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CHOOSING OF OPTIMAL START APPROXIMATION FOR LAPLACE EQUATION NUMERICALLY SOLVING

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ABSTRACT

In the last few years, repeatedly increased the role of simulation systems for solution of physical problems, particularly in the microwave and electronics. This article focuses on the promising methods for setting an initial approximation for the numerical solution of the Laplace equation. We investigate Dirichlet problem for a case of two-dimensional area with lime border, numerical scheme for solving this equation is widely knowns it finite difference method. One of the major stages in the algorithm for that numerical solution is choosing of start approximation, usually as the initial values of the unknown function are assumed to be zero, which may serve as a lead to a large number of iterations in finding the numerical solution. It is shown that there is a way to set a start approximation, which can significantly reduce the number of iterations in the solution of the Laplace equation.

Keywords: Laplace equation; approximation; net; Dirichlet problem; finite difference method.

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1. INTRODUCTION

The description of physical processes in many systems and devices, and devices is associated with a numerical solution of the Laplace equation. As Richard Feynman noted – physical phenomena that vary continuously in space and time are described by partial differential equations (PDE). The most important of these is Laplace's equation, which defines gravitational and electrostatic potentials as well as stationary flow of heat and ideal fluid [1]. Also, this equation can be used to describe the propagation of waves [2] the distribution of space charge in microwave technology [3- 6, 9] and other processes. There are many elegant analytical solutions to Laplace's equation in special geometries but nowadays real problems are usually solved numerically [7], and much more are methods of numerical solution of Laplace equation will list the main ones:

• The finite difference method

• Relaxation methods

• The finite element method

The finite difference method (FDM) is conceptually simple. The problems to which the method applies are specified by a PDE, a solution region (geometry), and boundary conditions. For more detailed derivations the reader may consult [10]. The finite difference method entails three basic steps:

• Divide the solution region into a grid of nodes. Grid points are typically arranged in a rectangular array of nodes.

• Approximate the PDE and boundary conditions by a set of linear algebraic equations (the finite difference equations) on grid points within the solution region.

• Solve this set of linear algebraic equations.

This method is good because it can be very easily implemented, but requires a large number of iterations to obtain the final solution [11].

Relaxation methods is a modification of the finite difference method the main idea is based on next sentence a relaxation step replaces the fun of every inner element by a better approximation based on the previous function value of the element and the value of its neighbors. This method strongly depends on the choice of the relaxation step and works well only on grid of small dimension [12, 13].

The finite element method (FEM) is a numerical technique for solving PDEs. FEM was originally applied to problems in structural mechanics. Unlike FDM, FEM is better suited for solution regions having irregularly shaped boundaries. The finite element analysis involves four basic steps:

• Divide the solution region into a finite number of elements. The most common elements have triangular or quadrilateral shapes. The collection of all elements should resemble the original region as closely as possible.

• Derive governing equations for a typical element. This step will determine the element coefficient matrix. Assemble all elements in the solution region to obtain the global coefficient matrix.

• Solve the resulting system of equations.

This method is great for large-scale grids, but it is very difficult to implement, and requires many resources. [14].

In [15] authors have proposed an original method for start approximation calculation in numerical solving of Laplace equation for space charge in charged electrical beam. The main idea based on a formula where the value of each inner node calculated the mean of the solution of one-dimensional Laplace equations, both along horizontal and vertical lines, and along diagonals intersecting at a given node.

This article shows a method to Laplace equation solution that removes the several shortcomings of these methods.

1.2 MATERIALS AND METHODS OF RESEARCH

Consider the two-dimensional Laplace equation for the area in Figure. 1 with known values of the function on the boundary of the domain:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0 \tag{1}$$

where U – unknown function. U can be found with relation to the known “cross” formula [11]

$$U_{i,j} = \frac{1}{4}(U_{i+1,j} + U_{i-1,j} + U_{i.j+1} + U_{i.j-1}) \tag{2}$$

We use FDM with modification.

Algorithm of the solution can entail next steps:

• Calculation of the boundaries.

• Using start approximation,

• Calculation of unknown function.

Usually value of the inner node of the grid assumed to be zero.

The essence of the proposed method of specifying the start approximation is as follows: at the beginning of the forward stroke at the stage defined by the initial values of the function at the nodes of the upper layer boundary cells (Fig. 2) according to the formula:

$$U_D = \frac{1}{3}(U_A + U_B + U_C) \tag{3}$$

Then we found values are searched for the values of the function of the cells of the second layer, and so on for subsequent layers. Then, at the stage of the reverse motion, the values of the function at the nodes of the boundary cells of the fourth layer (Number 4 in Fig. 2b) are determined similarly, and so on up to the upper layer. As a result, the value of the function used as the initial approximation is calculated as the average value obtained by forward and reverse for each node.

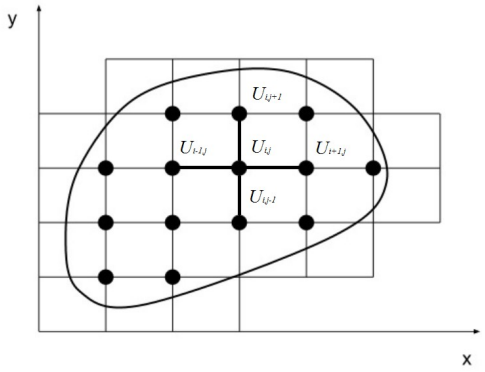

Fig.1. Area with border and inner nodes of the grid.

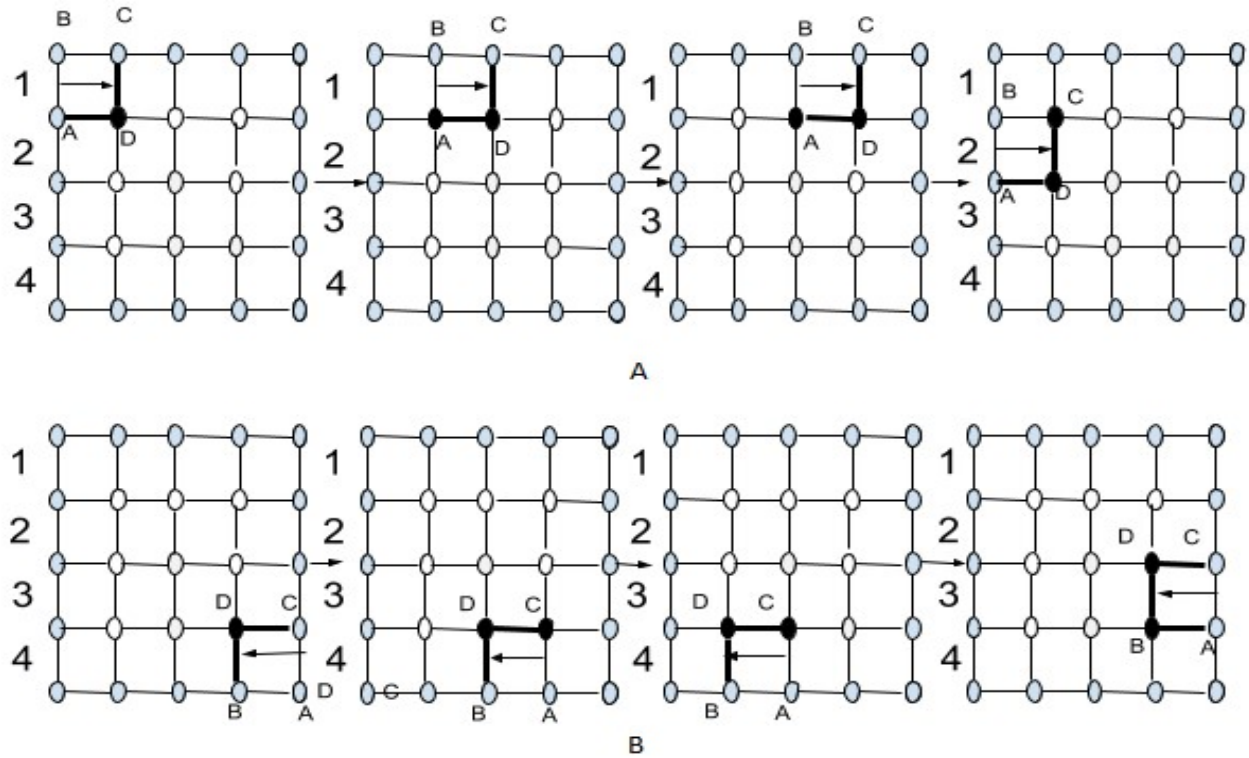

Fig.2. The proposed method of setting the initial approximation.
A) The forward motion of the start approximation; B) The reverse motion of the start approximation.

2. RESULTS AND DISCUSSION

The analysis was conducted to the scheme in Fig. 1, for area contains 100000 cells. We use are following boundary conditions:

$$U_{0,x} = x^2, U_{0,y} = y^2, U_{x,y} = 0, U_{y,x} = 0 \quad (4)$$

The solution of equation (1) with the boundary conditions (3) is shown in Fig. 3. In Fig. 4 shows the cross-sections of the final solution and the initial approximations given in different ways.

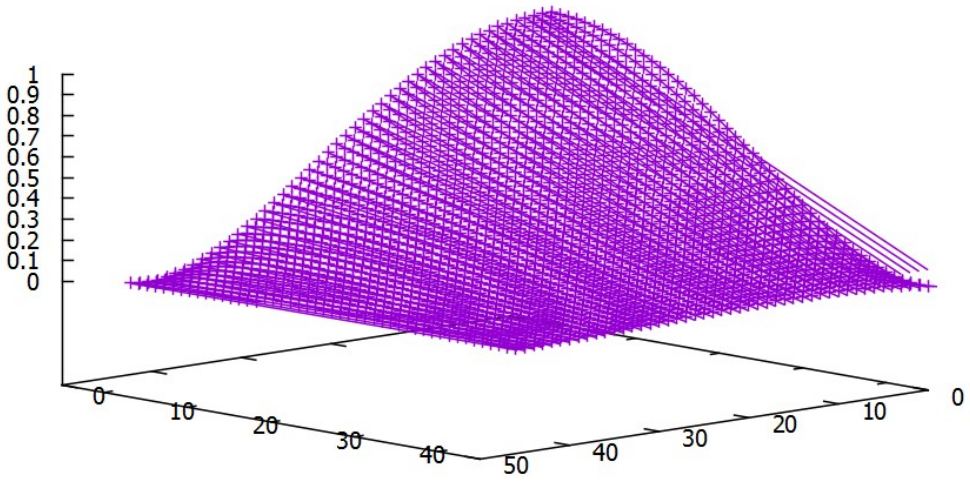

Fig.3. The final solution of Laplace equation.

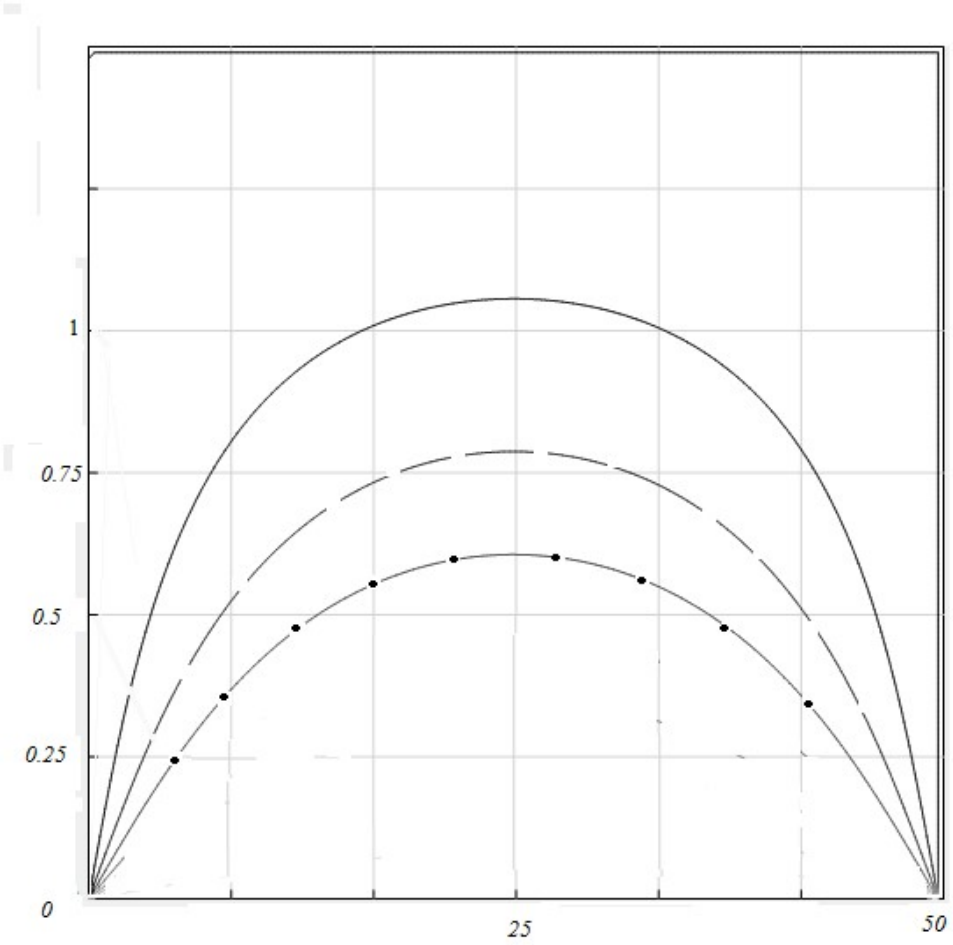

Fig.4. Cross section Solid line is the solution of Laplace equation. Dash dot Line is the initial approximation based on classic method after 1000 iterations. Dash line is the proposed initial approximation

Table 1. Comparison between methods

Area dimension	Classic FDM(value of inner node is zero)	Relaxation methods	Proposed method
10x10	30	28	28
50x50	361	318	342
100x100	1146	1001	1029
500x500	1407	1204	1316
1000x1000	3601	2008	1764
5000x5000	9601	11175	4840
10000x10000	21140	13186	10560
20000x20000	43840	23871	22711
30000x30000	57440	66381	27400
40000x40000	77221	100733	39761
50000x50000	100400	120879	60240
60000x60000	124476	140066	68761
100000x100000	3546807	7645807	1808871

3. CONCLUSION

As we can see, from the data in Table 1, and the graphs in Fig. 4, the proposed method has a gain, by the number of iterations. It should also be noted that the proposed method is more economical with respect to the cost of RAM (by 20% – 30%), in comparison with the classical.

In addition, it is necessary to note that on the grids of small dimension the proposed method loses the method of relaxations, however, on the grid of large dimension significantly exceed it.

In comparison with the method of finite differences, the proposed method wins on time execution of the algorithm, for example for a 1000000x1000000 grid, the execution time of the algorithm was 246 seconds versus 729, but on a grid of complex configuration with large differences in the values of the unknown function, the gain was 2 seconds.

The proposed method can be recommended in a wide range of physics tasks.

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