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THE NONABELIAN TENSOR SQUARE OF A BIEBERBACH GROUP WITH ELEMENTARY ABELIAN 2-GROUP POINT GROUP

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ABSTRACT

A Bieberbach group is a torsion free crystallographic. In this paper, one Bieberbach group with elementary abelian 2-group point group of the lowest dimension three is considered and its group presentation can be shown to be consistent polycyclic presentation. The main objective of this paper is to compute the nonabelian tensor square of one Bieberbach group with elementary abelian 2-group point group of dimension three by using the computational method of the nonabelian tensor square for polycyclic groups. The finding of the computation showed that the nonabelian tensor square of the group is abelian and the formula of the nonabelian tensor square of the Bieberbach group with elementary abelian 2-group of dimension 3, $S_1(3)$, can be extended in constructing the generalization of the formula of the nonabelian tensor square of the group up to dimension n.

Keywords: Bieberbach group; polycyclic groups; nonabelian tensor square.

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1. INTRODUCTION

1.1. Introduction

A Bieberbach group is a torsion free crystallographic group. This group is an extension of a free abelian group L of finite rank by a finite point group P which satisfies the short exact sequence

$$1 \longrightarrow L \xrightarrow{\varphi} G \xrightarrow{\phi} P \longrightarrow 1$$

such that $G/\varphi(L) \cong P$. New properties of crystallographic groups can be revealed by calculating the nonabelian tensor squares of the groups.

The nonabelian tensor square, $G \otimes G$ of a group G is generated by the symbols $g \otimes h$, for all $g, h \in G$, subject to relations

$$gg' \otimes h = ({}^{g}g' \otimes {}^{g}h)(g \otimes h) \quad \text{and} \quad g \otimes hh' = (g \otimes h)({}^{h}g \otimes {}^{h}h') \qquad [1]$$

for all $g, g', h, h' \in G$, where ${}^{g}g' = gg'g^{-1}$. Brown and Loday [1] have introduced the nonabelian tensor square as a specialization of more general nonabelian tensor products. Since then, many studies on computing the nonabelian tensor squares for various groups have been conducted. These include the 2-generator nilpotent of class two groups ([2], [3]), the free nilpotent groups [4] and the polycyclic groups [5].

The study of the nonabelian tensor squares of Bieberbach groups with certain point group have been started by Masri [6] who focused on Bieberbach groups with cyclic point group of order two. Next, other studies related to the computation of the nonabelian tensor squares of Bieberbach groups with other point groups have also been done by other researchers such as Mohd Idrus [7] and Wan Mohd Fauzi *et al.* [8] with the dihedral point group, Mat Hassim [9] with the cyclic group of order three and five and Tan *et al*. ([10], [11]) with the symmetric point group.

In this paper, we focus on a Bieberbach group with elementary abelian 2-groups, $C_2 \times C_2$ of lowest dimension 3, denoted by $S_1(3)$. This group is an extension of a finitely generated abelian group which is polycyclic. In other word, the group $S_1(3)$ is a polycyclic group. The consistent polycyclic presentation of group $S_1(3)$ is given as the following [12]:

$$S_1(3) = \left\langle a_0, a_1, l_1, l_2, l_3 \,\middle|\, \begin{array}{l} a_0^2 = l_1^{-1}, a_1^2 = l_2^{-1}, {}^{a_0}a_1 = a_1 l_1^{-1} l_2 l_3^{-1}, \\ {}^{a_0}l_1 = l_1, {}^{a_0}l_2 = l_2^{-1}, {}^{a_0}l_3 = l_3^{-1}, \\ {}^{a_1}l_1 = l_1^{-1}, {}^{a_1}l_2 = l_2, {}^{a_1}l_3 = l_3^{-1}, \\ {}^{l_1}l_2 = l_2, {}^{l_1}l_3 = l_3, {}^{l_2}l_3 = l_3 \end{array} \right\rangle . \quad (1)$$

1.2 Preliminaries

The computation of the nonabelian tensor square in this study involves a group $\nu(G)$ which was introduced by Rocco [13] as follows:

Definition 1 [13]

Let G be a group with presentation $\langle G \mid R \rangle$ and let G^{φ} be an isomorphic copy of G via the mapping $\varphi : g \to g^{\varphi}$ for all $g \in G$. The group $\nu(G)$ is defined to be $\nu(G) = \langle G, G^{\varphi} \mid R, R^{\varphi}, {}^{x}[g, h^{\varphi}] = [{}^{x}g, ({}^{x}h)^{\varphi}] = {}^{x^{\varphi}}[g, h^{\varphi}], \forall x, g, h \in G \rangle$.

Theorem 1 shows that the group $\nu(G)$ is related to the nonabelian tensor square.

Theorem 1 ([13], [14])

Let G be a group. The mapping $\sigma : G \otimes G \to [G, G^{\varphi}] \triangleleft \nu(G)$ defined by $\sigma(g \otimes h) = [g, h^{\varphi}]$ for all g, h in G is an isomorphism.

Therefore, all the tensor computations can be done through the commutator computation within the subgroup of $\nu(G)$, $[G, G^{\varphi}]$. Blyth and Morse [5] showed that if G is polycyclic, then $\nu(G)$ is also polycyclic as given in the following proposition.

Proposition 1 [5]

If G is polycyclic, then $\nu(G)$ is polycyclic.

In this study, the nonabelian tensor square of $S_1(3)$ is obtained by using the computational method for polycyclic group developed by Blyth and Morse [5]. Next, list of commutator identities in $\nu(G)$ with left conjugation are given as in the following. Let x, y and z be elements of a group G. Then

$$[xy, z] = {}^{x}[y, z] \cdot [x, z] \qquad (2)$$

$$[x, yz] = [x, y] \cdot {}^{y}[x, z] \qquad (3)$$

$$^{z}[x, y] = [{}^{z}x, {}^{z}y] \qquad (4)$$

Definition 2

The abelianization of a group G, $G^{ab} = G/G'$ is the quotient of group G by its derived subgroup, G'.

The next proposition shows the close relationship between the structure of the central subgroup of the nonabelian tensor square of group G, $\nabla(G)$ and G^{ab}.

Proposition 2 [15]

Let G be a group such that G^{ab} is finitely generated. Assume that G^{ab}. is the direct product of the cyclic groups $\langle x_i G' \rangle$, for $i = 1, \ldots, s$ and set $E(G)$ to be $\langle [x_i, x_j^{\varphi}] \mid i < j \rangle [G, G'^{\varphi}]$. Then the following hold:

(i) $\nabla(G)$ is generated by the elements of the set $\{[x_i, x_i^{\varphi}], [x_i, x_j^{\varphi}][x_j, x_i^{\varphi}] \mid 1 \le i < j \le s\}$;

(ii) $[G, G^{\varphi}] = \nabla(G)E(G)$.

The following propositions and theorem are some another commutator identities used in this paper.

Proposition 3([5], [13])

Let G be a group. Then the following relations hold in $\nu(G)$:

(i) $[g, g^{\varphi}]$ is central in $\nu(G)$ for all g in G;

(ii) $[g, g^{\varphi}] = 1$ for all g in G'.

Proposition 4[5]

Let g_1, g_2, g_3 and g_4 be elements of group G. Then in $\nu(G)$, $[[g_1, g_2],[g_3, g_4]^{\varphi}] = [[g_1, g_2{}^{\varphi}],[g_3, g_4{}^{\varphi}]]$.

Proposition 5 [15]

Let G be any group. Then the following hold:

(i) If $g_1 \in G'$ or $g_2 \in G'$, then $[g_1, g_2{}^{\varphi}]^{-1} = [g_2, g_1{}^{\varphi}]$.

(ii) $[Z(G),(G')^{\varphi}] = 1$.

Proposition 6 [5]

Let g and h be elements of G such that $[g,h]=1$. Then, in $\nu(G)$,

(i) $[g^n, h^{\varphi}] = [g, h^{\varphi}]^n = [g,(h^{\varphi})^n]$ for all integers n;

(ii) $[g^n,(h^m)^{\varphi}][h^m,(g^n)^{\varphi}] = ([g,h^{\varphi}][h,g^{\varphi}])^{nm}$;

Proposition 7 [16]

Let A and B be abelian groups. The properties of the ordinary tensor product of two abelian groups are given as in the following.

(i) $B_0 \otimes A \cong A$,

(ii) $B_0 \otimes B_0 \cong B_0$,

where B_0 is a cyclic group of infinite order.

Proposition 8 [6]

Let G be any Bieberbach group of dimension n with point group P and lattice group L. Let $B = G \times F_m^{ab}$ where F_m^{ab} be a free abelian group of rank m. Then B is a Bieberbach group of dimension $n+m$ with point group P.

Theorem 2 [17]

Let G be a group. Then there exists a commutator mapping $\kappa : G \otimes G \to G'$ which is defined by $\kappa(g \otimes h) = [g,h]$. The kernel of κ is in the centre of $G \otimes G$.

When G is abelian, $G \otimes G$ is just an ordinary tensor square for abelian groups. The following proposition gives the nonabelian tensor square of two abelian groups.

Proposition 9 [17]

Let G be any group such that $G = A \times B$. Then,

$$G \otimes G = (A \times B) \otimes (A \times B)$$

$$= (A \otimes A) \times (A^{ab} \otimes B^{ab}) \times (B^{ab} \otimes A^{ab}) \times (B \otimes B)$$

where $A^{ab} = A/A'$ and $B^{ab} = B/B'$ are the abelianizations of A and B respectively.

The derived subgroup, the abelianization and the central subgroup of the nonabelian tensor square of $S_1(3)$ are given in the following proposition.

Proposition 10 [18]

For group $S_1(3)$,

(i) The derived subgroup $S_1(3)' = \langle l_1^2, l_2^2, l_1 l_2^{-1} l_3 \rangle$

(ii) The abelianization, $S_1(3)^{ab} = \langle a_0 S_1(3)', a_1 S_1(3)' \rangle \cong C_4{}^2$.

Proposition 11 [18]

The central subgroup of the nonabelian tensor square of $S_1(3)$ is given as the following:

$$\nabla(S_1(3)) = \langle [a_0, a_0^{\varphi}], [a_1, a_1^{\varphi}], [a_0, a_1^{\varphi}][a_1, a_0^{\varphi}] \rangle \cong C_4 \times C_8^2.$$

2. RESULTS AND DISCUSSION

In this section, the nonabelian tensor square of $S_1(3)$, denoted as $S_1(3) \otimes S_1(3)$ is computed.

Theorem 3

The nonabelian tensor square of $S_1(3)$ is isomorphic to $C_4 \times C_8{}^2 \times C_0{}^3$, that is,

$$S_1(3) \otimes S_1(3) = \nabla(S_1(3)) E(S_1(3)) \cong C_4 \times C_8{}^2 \times C_0{}^3.$$

Proof. By Proposition 10, $S_1(3)' = \langle l_1^2, l_2^2, l_1 l_2^{-1} l_3 \rangle$ and by Proposition 2, $E(S_1(3)) = \langle [a_0, a_1{}^{\varphi}] \rangle [S_1(3), S_1(3)'^{\varphi}]$ where $[S_1(3), S_1(3)'^{\varphi}]$ is generated by generators $[a_0, l_1^{2\varphi}]$,

$[a_1, l_1^{2\varphi}]$, $[l_1, l_1^{2\varphi}]$, $[l_2, l_1^{2\varphi}]$, $[l_3, l_1^{2\varphi}]$, $[a_0, l_2^{2\varphi}]$, $[a_1, l_2^{2\varphi}]$, $[l_1, l_2^{2\varphi}]$, $[l_2, l_2^{2\varphi}]$, $[l_3, l_2^{2\varphi}]$, $[a_0, (l_1 l_2^{-1} l_3)^{\varphi}]$, $[a_1, (l_1 l_2^{-1} l_3)^{\varphi}]$, $[l_1, (l_1 l_2^{-1} l_3)^{\varphi}]$, $[l_2, (l_1 l_2^{-1} l_3)^{\varphi}]$, and $[l_3, (l_1 l_2^{-1} l_3)^{\varphi}]$. However, some of these generators can be expressed as a product of powers of other generators.

$$
\begin{aligned}
[a_1, l_1^{2\varphi}] &= [a_1, l_1^{\varphi}]^{l_1}[a_1, l_1^{\varphi}] && \text{by (3)} \\
&= [a_1, l_1^{\varphi}][a_1 l_1^{-2}, l_1^{\varphi}] && \text{since } {}^{l_1}a_1 = a_1 l_1^{-2} \\
&= [a_1, l_1^{\varphi}]^{a_1}[l_1^{-2}, l_1^{\varphi}][a_1, l_1^{\varphi}] && \text{by (2)} \\
&= [a_1, l_1^{\varphi}]^2[l_1^{2}, l_1^{-\varphi}] && \\
&= [a_1, l_1^{\varphi}]^2[l_1, l_1^{2\varphi}]^{-1} && \text{by Proposition 6(i)} \\
&= [a_1, l_1^{\varphi}]^2 && \text{since } [l_1, l_1^{2\varphi}] = 1
\end{aligned}
$$

$$
\begin{aligned}
[a_0, l_2^{2\varphi}] &= [a_0, l_2^{\varphi}]^{l_2}[a_0, l_2^{\varphi}] && \text{by (3)} \\
&= [a_0, l_2^{\varphi}][a_0 l_2^{-2}, l_2^{\varphi}] && \text{since } {}^{l_2}a_0 = a_0 l_2^{-2} \\
&= [a_0, l_2^{\varphi}]^{a_0}[l_2^{-2}, l_2^{\varphi}][a_0, l_2^{\varphi}] && \text{by (2)} \\
&= [a_0, l_2^{\varphi}]^2[l_2^{2}, l_2^{-\varphi}] && \\
&= [a_0, l_2^{\varphi}]^2[l_2, l_2^{2\varphi}]^{-1} && \text{by Proposition 6(i)} \\
&= [a_0, l_2^{\varphi}]^2 && \text{since } [l_2, l_2^{2\varphi}] = 1
\end{aligned}
$$

By using similar arguments, $[a_0, (l_1 l_2^{-1} l_3)^{\varphi}] = [a_0, a_1^{\varphi}]^2[a_1, l_1^{\varphi}]^{-1}([a_0, a_1^{\varphi}][a_1, a_0^{\varphi}])^{-2}$, $[a_1, (l_1 l_2^{-1} l_3)^{\varphi}] = [a_0, a_1^{\varphi}]^2[a_0, l_2^{\varphi}]$, $[l_3, (l_1 l_2^{-1} l_3)^{\varphi}] = [l_1, (l_1 l_2^{-1} l_3)^{\varphi}]^{-1}[l_2, (l_1 l_2^{-1} l_3)^{\varphi}]$ and $[l_1, (l_1 l_2^{-1} l_3)^{\varphi}] = [a_0, l_1^{2\varphi}]$, $[l_2, (l_1 l_2^{-1} l_3)^{\varphi}] = [a_1, l_2^{2\varphi}]$. However, $[a_0, l_1^{2\varphi}] = [a_0, a_0^{\varphi}]^{-4}$ and $[a_1, l_2^{2\varphi}] = [a_1, a_1^{\varphi}]^{-4}$. Next, it is can be shown that

$$
\begin{aligned}
[l_1, l_1^{2\varphi}] &= [l_1, l_1^{\varphi}][l_1, l_1^{\varphi}] && \text{by (3)} \\
&= [a_0^{-2}, l_1^{\varphi}][l_1, a_0^{-2\varphi}] && \text{since } a_0^2 = l_1^{-1} \\
&= [a_0, l_1^{2\varphi}]^{-1}[l_1^{2}, a_0^{\varphi}]^{-1} && \text{by Proposition 6(ii)}
\end{aligned}
$$

$=[a_0, l_1^{2\varphi}]^{-1}[[a_0, l_1^{2\varphi}]^{-1}]^{-1}$ by Proposition 5(i)

$=1$

$[l_2, l_1^{2\varphi}] = [l_2, l_1^{\varphi}][l_2, l_1^{\varphi}]$ by (3)

$= {}^{a_0}[l_2, l_1^{\varphi}][l_2, l_1^{\varphi}]$

$=[l_2^{-1}, l_1^{\varphi}][l_2, l_1^{\varphi}]$ since ${}^{a_0}l_2 = l_2^{-1}$

$=[l_2, l_1^{\varphi}]^{-1}[l_2, l_1^{\varphi}]$ by Proposition 6(i)

$=1$

By using similar arguments $[l_i, l_j^{2\varphi}] = 1$ for all $1 \le i,\ j \le 3$. Therefore, $[S_1(3), S_1(3)'^{\varphi}] = \langle [a_0, a_0^{\varphi}], [a_1, a_1^{\varphi}], [a_0, a_1^{\varphi}], [a_0, l_2^{\varphi}], [a_1, l_1^{\varphi}] \rangle$. However, $[a_0, a_0^{\varphi}]$ and $[a_1, a_1^{\varphi}]$ are the elements of $\nabla(S_1(3))$. Thus,

$$E(S_1(3)) = \langle [a_0, a_1^{\varphi}], [a_0, l_2^{\varphi}], [a_1, l_1^{\varphi}] \rangle.$$

By Proposition 2(ii),

$[S_1(3), S_1(3)^{\varphi}] = \nabla(S_1(3))E(S_1(3))$

$$= \langle [a_0, a_0^{\varphi}], [a_1, a_1^{\varphi}], [a_0, a_1^{\varphi}][a_1, a_0^{\varphi}], [a_0, a_1^{\varphi}], [a_0, l_2^{\varphi}], [a_1, l_1^{\varphi}] \rangle.$$

Next, the order of the six generators of $[S_1(3), S_1(3)^{\varphi}]$ will be determined. By Proposition 11, both $[a_0, a_0^{\varphi}]$ and $[a_1, a_1^{\varphi}]$ have order 8 while $[a_0, a_1^{\varphi}][a_1, a_0^{\varphi}]$ has order 4. By Theorem 2, $\kappa([a_0, a_1^{\varphi}]) = [a_0, a_1] = l_1 l_2^{-1} l_3$, $\kappa([a_0, l_2^{\varphi}]) = [a_0, l_2] = l_2^2$ and $\kappa([a_1, l_1^{\varphi}]) = [a_1, l_1] = l_1^2$. Since $[a_0, a_1^{\varphi}]$, $[a_0, l_2^{\varphi}]$ and $[a_1, l_1^{\varphi}]$ are all in $S_1(3)'$ and all the element in $S_1(3)'$ have infinite order, hence $[a_0, a_1^{\varphi}]$, $[a_0, l_2^{\varphi}]$ and $[a_1, l_1^{\varphi}]$ have infinite order.

Next, the six generators of $[S_1(3), S_1(3)^{\varphi}]$ will be shown to be independent. By Theorem 2, the generators of $[a_0, a_1^{\varphi}], [a_0, l_2^{\varphi}]$ and $[a_1, l_1^{\varphi}]$ are not in the kernel of κ. Hence, $[a_0, a_1^{\varphi}], [a_0, l_2^{\varphi}]$ and $[a_1, l_1^{\varphi}]$ cannot be a product of others or it is a contradiction that it would be in the kernel of κ. By order restrictions, $[a_0, a_0^{\varphi}]$ $[a_1, a_1^{\varphi}]$ and $[a_0, a_1^{\varphi}][a_1, a_0^{\varphi}]$ are independent generators of $[S_1(3), S_1(3)^{\varphi}]$.

By Proposition 2, $[S_1(3), S_1(3)^{\varphi}] = \nabla(S_1(3))E(S_1(3))$ Since $\nabla(S_1(3))$ is normal then all the generator commute to each other. Hence, $\nabla(S_1(3))$ is abelian. In order to show $E(S_1(3))$ is abelian, we need to show that all elements commute in $E(S_1(3))$.

$$
\begin{aligned}
[[a_0, a_1^{\varphi}],[a_0, l_2^{\varphi}]] &= [[a_0, a_1],[a_0, l_2]^{\varphi}] \\
&= [(l_1 l_2^{-1} l_3), l_2^{2\varphi}] \\
&= {}^{a_0}[l_1, l_2^{2\varphi}][l_2, l_2^{2\varphi}]^{-1\,a_1}[l_3, l_2^{2\varphi}]\,[l_1, l_2^{2\varphi}][l_2, l_2^{2\varphi}]^{-1}[l_3, l_2^{2\varphi}] \\
&= [l_1, l_2^{-2\varphi}][l_2, l_2^{2\varphi}]^{-1}[l_3^{-1}, l_2^{2\varphi}]\,[l_1, l_2^{2\varphi}][l_2, l_2^{2\varphi}]^{-1}[l_3, l_2^{2\varphi}] \\
&= [l_1, l_2^{2\varphi}]^{-1}[l_2, l_2^{2\varphi}]^{-1}[l_3, l_2^{2\varphi}]^{-1}\,[l_1, l_2^{2\varphi}][l_2, l_2^{2\varphi}]^{-1}[l_3, l_2^{2\varphi}] \\
&= [l_2, l_2^{2\varphi}]^{-2} \\
&= [l_2^{2}, l_2^{2\varphi}]^{-1} \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
[[a_0, a_1^{\varphi}],[a_1, l_1^{\varphi}]] &= [[a_0, a_1],[a_1, l_1]^{\varphi}] \\
&= [(l_1 l_2^{-1} l_3), l_1^{2\varphi}] \\
&= [l_1, l_1^{2\varphi}]^{a_0}[l_2, l_1^{2\varphi}]^{-1\,a_1}[l_3, l_1^{2\varphi}]\,[l_1, l_1^{2\varphi}][l_2, l_1^{2\varphi}]^{-1}[l_3, l_1^{2\varphi}] \\
&= [l_1, l_1^{2\varphi}][l_2^{-1}, l_1^{2\varphi}]^{-1}[l_3^{-1}, l_2^{2\varphi}]\,[l_1, l_1^{2\varphi}][l_2, l_1^{2\varphi}]^{-1}[l_3, l_1^{2\varphi}] \\
&= [l_1, l_1^{2\varphi}][l_2, l_1^{2\varphi}][l_3, l_1^{2\varphi}]^{-1}\,[l_1, l_1^{2\varphi}][l_2, l_1^{2\varphi}]^{-1}[l_3, l_1^{2\varphi}] \\
&= [l_1, l_1^{2\varphi}]^{2} \\
&= [l_1^{2}, l_1^{2\varphi}] \\
&= 1
\end{aligned}
$$

$$
[[a_0, l_2^{\varphi}],[a_1, l_1^{\varphi}]] = [[a_0, l_2],[a_1, l_1]^{\varphi}]
$$

$$=[l_2{}^2, l_1{}^{2\varphi}]$$

$$=[l_2, l_1{}^{2\varphi}][l_2, l_1{}^{2\varphi}]$$

$$= {}^{a_0}[l_2, l_1{}^{2\varphi}][l_2, l_1{}^{2\varphi}]$$

$$=[l_2{}^{-1}, l_1{}^{2\varphi}][l_2, l_1{}^{2\varphi}]$$

$$=[l_2, l_1{}^{2\varphi}]^{-1}[l_2, l_1{}^{2\varphi}]$$

$$=1$$

By similar arguments, $[[a_0, l_2^{\varphi}],[a_0, a_1^{\varphi}]]=1$, $[[a_1, l_1^{\varphi}],[a_0, a_1^{\varphi}]]=1$ and $[[a_1, l_1^{\varphi}],[a_0, l_2^{\varphi}]]=1$.

Since $[[x_1, y_1{}^{\varphi}],[x_2, y_2{}^{\varphi}]]=1$ for all $[x_1, y_1{}^{\varphi}],[x_2, y_2{}^{\varphi}]$ in $E(S_1(3))$, then we can conclude that $E(S_1(3))$ is abelian. Therefore, we can conclude that

$$S_1(3)\otimes S_1(3)=\nabla(S_1(3))E(S_1(3))\cong C_4\times C_8^2\times C_0^3.$$

is abelian. □

Next, Theorem 4 gives the generalization of the nonabelian tensor square of Bieberbach group with elementary abelian 2-group point group up to dimension *n*.

Theorem 4

For the Bieberbach group of $S_1(n)$,

$$S_1(n)\otimes S_1(n)\cong C_4{}^{4n-11}\times C_8{}^2\times C_0{}^{n^2-6n+12} \quad \text{for} \quad n\geq 4.$$

Proof. By Proposition 8, $S_1(n)=S_1(3)\times F_{n-3}^{ab}$ for $n\geq 3$. Then by Proposition 9,

$$S_1(n)\otimes S_1(n)=(S_1(3)\otimes S_1(3))\times(S_1(3)\times F_{n-3}^{ab})\times(F_{n-3}^{ab}\otimes S_1(3)^{ab})\times(F_{n-3}^{ab}\otimes F_{n-3}^{ab}).$$

By Theorem 3, $S_1(3)\otimes S_1(3)\cong C_4\times C_8^2\times C_0^3$. Then, by Proposition 10 (ii), we have $S_1(3)^{ab}\cong C_4\times C_4$. By using Proposition7(i),

$$S_1(3)^{ab}\otimes F_{n-3}^{ab}\cong(C_4\times C_4)\otimes C_0^{n-3}$$

$$=(C_4\otimes C_0^{n-3})\times(C_4\otimes C_0^{n-3})$$

$$=C_4^{n-3}\times C_4^{n-3}$$

And by symmetry,

$$F_{n-3}^{ab} \otimes S_1(3)^{ab} = C_4^{n-3} \times C_4^{n-3}.$$

Finally, by Proposition 7(ii) we have,

$$F_{n-3}^{ab} \otimes F_{n-3}^{ab} = C_0^{n-3} \times C_0^{n-3} = C_0^{(n-3)^2}.$$

By collecting terms, then $S_1(n) \otimes S_1(n)$

$$\cong C_4 \times C_8^2 \times C_0^3 \times C_4^{n-3} \times C_4^{n-3} \times C_4^{n-3} \times C_4^{n-3} \times C_0^{(n-3)^2}$$

$$= C_4^{1+(n-3)+(n-3)+(n-3)+(n-3)} \times C_8^2 \times C_0^{3+(n-3)^2}$$

$$= C_4^{4n-11} \times C_8^2 \times C_0^{n^2-6n+12}$$

which completes the proof. □

3. CONCLUSION

In this paper, the nonabelian tensor square of a Bieberbach group with elementary abelian 2-group point group, $S_1(3) \otimes S_1(3)$ is computed and is shown to be abelian. Then, the generalization of the nonabelian tensor square of $S_1(3)$ of dimension n is constructed. The findings of this research can be used for further research in computing and generalizing the other homological functors of this group.

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