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SINUSOIDAL VELAROIDAL SHELL – NUMERICAL MODELLING OF THE NONLINEAR BUCKLING RESISTANCE

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ABSTRACT

Many works are devoted to linear and nonlinear analyses of shells of classical form. But for thin shells of complex geometry, many things remained to do. Four different sources of nonlinearity exist in solid mechanics. The geometric nonlinearity, the material nonlinearity, the kinetic nonlinearity and the force nonlinearity. The nonlinearity, applied to a sinusoidal velaroidal shell with the inner radius r_0, the outer variables radii from *10m* to *20m* and the number of waves *n=8,* will give rise to the investigation of its nonlinear buckling resistance. The building material is a high-performant concrete. The investigation emphasizes more on the material and the geometric nonlinearities. The result of the investigation is the buckling force of the shell under self-weight and uniformly vertically distributed load on its area, the corresponding numerical values of displacements and the buckling mode.

Keywords: Nonlinear analysis, nonlinear buckling resistance, numerical modelling, sinusoidal velaroidal shell, geometric nonlinearity, material nonlinearity, kinematic nonlinearity, force nonlinearity.

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1. INTRODUCTION

1.1 Shells of complex geometry and nonlinearity

The nonlinear analysis of thin-walled shells is not a rarity. Particularly the nonlinear strength one. Many works are devoted to linear and nonlinear analyses of shells of classical form: cylindrical, spherical, hemispherical, shallow, conical…But for thin shells of complex geometry, many things remained to do, even using commercial softwares.

The concept of shells of complex geometry appears when the coefficients of the fundamental quadratic forms of their middle surfaces are functions of the curvilinear coordinates. They are, most of them, constant for classical thin shells. The works [10, 11] are encyclopedic editions on analytic and differential geometry of regular analytical surfaces, which have found application in some parts of mathematics or in different branches of techniques and buildings. These surfaces can be used to design both classical and thin-walled shells of complex geometry with further linear or nonlinear analyses.

Concerning nonlinearity [12], although there are many ways of categorizing different nonlinearities, four different sources of nonlinearity exist in solid mechanics. Figure 1 illustrates the occurrence of these nonlinearities in their relations among applied loads, stresses, strains, displacements, and boundary conditions.

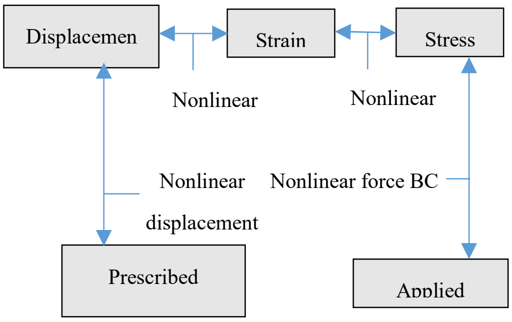


Fig. 1. Nonlinearities in solid mechanics

Geometric Nonlinearity

Geometric nonlinearities, in general, represent the cases when the relations among kinematic quantities (i.e., displacement, rotation, and strains) are nonlinear. Such nonlinearities often occur when deformation is large.

Material Nonlinearity

Material nonlinearity represents the case when the relation between stress and strain is not linear. This relation often referred to as the constitutive relation.

Kinematic Nonlinearity

Kinematic nonlinearity, called boundary nonlinearity, as the displacement boundary conditions depend on the deformations of the structure. In general, structural equations solve for unknown displacements in the domain with given applied loads and prescribed displacement boundary conditions. When the boundary conditions change as a function of displacements, both the displacements and boundary conditions are unknown.

Force Nonlinearity

Like kinematic nonlinearity, force nonlinearity occurs when the applied forces depend on deformation. Since force is a vector, its magnitude and/or direction can change according to the deformation of a structure. Force nonlinearity is often accompanied by geometric nonlinearity. The most common example in solid mechanics is pressure loads of fluids.

1.2 The analysis of achievements and publications on velaroidal surfaces

Velaroidal surfaces in Russia unlike countries of Western Europe and America [1-3] don't enjoy wide popularity, except for the parabolic velaroid which form is transferred to covering of "Darbazi" [14, 15].This name occurred from the name of far historic Georgian covering.

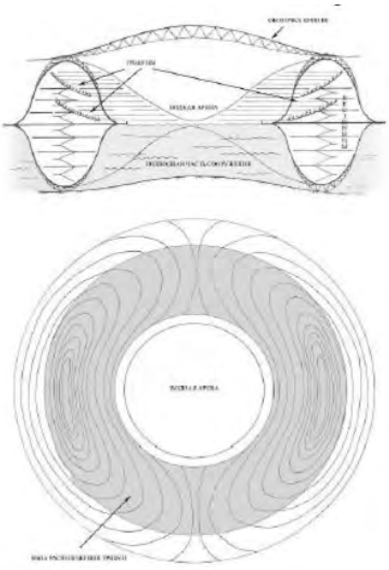


Fig. 2

Following E. Torroja and F. Candela's manuals, it is necessary to have shown possibilities of thin-walled spatial shell structures and to increase interest to design of wide-span spatial structures taking into consideration the emergence of new materials such as fibre concrete and the fibrous reinforced polymeric composites, and taking in view the extending progress in numerical methods of analysis.

A group of students of Engineering Faculty of the Peoples' Friendship University of Russia training in architecture became interested in results of geometrical researches of surfaces of velaroidal type. Within students' scientific society, they developed offers on application of the presented materials in landscape architecture of artificial and natural objects, in designing products for a decor and for shape generation of public buildings (Fig. 2).

In monograph [15], the author presents the nonlinear analysis as a necessity. (a) In designing high performance and efficient components of certain industries (e.g. aerospace, defense and nuclear). (b) In assessing functionality (e.g., residual strength and stiffness of structural elements) of existing systems that exhibit some types of damage and failure. (c) In establishing causes of system failure. (d) In simulating true material behavior of processes, and (c) research to gain a realistic understanding of physical phenomena. The following features of nonlinear analysis should be noted:

a. The principle of superposition does not hold.
b. Analysis can be carried out for one load case at a time.
c. The history (or sequence) of loading influences the response.
d. The initial state of the system (e.g. prestress) may be important.

Let us apply this theoretical aspect of the nonlinearity to a sinusoidal velaroidal shell, which is one of thin shells of complex geometry.

A sinusoidal velaroid generates by two families of half waves of the sinusoids lying in mutually perpendicular planes and facing by convexities into the same side [3]. Each set of sinusoids has the identical period. Sinusoidal velaroid is boundered by a flat rectangular contour.

Let's consider a sinusoidal velaroidal shell with the inner radius $r_0=1m$, the outer radius $R=20m$ and the number of waves $n=8$. An example of such a shell is shown in Fig. 2 and 3. [4], [5], [6]. When $r_0>0$, there is a hole at the center of the shell. It was assumed that the investigated shell is made of reinforced concrete.

Analysis of the stress-strain state and stability of the shell were performed by the finite element method (FEM) in certified software complexes Lira 10.4 and Stark ES 2015. The calculation was carried out in linear and nonlinear statement.

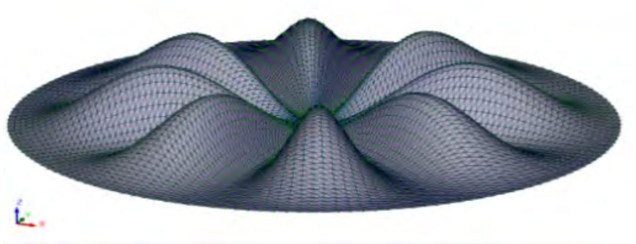


Fig.3. A sinusoidal velaroidal shell, $r_0=0$

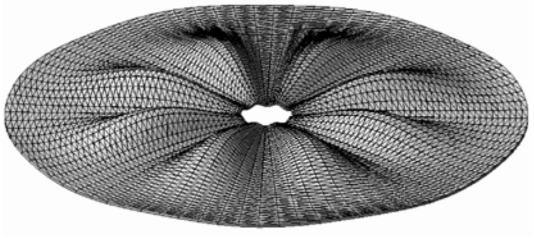


Fig.4. A FEM shell model

A FEM model of the shell (Fig. 4) consisted of 6400 elements and 3280 nodes, the total number of unknown – 18991. For surface modelling, was used flat shell elements having 6 degrees of freedom in each of their nodes. Boundary conditions – hinged support on the outer and inner contours.

2. RESULTS AND DISCUSSION

The stability analysis was carried out for different shell thicknesses (10, 15 and 20cm).

In the linear calculation was considered elastic performance of the material (concrete) in accordance with the linear graph shown in figure 5. While not considered redistribution of effort and the formation of cracks on the deformability and stability of the shell. Linear analysis was conducted with the following parameters of material (concrete): the initial modulus of elasticity $E=3\cdot10^4$ *MPa*; Poisson's ratio $\nu=0.18$; density $2.5\ t/m^3$.

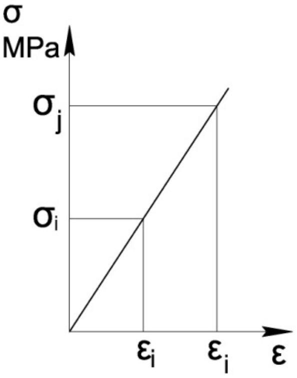


Fig. 5. For linear analysis: the law of deformation of the material

σ_i (σ_j) – stresses (MPa) and the corresponding elastic strains ε_i (ε_j)

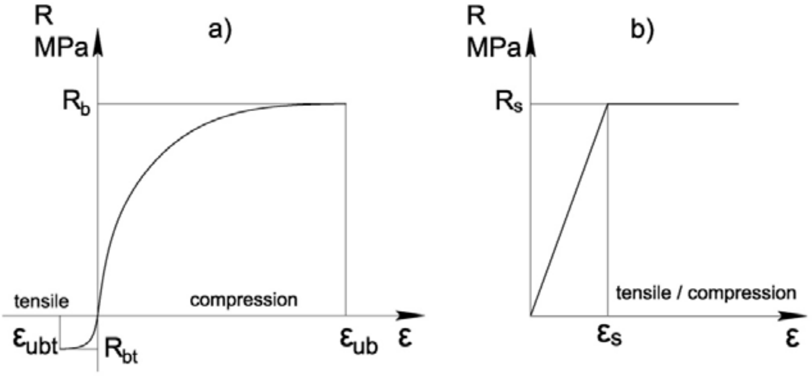


Fig. 6. For nonlinear analysis: a) the law of deformation of concrete; b) the law of the deformation of reinforcement

The performance of the considered shell, taking into account the non-elastic material properties in a nonlinear formulation is of special interest. In this regard, was performed a nonlinear calculation of the shell based on the modified method of Newton – Raphson.

In order to take into account the real properties of two-component material (reinforced concrete) were used nonlinear laws of deformation of concrete and reinforcing steel (Fig. 6).

It was performed cycles of nonlinear calculation with different shell thicknesses (10 cm, 15 cm and 20 cm) and different percentage of reinforcement (p = 2% p = 3%). In each cycle, effort, movement were determined, and buckling was studied.

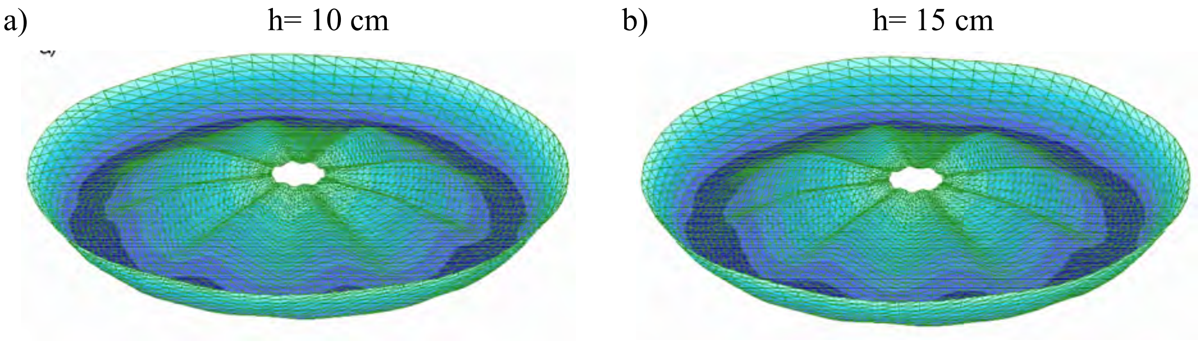
The calculations were carried out on the main combination of loads:

$$q = g + S \quad [t/m^2]; \tag{1}$$

The main results of the calculations are summarized in table 1.

The results of the analysis		The thickness of the shell h, cm	Type of analysis		
			Linear	Nonlinear	
				p=2%	p=3%
The maximum displacement w, mm		10	79.10	-	379.48
		15	48.80	260.31	189.13
		20	34.40	204.03	152.55
Factor of safety, K_S	1 Form	10	12.136	-	2.530
		15	32.219	6.040	8.313
		20	63.156	10.652	14.247
	2 Form	10	12.170	-	2.537
		15	32.330	6.061	8.341
		20	64.459	10.876	14.546
	3 Form	10	12.186	-	2.540
		15	32.363	6.067	8.350
		20	64.535	10.893	14.569

Figure 7 shows vertical displacements isofields corresponding to the thicknesses of the shell 10cm and 15cm.



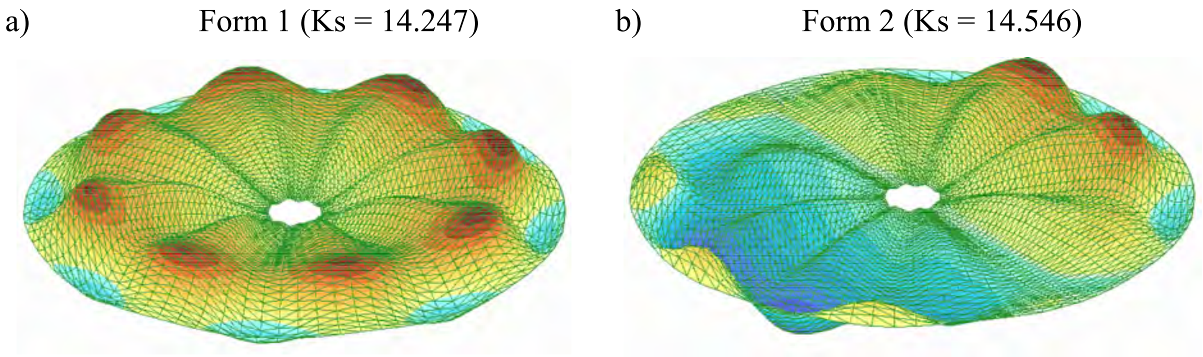
a) h= 10 cm b) h= 15 cm

Fig.7. Vertical displacements isofields:

a) when the shell thickness is *h = 10 cm* (*w = 379,48 mm*),

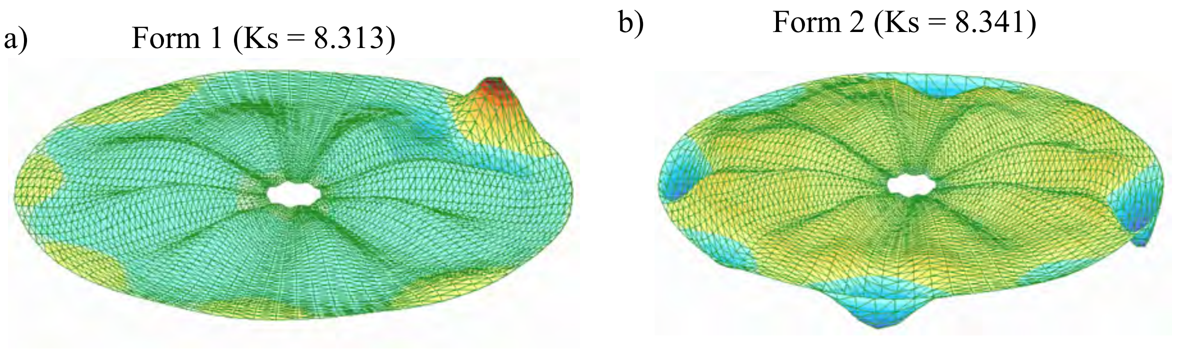
b) when the shell thickness is *h = 15 cm* (*w = 189,13 mm*)

In figures 8 and 9 are shows the buckling for shells with a thickness of 20 cm and 15 cm with the same reinforcement ratio (*p = 3%*).



a) Form 1 (Ks = 14.247) b) Form 2 (Ks = 14.546)

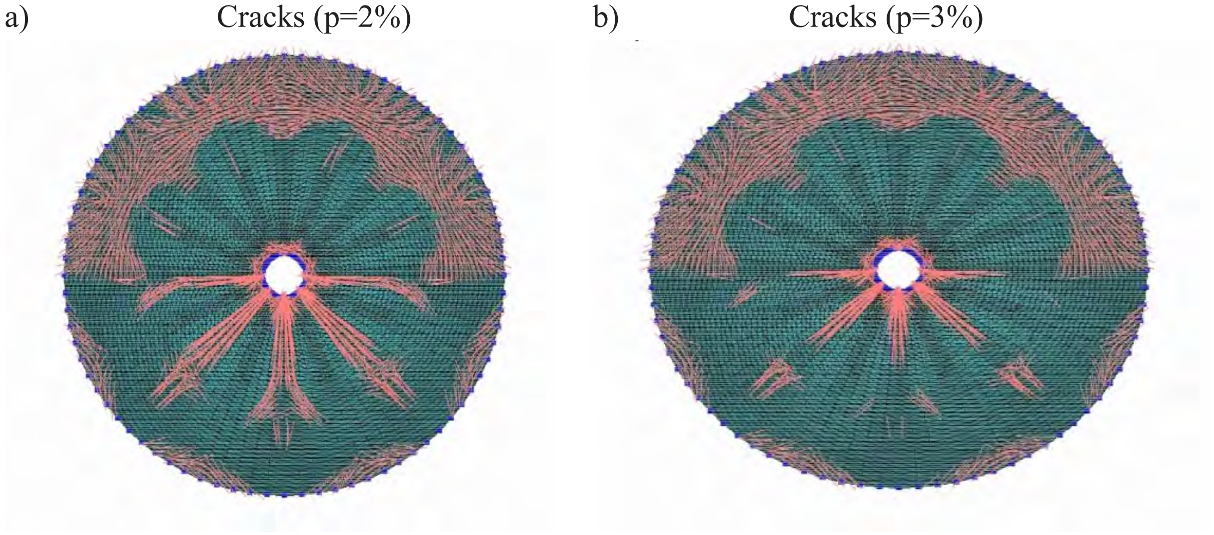
Fig. 8. Forms of loss of stability at *h = 20 cm* and *p = 3%:* a) form 1; b) form 2



a) Form 1 (Ks = 8.313) b) Form 2 (Ks = 8.341)

Fig. 9. Forms of loss of stability at *h = 15 cm* and *p = 3%:* a) form 1; b) form 2

Figure 10 shows the peculiarity of crack formation at different percentages of shell reinforcement. In both cases, the crack formation is not symmetric, although the structure and the loading are symmetric. The crack formation in the central zone of the shell decreases as the percentage of reinforcement increases. This appears in one half the shell were the crack formation is rare at the edge.


a) Cracks (p=2%)

b) Cracks (p=3%)

Fig. 10. Picture of destruction: a) $p = 2\%$, b) $p = 3\%$

3. CONCLUSION

The study has revealed certain defects of linear formulation of the problem, namely, the low displacement values and consequently, overestimated the coefficient of stability. Incorrect interpretation of results can give a false idea about the excessive reliability of the structure.

It turned out that displacements in nonlinear analysis exceed more than 4 times the corresponding displacements of the linear calculation. Thus, the coefficient of stability for the shell thickness of 20 cm for 1 form 63.156 (linear analysis) has decreased to 14,247 (non-linear analysis, $p = 3\%$).

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