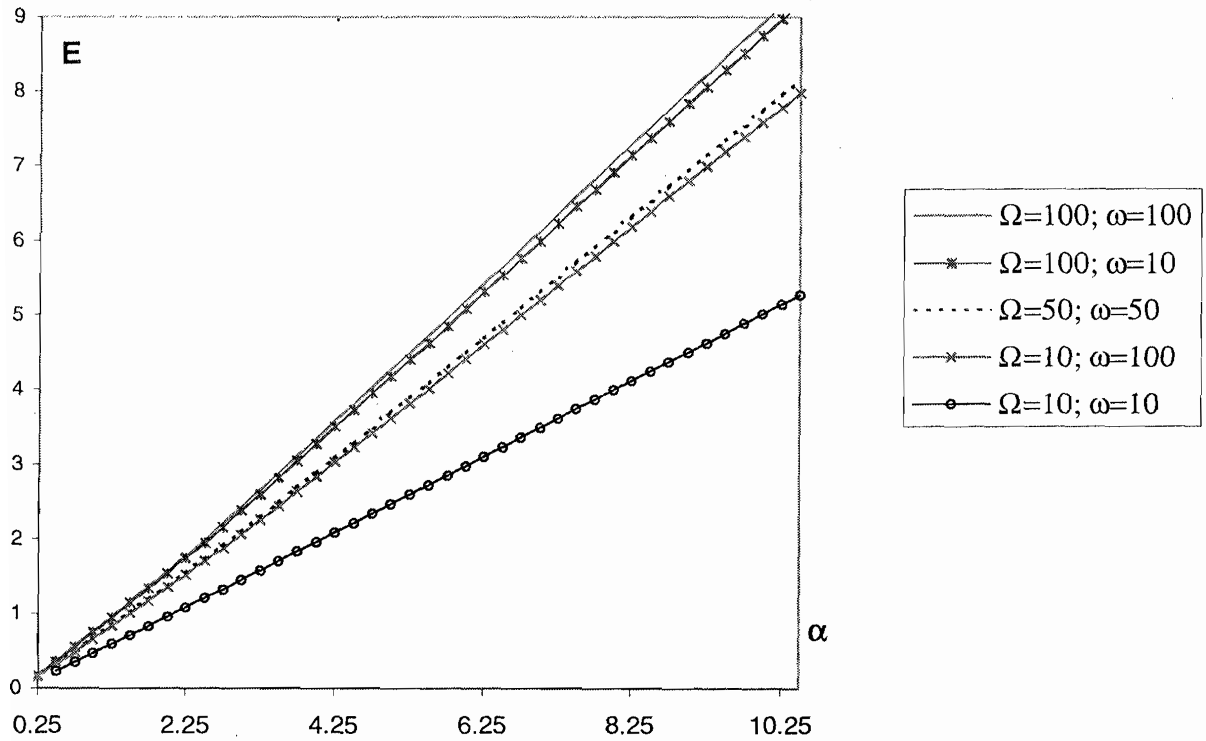


Figure - 4: Plot of polaron mass versus Fröhlich electron -phonon coupling constant

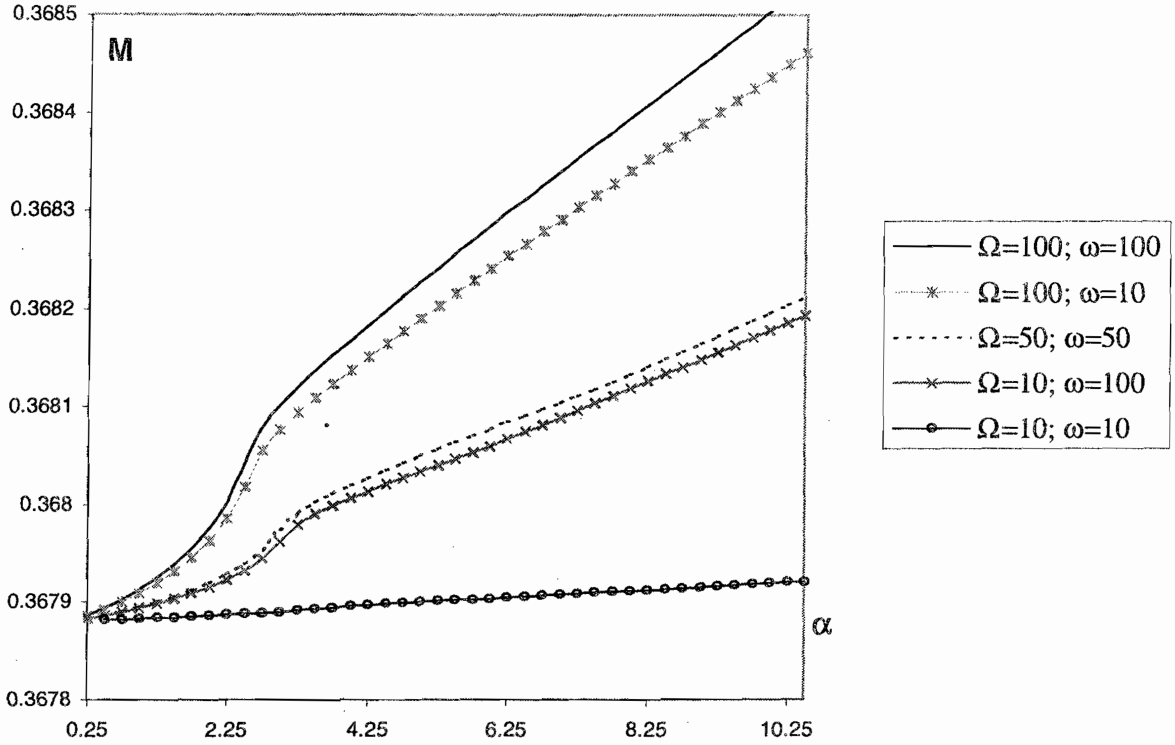


Figure - 5: Plot of polaron energy versus cyclotron frequency

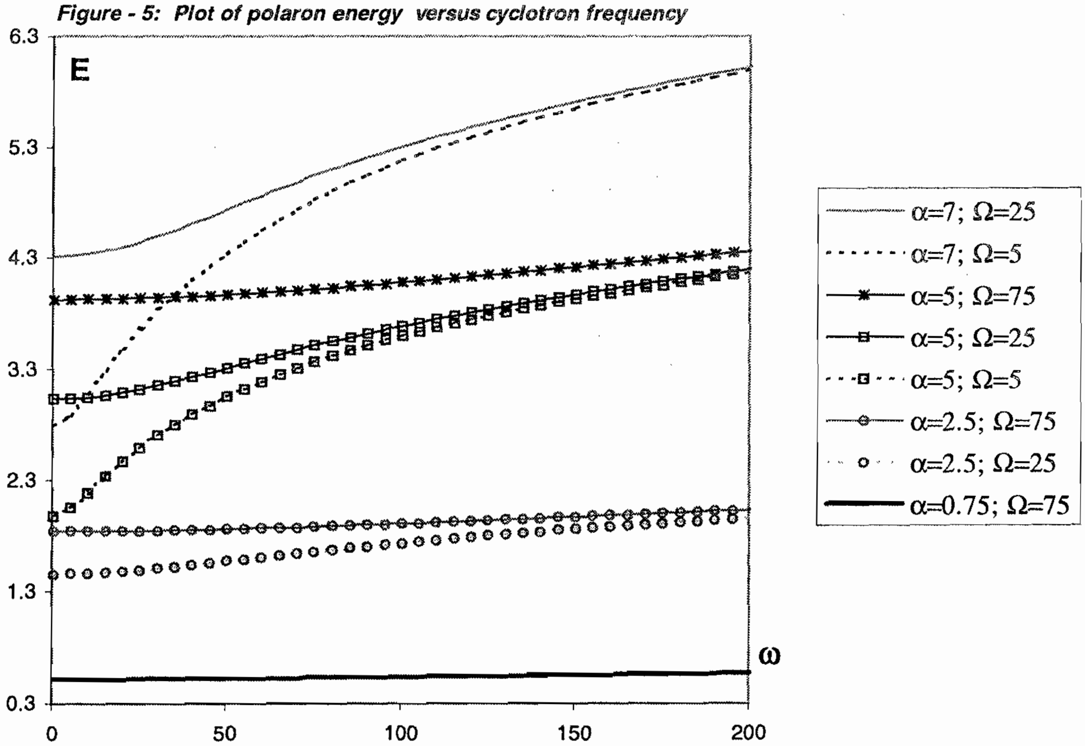


Figure - 6: Plot of polaron effective mass versus cyclotron frequency

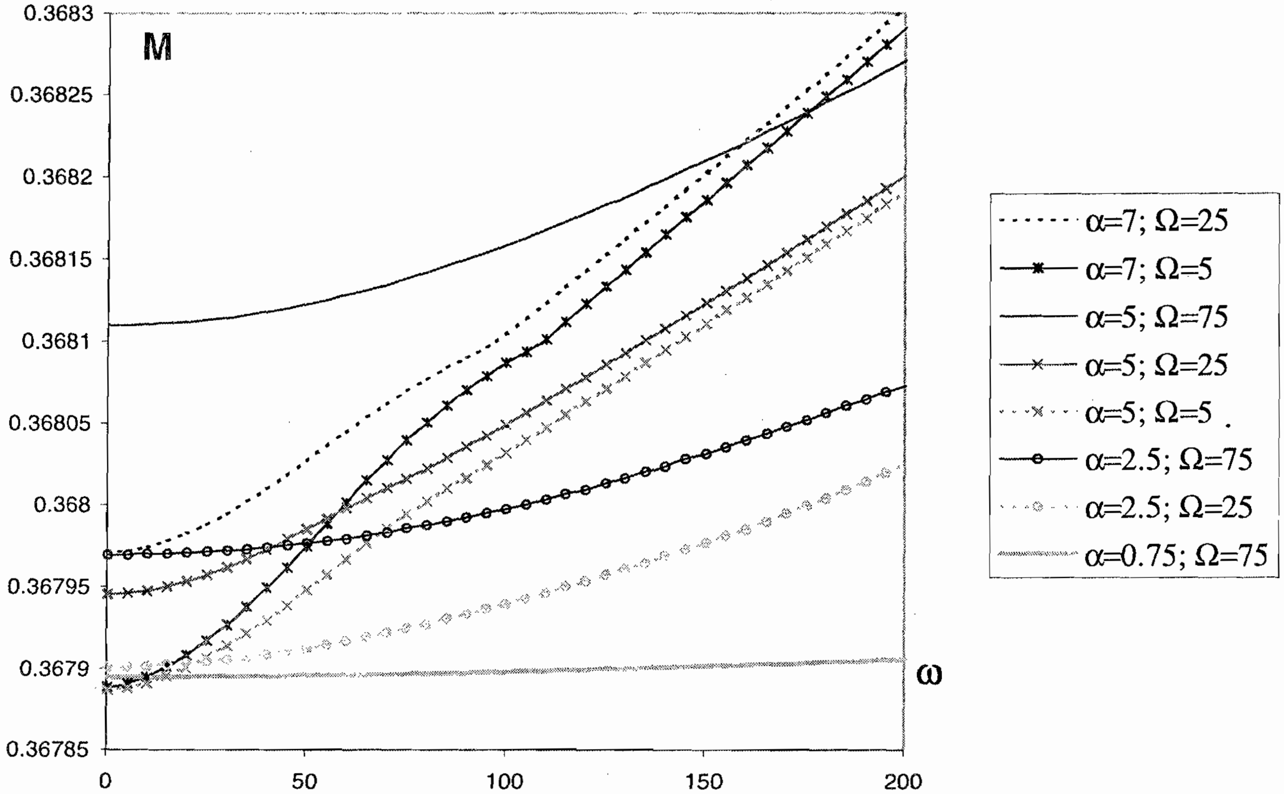


Figure - 7: Plot of polaron energy versus confinement frequency

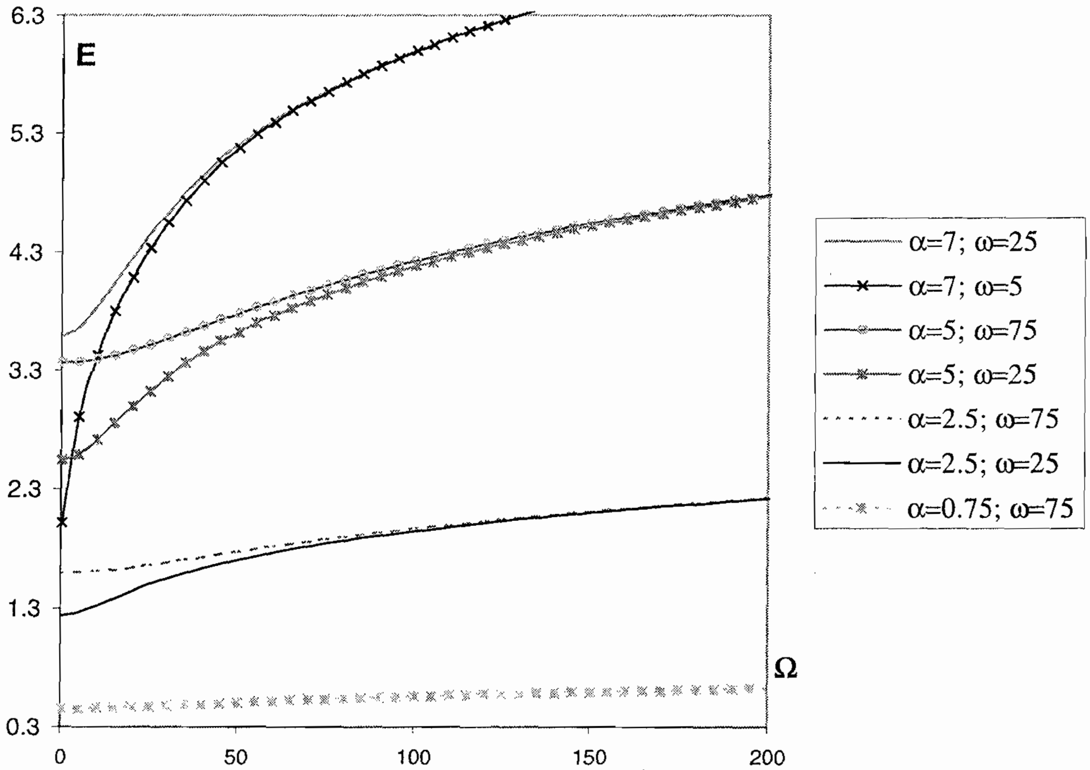
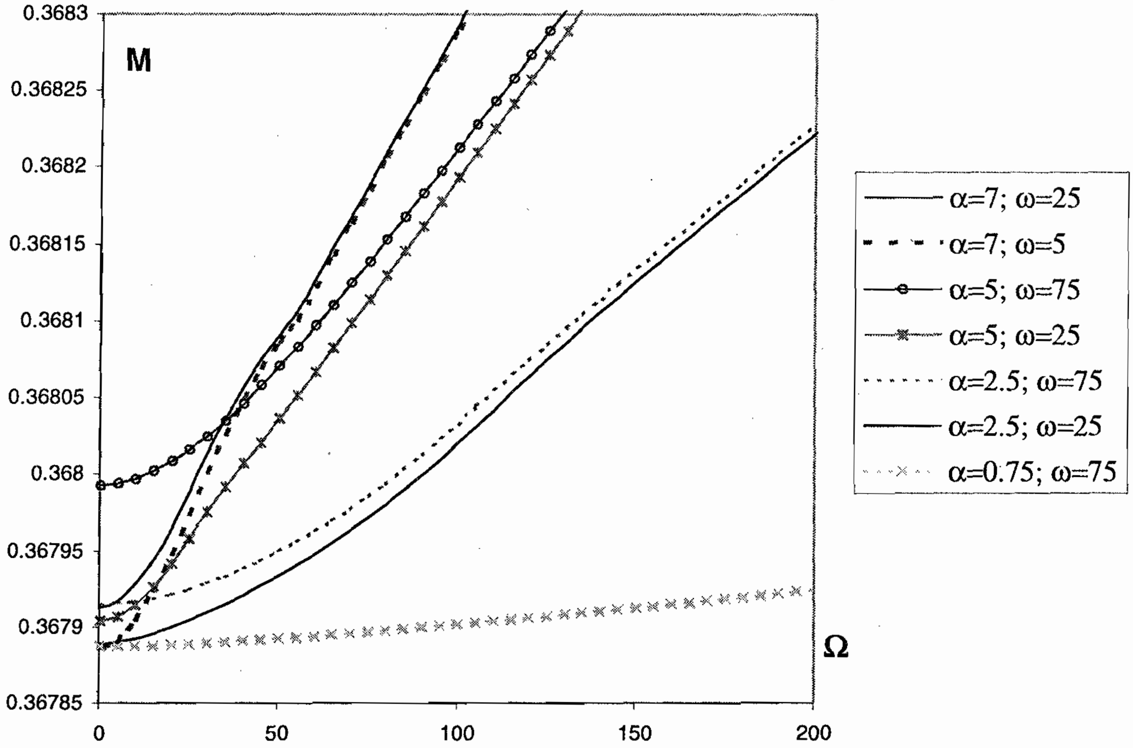


Figure - 8: Plot of polaron effective mass versus confinement frequency



The Ideal Structure Theorem for $D\alpha$

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ABSTRACT

In this paper we describe completely the closed ideals of the Banach algebra of functions f analytic in the unit disc such that their Taylor coefficients $\hat{f}$ satisfy the condition

$$\sum_{n \in IN} (1+n)^{\alpha} \mid \hat{f}(n) \mid^2 := \mid\mid f \mid\mid^2 < +\infty$$

for $2n+1 > \alpha > 2n-1$, ($n \in IN^*$ fixed), when the skeletons of the closed ideals under consideration are at most countable.

Keywords: Closed Ideals, Banach Algebras, K-algebra, Skeleton, Inner factors, Standard Ideals

RESUME

Dans cet article nous décrivons complètement les idéaux fermés de l'algebre de Banach D_α de fonctions f analytiques dans le disque unitaire dont les coefficients de Taylor $\hat{f}$ satisfient la condition

$$\sum_{n \in IN} (1+n)^{\alpha} \mid \hat{f}(n) \mid^2 := \mid\mid f \mid\mid^2 < +\infty$$

pour $2n+1>\alpha>2n-1$, ($n \in IN^*$ est fixe), quand les squelettes des idéaux considerés sont au plus denombrables.

Mots clés: Idéaux fermés, Algebres de Banach, K-algebre, Squelette, Facteurs intérieurs, Idéaux étandards.