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MATHEMATICAL RAINFALL MODEL FOR HYDROGRAPHIC DEMARCATION OF MANABI (ECUADOR)

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ABSTRACT

To determine the average annual rainfall at a specific site in a basin, it is necessary to count with rainfall information in the study area and topographic information. Counting with base information, is comes to the filling of them data missing with the help of any of them methods statistical existing for the effect. Thiessen polygons are built. Based on this procedure, may be elaborated isohyet or curves of equal rainfall, that are an input that can help also to estimate of precipitation in a specific geographical site. Here is a methodology that allows creating with the help of geographic information systems (GIS), a mathematical model to estimate very accurately the values of rainfall based only on the geographical coordinates. To achieve this objective, the basins of the Hydrographic Demarcation of Manabí have been chosen to develop the indicated mathematical model, which can be applied to other basins in the world.

Keywords: multiple regression; mathematical model; GIS; Hydrology; rainfall.

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1. INTRODUCTION

1.1. Introduction

By mathematical model referred to a description, in mathematical language, an object that exists in a no-mathematical universe and employing some kind of mathematical tokenism to express relations, substantive propositions of facts, variables, parameters, entities, and relationships between variables.

Mathematical models are called "phenomenological", when they try to stochastic type of hydrometeorological events and similar variables that are regularly measured at stations and special sites designed to this effect. [2, 7, 10].

In the research "Hydrological basis for the control and optimization of the use of the water resources of the Hydrographic Demarcation of Manabí"[6] the average annual precipitations were determined on the basis of records of the monthly precipitations for the period 1963 - 2013, which served as Input for the elaboration of the map of isohyets. [1, 3]. This map serves to graphically estimate annual mean precipitation in any site of the Manabí Demarcation, which has a subjective character with the probability of having errors in estimates. For this reason, a mathematical model has been developed for the determination of the mentioned precipitations in an objective way.

1.2. MATERIALS AND METHODS

Spatial modeling consists of three variables: 1) Rainfall, dependent variable *z*; 2) Longitude in the UTM system WGS84, independent variable *x*; and, 3) Latitude in the UTM system WGS84, independent variable *y*.

The variables *x* and *y* are taken directly from topographic maps; and the variable z, is determined with the help of the elements of geographic information systems (GIS). It is taken as the mathematical basis of modeling to the "multiple linear correlation" [4], which has an equation with 3 variables and 3 coefficients:

$$z = a + b_1 x + b_2 y \qquad (1)$$

Where: a, b_1, b_2 – are coefficients to be found; z – average annual rainfall *P*, mm; x – East longitude, m; y – North latitude, m.

To obtain the coefficients a, b_1 and b_2 of the multiple linear regression equation (1), using the least squares method, it is necessary to solve the following 3 equations:

$$\sum z = na + b_1 \sum x + b_2 \sum y \qquad (2)$$

$$\sum xz = a \sum x + b_1 \sum x^2 + b_2 \sum xy \qquad (3)$$

$$\sum yz = a \sum y + b_1 \sum xy + b_2 \sum y^2 \qquad (4)$$

The system of equations (2), (3), (4) can be solved by using specialized software such as AD +, SPSS, R, Minitab, Mathematica, Excel, among others.

The multiple correlation coefficient *R* is a dimensionless parameter whose absolute value can be between 0 and 1. The closer it gets to the unit, the greater the degree of associativity between the variables; and, as it approaches zero, the linear relationship tends to disappear. This coefficient is calculated with the formula [4, 11]:

$$R = \sqrt{\frac{r_{zx}^2 + r_{zy}^2 - 2r_{zx}r_{zy}r_{xy}}{1 - r_{xy}^2}} \qquad (5)$$

Where: r_{zx} is correlation coefficient for the series z and x, r_{zy} is correlation coefficient for the series z and y, r_{xy} is correlation coefficient for the series x and y.

Another statistical parameter that measures the data correlation is the standard error. The smaller the value, the better data fit. It is estimated with the formula:

$$s = \sqrt{\frac{\sum (Z - \check{Z})^2}{n - m - 1}} \qquad (6)$$

Where: s is standard error, Z is observed data, $\check{Z}$ is calculated data using the formula obtained, n is number of data analyzed, m is number of independent variables.

To solve the system of equations (2), (3) and (4), the following determinants must be calculated:

$$D = \begin{vmatrix} n & \sum x & \sum y \\ \sum x & \sum x^2 & \sum xy \\ \sum y & \sum xy & \sum y^2 \end{vmatrix} \qquad (7)$$

$$D_a = \begin{vmatrix} \sum z & \sum x & \sum y \\ \sum zx & \sum x^2 & \sum xy \\ \sum zy & \sum xy & \sum y^2 \end{vmatrix} \qquad (8)$$

$$D_{b1} = \begin{vmatrix} n & \sum z & \sum y \\ \sum x & \sum zx & \sum xy \\ \sum y & \sum zy & \sum y^2 \end{vmatrix} \qquad (9)$$

$$D_{b2} = \begin{vmatrix} n & \sum x & \sum z \\ \sum x & \sum x^2 & \sum zx \\ \sum y & \sum xy & \sum zy \end{vmatrix} \qquad (10)$$

Finally, the coefficients are calculated with the following formulas:

$$a = \frac{D_a}{D}, \quad b_1 = \frac{D_{b1}}{D}, \quad b_2 = \frac{D_{b2}}{D} \qquad (11)$$

Geographic information systems (GIS). Most of the elements that exist in nature can be represented by geometric figures, such as points, lines and polygons, ie vectors, or in the form of meshes with information (RASTERS). The vector and the raster (Fig. 1) are forms to represent the space, which help to interpret the elements investigated depending on their nature.

The main elements of which GIS are formed are vectors and rasters, which are elaborated taking into account certain principles and laws of Geoinformatics and the methods used in this science [5, 8, 9].

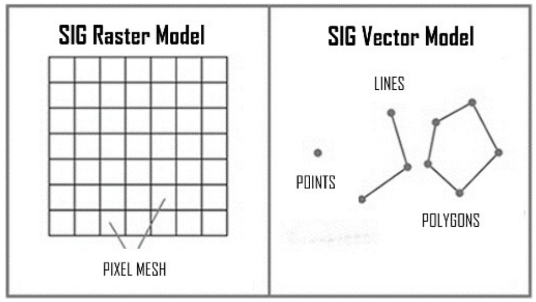

Fig.1. Raster and vector

Geographic information systems are closely linked to geo-information technologies, which can be defined as the set of programs and technological means to obtain new types of information about the surrounding world. The mentioned technologies are developed to increase the effectiveness of the processes of direction, conservation and presentation of the information, being constituted in a fundamental support in the decision making [5]. In this way, within the open source applications, it has as main QGIS and GvSIG, which can be downloaded free of charge from http://www.qgis.org/en/site/ and http: //www.gvsig.com/en, respectively. On the other hand, in the market there is the commercial application ArcGIS, which is widely distributed worldwide.

Basic information. For the development of the mathematical model; as a base information, we have: 1) Digital maps of the Hydrographic Demarcation of Manabi with its corresponding subdivision in microbasins according to the Pfafstetter methodology, 2) Multiannual average rainfall of 34 rainfall stations in the province of Manabí.

Fig. 2 shows the rainfall stations with their codes, average annual rainfall in millimeters, and the Pfafstetter coding of the microbasins of the Manabi Demarcation; in Table 1 the main data of its geographical location.

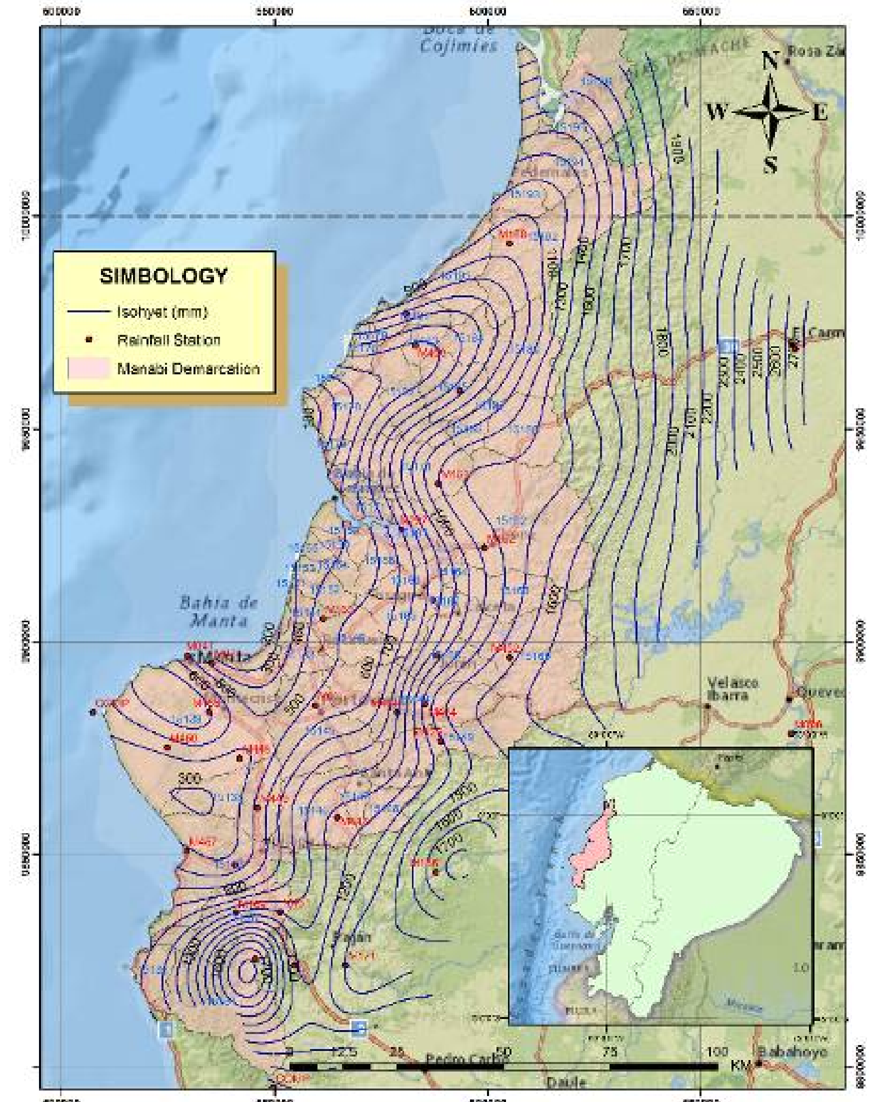

Fig.2. Isohyet map of annual average rainfall

Data processing. It is necessary to obtain one equation for each microbasin with the following calculation procedure: 1) With the data of the annual average rainfall and the geographic location of the rainfall stations, to prepare a raster precipitation map for the Hydrographic Demarcation of Manabi; 2) In the raster created, locate enough arbitrary points in such a way as to cover the areas of each of the 56 microbasins; 3) Using the GIS interpolation techniques, for all the plotted points, estimate the corresponding precipitation values; 4) For each microbasin export to Excel format the data of precipitation, as well as UTM coordinates, of the points that are inside; 5) Using multiple linear correlation, obtain the coefficients of equation (1) for each micro-basin.

Table 1. Rainfall stations, geographic location, annual average rainfall

№	Code Station	X-UTM	Y-UTM	Rainfall (mm)
1	M005	559523	9884982	528.0
2	M006	671167	9878373	2156.7
3	M026	684860	9947353	2768.8
4	M047	529608	9896745	397.2
5	M074	535232	9894995	270.2
6	M160	671939	9968948	2650.3
7	M162	599186	9922067	1233.4
8	M163	588400	9937145	1190.7
9	M165	561350	9905400	454.1
10	M166	587791	9845734	1657.2
11	M167	580800	9977125	778.8
12	M168	605098	9993552	1036.9
13	M169	540911	9836412	990.6
14	M171	566617	9823940	1308.6
15	M296	587159	9909725	847.6
16	M297	579744	9926307	705.8
17	M298	568607	9871041	859.8
18	M446	593441	9959038	767.5
19	M447	564710	9858637	1024.3
20	M448	541813	9872580	378.4
21	M449	545983	9860943	530.1
22	M450	524785	9875161	443.1
23	M451	551325	9836471	994.6
24	M452	605084	9896272	1472.6
25	M453	534613	9883481	609.9
26	M454	578716	9883443	891.3

27	M455	540758	9847496	459.0
28	M456	582963	9969540	480.3
29	M457	529325	9850844	416.0
30	M458	554628	9823913	1131.9
31	M459	545421	9825480	1671.5
32	M462	588084	9896706	1058.6
33	M464	585361	9885407	1234.5
34	MA29	589006	9876563	1287.8

2. RESULTS AND DISCUSSION

Fig. 3 shows the annual average rainfall raster generated by the GIS, and in Table 2, the values of the coefficients of equation (1) for the 56 micro-basins for the calculation of annual mean rainfall anywhere of the Hydrographic Demarcation of Manabi.

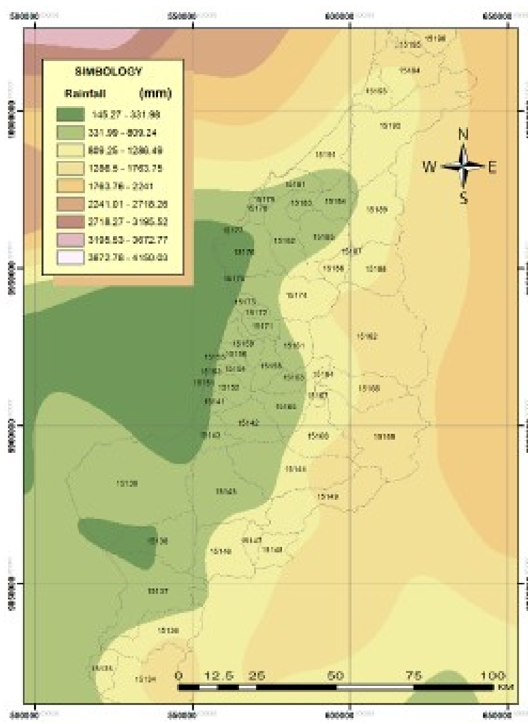

Fig. 3. Annual average rainfall raster

Table 2: Values of the coefficients of the equations to determine the annual average rainfall of the Hydrographic Demarcation of Manabi

№	Pfafstetter basin code	*a*	b_1	b_2	*R*
1	15134	-256793.99	0.0203	0.0252	0.817
2	15135	34391.54	0.0198	-0.0045	0.962
3	15136	481201.28	0.0205	-0.0500	0.928
4	15137	210107.75	0.0083	-0.0217	0.935
5	15138	53644.13	0.0140	-0.0062	0.900
6	15139	-21570.60	-0.0066	0.0026	0.895
7	15141	3990.94	0.0238	-0.0017	0.996
8	15142	31060.12	0.0093	-0.0036	0.934
9	15143	30130.91	0.0188	-0.0041	0.970
10	15144	63627.57	0.0329	-0.0083	0.981
11	15145	137089.49	0.0129	-0.0145	0.985
12	15146	63761.55	0.0208	-0.0076	0.973
13	15147	146077.05	0.0043	-0.0150	0.997
14	15148	171370.71	0.0184	-0.0183	0.993
15	15149	-18891.26	0.0207	0.0008	0.976
16	15151	-29460.50	0.0194	0.0019	1.000
17	15152	49671.91	0.0129	-0.0057	0.986
18	15153	-13352.34	0.0150	0.0005	0.999
19	15154	47419.94	0.0093	-0.0053	0.996
20	15155	20142.53	0.0094	-0.0025	0.999
21	15156	26278.35	0.0083	-0.0031	1.000
22	15158	-18859.25	0.0139	0.0012	0.908
23	15159	-12273.00	0.0122	0.0006	0.993
24	15161	-186150.62	0.0355	0.0167	0.997
25	15162	4857.58	0.0131	-0.0012	0.924

26	15163	-71330.30	0.0278	0.0056	0.991
27	15164	-75154.79	0.0310	0.0058	1.000
28	15165	27703.66	0.0234	-0.0041	0.982
29	15166	28108.33	0.0223	-0.0041	0.997
30	15167	26903.27	0.0308	-0.0045	0.999
31	15168	162597.75	0.0286	-0.0180	0.986
32	15169	65321.57	0.0181	-0.0076	0.975
33	15171	-129670.51	0.0277	0.0115	0.996
34	15172	-58369.00	0.0303	0.0042	0.997
35	15173	52136.81	0.0305	-0.0069	0.998
36	15174	38688.98	0.0341	-0.0058	0.993
37	15175	103563.83	0.0326	-0.0122	0.999
38	15176	10462.80	0.0280	-0.0026	0.954
39	15177	-217037.38	0.0226	0.0205	0.987
40	15178	-372721.08	0.0005	0.0374	0.996
41	15179	-446460.64	-0.0146	0.0457	0.999
42	15181	-411614.93	-0.0134	0.0421	0.992
43	15182	82896.64	0.0286	-0.0099	0.864
44	15183	-202158.78	0.0011	0.0203	0.990
45	15184	-34165.33	0.0218	0.0022	0.972
46	15185	246825.88	0.0237	-0.0261	0.989
47	15186	265622.65	0.0150	-0.0275	0.996
48	15187	222396.95	0.0212	-0.0235	1.000
49	15188	75603.58	0.0126	-0.0082	0.954
50	15189	93988.42	0.0298	-0.0111	0.989
51	15191	-247398.93	-0.0100	0.0255	0.982
52	15192	-84988.51	0.0175	0.0075	0.978
53	15193	-194177.77	-0.0077	0.0200	0.995
54	15194	-162562.75	0.0126	0.0156	0.950

55	15195	-169326.79	0.0090	0.0165	0.988
56	15196	-153805.63	0.0111	0.0148	0.974

Application example. Estimate the annual average precipitation for the micro-basin 15174 corresponding to the **coordinates** UTM 588567 E, 9938603 N.

From Table 2 we take the coefficients a, b1 and b2 for micro-basin 15174: a=38688.983, b1=0.0341316 and b2=-0.005796. The equation for the estimation of precipitation would be:

$$z = 38688.983 + 0.0341316x - 0.0057961y \quad (12)$$

By replacing the values of the given coordinates and making the respective calculation, we obtain that the precipitation is equal to 1173 mm, which can be corroborated graphically with the existing isohyet map (Fig. 2).

3. CONCLUSION

For the first time, 56 equations have been obtained for the estimation of annual mean precipitation according to the geographical coordinates.

The obtained equations are considered very reliable since there are high values of multiple correlation coefficients in the limits of 0.8170 - 0.9997.

4. ACKNOWLEDGEMENTS

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