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On Intuitionistic Gem Sets and Intuitionistic Turing Points

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ABSTRACT

Being familiar with intuitionistic set theory and with the introduction of neutrosophic theory as a generalization of fuzzy set theory and Atanassov's (IFS) theory, it is well known that there are numerous applications of intuitionistic, fuzzy, and neutrosophic sets in many fields such as topology, graph theory, image processing, algebra, medicine, etc. As a combination of the concept of Gem set and turing points with neutrosophic crisp sets, different sets and points, namely neutrosophic crisp Gem sets and neutrosophic turing points, are introduced and studied. In the same fashion, the concept of blending a Gem set with an intuitionistic set is introduced in this article in order to develop more concepts and theories on intuitionistic topological spaces. The main intention of this research article is to introduce the new types of intuitionistic sets and limit points in intuitionistic topological space, namely, intuitionistic Gem sets and intuitionistic turing points, respectively. In detail, the characteristics of intuitionistic Gem sets and a few counterexamples are discussed. As a result, the properties of the intutionistic Gem set and the relation between the mappings on intutionistic Gem sets are analysed.

Keywords: Intuitionistic ideal**,** Intuitionistic Gem sets, Intuitionistic Turing points, Vanishing intuitionistic Turing point.

1. INTRODUCTION

Intuitionistic ideals and intuitionistic fuzzy ideals (Bakhadach et al., 2016) are applied in geographic information systems, decision-making processes, and optimization techniques (Salama et al., 2014). The concept of Gem sets, turing points (Al-Swidi and Al-Nafee, 2013; Al-Nafee and Hamed, 2019), ideals (Al-Swidi and Al-Sada, 2012), separation axioms (Rathinam and Elango, 2014) in topological spaces (Munkers, 2013) and its extension such as soft Gem sets (Al-Nafee and Al-Swidi, 2018), soft turing points and soft ideals in soft topological spaces were introduced. This article is framed in a view that it will promote future studies in applications as mentioned earlier. Special points in natural intuitionistic sets, such as intuitionistic points, vanishing intuitionistic points, intutionistic fuzzy topological space (Coker, 1996; Coker, 2000) and intutionistic fuzzy ideals are focused in this paper in order to introduce the new points namely intuitionistic turing points and intuitionistic vanishing turing

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points. The idea of crisp neutrosophic Gem sets and crisp neutrosophic turing points (Al-Nafee et al., 2020), induced the idea to bring forth the concept of intuitionistic Gem sets, intuitionistic turing points and vanishing intuitionistic turing points.

Intuitionistic points form as the base for the neighborhood structures in intuitionistic topological spaces; similarly, neutrosophic crisp sets (Kroumov et al., 2014) and points form the base for neighborhood structures in neutrosophic crisp topological spaces. Neutrosophic crisp bitopological spaces (Al-Hamido, 2018), binary neutrosophic crisp sets and points (Rose Venish et al., 2023) and neutrosophic simply soft open sets in soft topological space (Das and Pramanik, 2020) were also introduced, which are useful for this new study. The notion of fuzzy set (Zadeh, 1965) and intuitionistic fuzzy points (IFP) was introduced earlier and its extensions were studied (Sardar et al., 2011). They are used in many areas to deal with quasicoincidence for intuitionistic fuzzy points (Lupianez, 2005) and to obtain few relations between the intuitionistic fuzzy ideals of a semigroup S and the set of all IFPs of S (Khan and Anis, 2013; Jun and Song, 2006). All these imply the importance of intuitionistic turing points and vanishing intuitionistic turing points that are introduced in this paper. Future studies can be done in the above-discussed fields based on the special points introduced in this paper. Following the introduction, preliminary concepts, such as basic definitions and proposition are established, after which intuitionistic Turing points and vanishing intuitionistic Turing points are defined. The intuitionistic Gem set is then introduced and its properties are analyzed using counter examples, leading to the conclusion.

2. PRELIMINARIES

Definition 2.1 (Coker, 1996) Suppose T be a non-empty fixed set, an intuitionistic set (IS) A is an object having the form $A = \langle T, A_1, A_2 \rangle$, where A_1 and A_2 are subsets of T satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of members of A, while A_2 is called the set of non members of A.

Definition 2.2 (Coker, 1996) An intuitionistic topology on a non-empty set T is a family η of IS's in T satisfying:

1. $T, \phi \in \eta$.
2. $A_1 \cap A_2 \in \eta$ for any $A_1, A_2 \in \eta$.
3. $\cup A_i \in \eta$ for any arbitrary family $\{A_i : i \in J\} \subseteq \eta$.

(T,η) is said to be intuitionistic topological space (ITS) and IS in η is said to be as an intuitionistic open set (IOS) in T, the complement of IOS is said to be intuitionistic closed set (ICS).

Definition 2.3 (Coker, 1996) Suppose T be a non empty set and $p \in T$ a fixed element in T. IS $\underset{\sim}{p} = \langle T, \{p\}, \{p\}^c \rangle$ is an intuitionistic point (IP) in T and IS $(\underset{\approx}{p}) = \langle T, \phi, \{p\}^c \rangle$ is said to be a vanishing intuitionistic point (VIP) in T.

Definition 2.4 (Coker, 1996) Suppose (T,η) be an ITS and let $A = \langle T, A_1, A_2 \rangle$ be an IS in T, then the interior and closure of A are :

Iint(A) = $\bigcup\{$ K : K be an IOS in X and K $\subseteq$ A$\}$.

Icl(A) = $\bigcap\{$ J : J be an ICS in X and A $\subseteq$ J $\}$.

Definition 2.5 (Coker, 1996) Suppose I_I be a non empty collection of intuitionistic sets over T. Then I_I is called intuitionistic ideal on T if,

- $C \in I_I$ and $D \in I_I$ then $C \cup D \in I_I$
- $C \in I_I$ and $D \subseteq C$ then $D \in I_I$.

Definition 2.6 (Al-Swidi and Al-Nafee, 2013) For a topological space (T, η), $T \in T, Y \subseteq T$, we define an ideal ${}^{Y}I_T$ w.r.t the subspace (Y, η_Y) as : ${}^{Y}I_T = \{G \subseteq Y : T \in (T - G)\}$.

Definition 2.7 (Al-Swidi and Al-Nafee, 2013) For a topological space (T, η), $Y \subseteq T$, for each $G \neq \phi$, then ${}^{Y}I_T = \{G \subseteq Y : T \in (T - G)$, for every $p \in Y\}$ = $\{G \subseteq Y : T \in (Y - G)$, for every $p \in Y\}$.

Definition 2.8 (Al-Swidi and Al-Nafee, 2013) Gem set is defined as:for a topological space (T, η), $Y \subseteq T$ and $A \subseteq Y$, we define ${}^{Y}A^{*T}$ w.r.t the subspace (Y, T_Y) as follows: for $A \subseteq Y$, ${}^{Y}A^{*T} = \{y \in Y : G \cap A \notin I_Y$, for every $G \in \eta_y\}$, where $T_Y(Y) = \{G \in T_Y : y \in G\}$.

Proposition 2.9 (Al-Swidi and Al-Nafee, 2013) Let (T, η) be a topological spaces and let A, B be subsets of T, $p \in T$. Then

- $\phi^{*p} = \phi$
- $T^{*p} = T$, whenever $I_p = \phi$
- $A \subset B \Rightarrow A^{*p} \subset B^{*p}$
- For another ideal $I_{p_1} \supset I_{P_2}$ on T, $A^{*p_1} \subset A^{p_2}$
- If $p \in T$, then $p \in A$ iff $p \in A^{*p}$
- If $p \in A$, then $(p^{*T})^{*p} = A^{*p}$
- If $p_1 \in A, p_2 \in B$ such that $p_1 \neq p_2$ then $A^{*p_1} \cap B^{*p_2} = \phi$
- If $p_1, p_2 \in T$ such that $p_1 \neq p_2$, then $p_2 \in \{p_1\}^c \Rightarrow p_1 \notin \{p_1\}^{*p_2}$ and $p_2 \notin \{p_2\}^{*p_1}$
- $A^{*p_1} \cup B^{*p_2} = (A \cup B)^{*p_1}$

- $(A \cup B)^{*p_1} \subset A^{*p_1} \cap B^{*p_2}$
- $A^{*p} \subset cl(A)$.

3. INTUITIONISTIC TURING POINT

We introduce in this section intuitionistic turing points and vanishing intuitionistic turing points and few of its properties are discussed.

Definition 3.1. Suppose (T, η) ITS, $p \in T$, we define an intuitionistic ideal with respect to an intuitionistic point p as follows: $I_I\ (p) = \{D \in \eta: p \in (D)^c\}$.

Example 3.2. Let (T, η) be an ITS such that $T = \{1,2,3\}$ $\eta = \{\phi, T, A_1, A_2, A_3, A_4\}$ $p = \langle T, \{1\}, \phi\rangle$ such that $A_1 = \langle T, \{1\}, \{2\}\rangle$ $A_2 = \langle T, \{3\}, \{1\}\rangle$ $A_3 = \langle T, \{1,3\}, \phi\rangle$ $A_4 = \langle T, \phi, \{1,2\}\rangle$ $A_1^c = \langle T, \{2\}, \{1\}\rangle$ $A_2^c = \langle T, \{1\}, \{3\}\rangle$ $A_3^c = \langle T, \varphi, \{1,3\}\rangle$ $A_4^c = \langle T, \{1,2\}, \varphi\rangle$. Then $I_I(\underset{\sim}{p}) = \{\underset{\sim}{\varphi}, A_4\}$

Definition 3.3. Suppose (T, η) be an ITS , $\underset{\sim}{p} \in (T, \eta)$, $Y \subseteq T$, we define an intuitionistic ideal ${}^{Y}I_I(\underset{\sim}{p})$ with respect to the subspace (Y, η_Y) as follows: ${}^{Y}I_I(\underset{\sim}{p}) = \{D \in \eta_Y: \underset{\sim}{p} \in (T - D)\}$.

Remark 3.4. Suppose (T, η) be an ITS, $Y \subseteq T$, for every $D \neq \varphi$ and $\underset{\sim}{p} \in IP$ in T, then:

$$ {}^{Y}I_I\left(\underset{\sim}{p}\right) = \{D \in \eta_Y: \underset{\sim}{p} \in (T - D)\} $$

$= \{D \in \eta_Y: \underset{\sim}{p} \in (Y - D)\}$.

Proof. Assume that (T, η) be an intuitionistic topological with , $Y \subseteq T$. Using the defenition of intuitionistic ideal ${}^{Y}I_I(\underset{\sim}{p})$ with respect to the subspace (Y, η_Y), we have,

${}^{Y}I_I(\underset{\sim}{p}) = \{D \in \eta_Y: \underset{\sim}{p} \in (T - D)\}$

$= \{D \in \eta_Y$: for every $\underset{\sim}{p} \in Y, \underset{\sim}{p} \notin D\} = \{D \in \eta_Y$: for every $\underset{\sim}{p} \in Y, \underset{\sim}{p} \in (Y - D)\}$.

Remark 3.5. Let (T, η) be an ITS, $Y \subseteq T$, for each $D \neq \underset{\sim}{\varphi}$ and $\underset{\sim}{p} \in IP$ in T, then:

$$ {}^{Y}I_I\left(\underset{\sim}{p}\right) = \{D \in \eta_Y: \underset{\sim}{p} \in (T - D)\} $$

$= \{D \cap Y$ for each $D \neq \underset{\sim}{\varphi} \in I_I \underset{\sim}{p}\}$.

Proof. Assume that (T, η) be an intuitionistic topological with , $Y \subseteq T$. Using the defenition of intuitionistic ideal ${}^{Y}I_I(\underset{\sim}{p})$ with respect to the subspace (Y, η_Y), we have,

$$ {}^{Y}I_I(\underset{\sim}{p}) = \{D \in \eta_Y: \underset{\sim}{p} \in (T - D)\} $$

$= \{D \in \eta_Y$: for every $\underset{\sim}{p} \in Y, \underset{\sim}{p} \notin D\} = \{D \cap Y$ for each $D \neq \underset{\sim}{\varphi} \in I_I \underset{\sim}{p}\}$.

Definition 3.6. Suppose (T,η) be an ITS, $\underset{\sim}{p} \in IP's$ in T and I_I be an intuitionistic turing point of T if $D^c \in I_I$for every $D \in \eta_Y, \eta_Y$ is the collection of all intuitionistic open sets of IP $\underset{\sim}{p}$.

Example 3.7. Suppose (T,η) be ITS s.t $T = \{a,b,c\}$ $\eta = \{\underset{\sim}{\varphi}, \underset{\sim}{T}, A_1, A_2\}$

$p_1 = \langle T, \{c\}, \{a\}\rangle$, $p_2 = \langle T, \{a\}, \{b\}\rangle$ such that $A_1 = \langle T, \{a\}, \{c\}\rangle$ $A_2 = \langle T, \{a,b\}, \varphi\rangle$

$$A_1^c = \langle T, \{c\}, \{a\}\rangle \;\; A_2^c = \langle T, \varphi, \{a, \mathrm{b}\}\rangle$$

Then $I_I(p_1) = \{\varphi, A_1\}$ and $I_I(A_2) = \{\underset{\sim}{\varphi}\}$, here p_1 is an intuitionistic turing point of an intuitionistic ideal $I_I(p_1)$ but not p_2. Since $D^c \in I_I$.

Example 3.8. Let(T,η) be an ITS, $\underset{\sim}{p} \in IP's$ in T and $I_I(\underset{\sim}{p}) = \{D \in \eta : \underset{\sim}{p} \in (D)^c\}$ be an intuitionistic ideal of T. Then $\underset{\sim}{p}$ is an intuitionistic turing point of $I_I(\underset{\sim}{p})$

Proof. Given (T,η), $\underset{\sim}{p} \in IP's$ in T and $I_I(\underset{\sim}{p}) = \{D \in \eta : \underset{\sim}{p} \in (D)^c\}$ be an intuitionistic ideal of T. By the definition of intuitionistic turing point, $\underset{\sim}{p}$ is an intuitionistic turing point of $I_I(\underset{\sim}{p})$.

Definition 3.9. Suppose (T,η) be an ITS, $\underset{\approx}{p} \in T$, we define an intuitionistic ideal with respect to a vanishing intuitionistic point $\underset{\approx}{p}$ as follows: $I_I(\underset{\approx}{p}) = \{D \in \eta : \underset{\approx}{p} \in (D)^c\}$.

Example 3.10. Let (T,η) be an ITS such that $T = \{1,2,3\}$ $\eta = \{\underset{\sim}{T}, \underset{\sim}{\varphi}, A_1, A_2, A_3, A_4\}$

$\underset{\approx}{p_1} = \langle T, \varphi, \{1,3\}\rangle$, $\underset{\approx}{p_2} = \langle T, \varphi, \{3\}\rangle$ such that $A_1 = \langle T, \{1\}, \{2\}\rangle$

$A_2 = \langle T, \{3\}, \{1\}\rangle$ $A_3 = \langle T, \{1,3\}, \varphi\rangle$ $A_4 = \langle T, \varphi, \{1,2\}\rangle$

$A_1^c = \langle T, \{2\}, \{1\}\rangle$ $A_2^c = \langle T, \{1\}, \{3\}\rangle$ $A_3^c = \langle T, \varphi, \{1,3\}\rangle$ $A_4^c = \langle T, \{1,2\}, \varphi\rangle$

Then $I_I(\underset{\approx}{p_1}) = \{\underset{\sim}{\varphi}, A_1, A_2, A_3, A_2, A_4\}$ and $I_I(\underset{\approx}{p_2}) = \{\underset{\sim}{\varphi}, A_2, A_4\}$.

Definition 3.11. Suppose (T,η) be an ITS, $\underset{\approx}{p} \in (T,\eta)$, $Y \subseteq T$,we define an intuitionistic ideal ${}^{Y}I_I(\underset{\approx}{p})$ with respect to the subspace (Y,η_Y) as follows: ${}^{Y}I_I(\underset{\approx}{p}) = \{D \in \eta_Y : \underset{\approx}{p} \in (T - D)\}$.

Remark 3.12. Let (T,η) be an ITS, $Y \subseteq T$, for each $D \neq \underset{\sim}{\varphi}$ and $\underset{\approx}{p} \in IP$ in T, then:

$${}^{Y}I_I(\underset{\approx}{p}) = \left\{D \in \eta_Y : \underset{\approx}{p} \in (T - D)\right\} = \{D \in \eta_Y : \underset{\approx}{p} \in (Y - D)\}.$$

Proof. Assume that (T,η) be an intuitionistic topological space. Consider the vanishing intuitionistic point $\underset{\approx}{p} \in IP$ in T. Hence ${}^{Y}I_I(\underset{\approx}{p}) = \left\{D \in \eta_Y : \underset{\approx}{p} \in (T - D)\right\}$

$= \{D \in \eta_Y$: for every $\underset{\approx}{p} \in Y, \underset{\approx}{p} \notin D\}$ which implies ${}^{Y}I_I\left(\underset{\approx}{p}\right) =$ $\{D \in \eta_Y$: for every $\underset{\approx}{p} \in Y, \underset{\approx}{p} \in (T - D)\}$.

Remark 3.13. Let (T,η) be an ITS, $Y \subseteq T$, for each $D \neq \underset{\approx}{\varphi}$ and $\underset{\approx}{p} \in IP$ in T, then:

$${}^{Y}I_I\left(\underset{\approx}{p}\right) = \left\{D \in \eta_Y : \underset{\approx}{p} \in (T - D)\right\} = \{D \cap Y \text{ for each } D \neq \varphi \in I_I\, \underset{\approx}{p}\}.$$

Proof. Assume that (T,η) be an ITS, $Y \subseteq T$. Consider the vanishing intuitionistic point, $\underset{\approx}{p} \in IP$ in T, then by the remark 3.12, we have ${}^{Y}I_I\left(\underset{\approx}{p}\right) = \left\{D \in \eta_Y : \underset{\approx}{p} \in (T - D)\right\}$

$= \{D \in \eta_Y$: for each $\underset{\approx}{p} \in Y, \underset{\approx}{p} \notin D\}$ which implies ${}^{Y}I_I(\underset{\approx}{p}) = \{D \cap Y$ for each $D \neq \varphi \in I_I \underset{\approx}{p}\}$.

Definition 3.14. Let (T,η) be an ITS , $\underset{\approx}{p} \in IP's$ in T and I_I be an vanishing intuitionistic turing point of I_I if $D^c \in I_I$ for every $D \in \eta_p, \eta_p$ is the collection of all intuitionistic open sets of vanishing intuitionistic point $\underset{\approx}{p}$.

Example 3.15. Let (T,η) be an ITS s.t $T = \{1,2,3\}$ $\eta = \{\underset{\sim}{T}, \underset{\sim}{\varphi}, A_1, A_2, A_3, A_4\}$
$\underset{\approx}{p} = \langle T, \varphi, \{1,3\}\rangle$ such that $A_1 = \langle T, \{1\}, \{2\}\rangle$ $A_2 = \langle T, \{3\}, \{1\}\rangle$ $A_3 = \langle T, \{1,3\}, \varphi\rangle$ $A_4 = \langle T, \varphi, \{1,2\}\rangle$ $A_1^c = \langle T, \{2\}, \{1\}\rangle$, $A_2^c = \langle T, \{1\}, \{3\}\rangle$, $A_3^c = \langle T, \varphi, \{1,3\}\rangle$, $A_4^c = \langle T, \{1,2\}, \varphi\rangle$
Then $I_I(\underset{\approx}{p}) = \{\underset{\sim}{\varphi}, A_1, A_2, A_3, A_4\}$ and $\underset{\approx}{p}$ is not a vanishing intuitionistic turing point.

Remark 3.16. Let (T,η) be an ITS , $T,\eta \in IP's$in T and $I_I(\underset{\approx}{p}) = \{D \in \eta : \underset{\approx}{p} \in (D)^c\}$ be an intuitionistic ideal of T. Then $\underset{\approx}{p}$ is a vanishing intuitionistic turing point of $I_I(\underset{\approx}{p})$.

Proof. Given (T,η), an ITS. $\underset{\approx}{p} \in IP's$ in T and $I_I(\underset{\approx}{p}) = \{D \in \eta : \underset{\approx}{p} \in (D)^c\}$ be an intuitionistic ideal of T. By the definition of vanishing intuitionistic turing points, $\underset{\approx}{p}$ is a vanishing intuitionistic turing point of $I_I(\underset{\approx}{p})$.

4. INTUITIONISTIC GEM SETS

Definition 4.1. Suppose (T,η) be an ITs $\underset{\sim}{p} \in IPs$ in T, $I_I(\underset{\sim}{p})$ be an intuitionistic ideal on (T,η) and $D \subseteq (T,\eta)$ we define the intuitionistic Gem set $_ID^{*\underset{\sim}{p}}$ w.r.t space (T,η) as:
$_ID^{*\underset{\sim}{p}} = \{\underset{\sim}{p} \in IPs$ in T;$F \cap D \notin I_I(\underset{\sim}{p})$ for every $D \in \eta_p$,η_p is collection of all IOS of an intuitionistic point $\underset{\sim}{p}$.

Example 4.2. Let (T,η) be an ITs such that $T = \{1,2,3\}$
$\eta = \{\underset{\sim}{T}, \underset{\sim}{\varphi}, A_1, A_2, A_3, A_4\}$ $\underset{\sim}{p} = \langle T, \{1\}, \varphi\rangle, D = \langle T, \{3\}, \{1\}\rangle$ where $A_1 = \langle T, \{1\}, \{2\}\rangle$
$A_2 = \langle T, \{3\}, \{1\}\rangle$ $A_3 = \langle T, \{1,3\}, \varphi\rangle$ $A_4 = \langle T, \varphi, \{1,2\}\rangle$ $A_1^c = \langle T, \{2\}, \{1\}\rangle$
$A_2^c = \langle T, \{1\}, \{3\}\rangle$ $A_3^c = \langle T, \varphi, \{1,3\}\rangle$ $A_4^c = \langle T, \{1,2\}, \varphi\rangle$. Then $I_I(\underset{\sim}{p}) = \{\varphi, A_4\}$
$_ID^{*\underset{\sim}{p}} = \langle T, \{1\}, \varphi\rangle$

Theorem 4.3. Let (T,η) be an ITS, $\underset{\sim}{p}$ be an IP in T and suppose D, C be subsets of (T,η). Then

1. $\varphi^{*\underset{\sim}{p}} = \varphi$
2. $T^{*\underset{\sim}{p}} = T$ whenever $I_I(\underset{\sim}{p}) = \varphi$
3. $C \subseteq D \Rightarrow IC^{*\underset{\sim}{p}} \subseteq ID^{*\underset{\sim}{p}}$
4. $\underset{\sim_1}{p}, \underset{\sim_2}{p} \in IPs$ in T with $I_I(\underset{\sim_2}{p}) \supseteq I_I(\underset{\sim_1}{p})$ then $ID^{*\underset{\sim_2}{p}} \subseteq ID^{*\underset{\sim_1}{p}}$

5. $\underset{\sim}{p} \in D$ iff $\underset{\sim}{p} \in ID^{*\underset{\sim}{p}}$
6. If $\underset{\sim}{p} \in D$ then $(ID^{*\underset{\sim}{p}})^{*\underset{\sim}{p}}$
7. If $\underset{\sim 1}{p} \in D,\ \underset{\sim 2}{p} \in C$ with $\underset{\sim 1}{p} \neq \underset{\sim 2}{p}$,$D \cap C = \varphi$ then $ID^{*\underset{\sim 1}{p}} \cap IC^{*\underset{\sim 2}{p}} = \varphi$
8. If $\underset{\sim 1}{p}, \underset{\sim 2}{p} \in T$ with $\underset{\sim 1}{p} \neq \underset{\sim 2}{p}$, then $\underset{\sim 2}{p} \in (\underset{\sim 1}{p})^c \Rightarrow \underset{\sim 1}{p} \notin (\underset{\sim 1}{p})^{*\underset{\sim 1}{p}}$ and $\underset{\sim 2}{p} \notin (\underset{\sim 1}{p})^{*\underset{\sim 1}{p}}$.

Proof. By the definition of intuitionistic Gem set $I_I D^{*\underset{\sim}{p}}$, (1), (2) and (5) are true. Given D,C are subsets of (T, η). Consider $C \subseteq D$, applying the definition of Gem sets to the subsets C and D, we can find $I_I C^{*\underset{\sim}{p}} \subseteq I_I D^{*\underset{\sim}{p}}$. Hence (3) is proved. For proving (4) assume that $\underset{\sim 1}{p}, \underset{\sim 2}{p} \in IP's$ in T with $I_I(\underset{\sim 2}{p}) \supseteq I_I(\underset{\sim 1}{p})$ applying the definition $I_I D^{*\underset{\sim}{p}}$ sets and (3) then we can prove that $I_I D^{*\underset{\sim 2}{p}} \subseteq I_I D^{*\underset{\sim 1}{p}}$. Assume $\underset{\sim}{p} \in D$, using the definition of Gem set twice on D and (5) is proved that $(I_I D^{*\underset{\sim}{p}})^{*\underset{\sim}{p}}$. Therefore (6) is proved. Let $\underset{\sim 1}{p} \in D, \underset{\sim 2}{p} \in C$ with $\underset{\sim 1}{p} \neq \underset{\sim 2}{p}$, $D \cap C = \varphi$ by definition of Gem set and by (1), $I_I D^{*\underset{\sim 1}{p}} \cap I_I D^{*\underset{\sim 2}{p}} = \varphi$. Hence (7) is true. Given $\underset{\sim 1}{p}, \underset{\sim 2}{p} \in T$ with $\underset{\sim 1}{p} \neq \underset{\sim 2}{p}$ and assume that $\underset{\sim 2}{p} \in (\underset{\sim 1}{p})^c$ Therefore (8) is proved.

Theorem 4.4. Let (T, η) be an ITS ,$\underset{\sim 1}{p} \in IP's$ in T and D, C be subsets of (T, η). Then $I_I \mathrm{D}^{\underset{\sim 1}{p}} \cup I_I C^{\underset{\sim 1}{p}} = I_I(D \cup C)^{*\underset{\sim 1}{p}}$.

Proof. Assume that (T, η) be an intuitionistic topological space and $\underset{\sim 1}{p} \in IP's$ in T and D, C be subsets of (T, η). W.K.T, $D \subset (D \cup C)$ and $C \subset (D \cup C)$ then by theorem 4.3 (3), we find $I_I D^{\underset{\sim 1}{p}} \subset I_I(D \cup C)^{*\underset{\sim 1}{p}}$ and $I_I D^{\underset{\sim 1}{p}} \subset I_I(D \cup C)^{*\underset{\sim 1}{p}}$, for any $\underset{\sim 1}{p} \in IP's$ in T. Hence

$$I_I D^{*\underset{\sim 1}{p}} \cup I_I C^{*\underset{\sim 1}{p}} \subset I_I(D \cup C)^{\underset{\sim 1}{p}} \cdots\cdots\cdots 1$$

For converse part, let $\underset{\sim 2}{p} \notin I_I D^{*\underset{\sim 1}{p}}$.Then $\exists$ an IOS U containing $\underset{\sim 1}{p}$ with $D \cap U \in I_I(\underset{\sim 1}{p})$. In the same way, if $\underset{\sim 2}{p} \notin I_I C^{\underset{\sim 1}{p}}$ then $\exists$ IOS V containing $\underset{\sim 1}{p}$, with $C \cap V \in I_I(\underset{\sim 1}{p})$. By hereditary property of intuitionistic ideal we get, $D \cap U \cap V \in I_I(\underset{\sim 1}{p})$ and $D \cap U \cap V \in I_I(\underset{\sim 1}{p})$. Hence $\underset{\sim}{p} \notin I_I(D \cap C)^{*\underset{\sim 1}{p}}$. So,

$$I_I D^{*\underset{\sim 1}{p}} \cup I_I C^{*\underset{\sim 1}{p}} \supset I_I(D \cup C)^{\underset{\sim 1}{p}} \cdots\cdots\cdots 2$$

From 1 and 2 we get $I_I D^{\underset{\sim 1}{p}} \cup I_I C^{\underset{\sim}{p}} = I_I(D \cup C)^{*\underset{\sim 1}{p}}$.

Theorem 4.5. Suppose (T,η) be an ITS, $\underset{\sim 1}{p} \in IP's$ in T and D, C be subsets of (T,η).Then $I_I D^{\underset{\sim 1}{p}} \cap I_I C^{\underset{\sim 1}{p}} \supset I_I(D \cap C)^{*\underset{\sim 1}{p}}$.

Proof. Assume that (T,η) be an intuitionistic topological space, $\underset{\sim 1}{p} \in IP's$ in T and D, C be subsets of (T,η). W.K.T, $D \cap C \subset D$ and $D \cap C \subset C$, then by theorem 4.3 (3), $I_I(D \cap C)^{*\underset{\sim 1}{p}} \subset I_I D^{*\underset{\sim 1}{p}}$ and $I_I(D \cap C)^{*\underset{\sim 1}{p}} \subset I_I C^{*\underset{\sim 1}{p}}$. Hence $I_I D^{\underset{\sim 1}{p}} \cap I_I C^{\underset{\sim 1}{p}} \supset I_I(D \cap C)^{*\underset{\sim 1}{p}}$ for any $\underset{\sim 1}{p} \in IP's$ in T.

Theorem 4.6. Suppose (T,η) be an ITS, $\underset{\sim 1}{p} \in IP's$ in T for each IOS U $\supseteq \underset{\sim}{p}$, then $\underset{\sim 1}{p}^{*\underset{\sim 1}{p}} \subset U$.

Proof. Consider the intutionistic topological space (T,η) and , $\underset{\sim 1}{p} \in IP's$ in T for each IOS U $\supseteq \underset{\sim}{p}$. Assume that $\underset{\sim 2}{p} \notin \underset{\sim}{p}$, so $\underset{\sim 1}{p} \notin \underset{\sim 2}{p}$, then we find $U \cap \underset{\sim 2}{p} = \underset{\sim}{p} \in I_I(\underset{\sim 1}{p})$ that means $\underset{\sim 2}{p} \notin (\underset{\sim 1}{p})^{*\underset{\sim 1}{p}}$. Thus $(\underset{\sim 1}{p})^{*\underset{\sim 1}{p}} \subset \underset{\sim}{p}$.

Theorem 4.7. Let (T,η) be an ITS, $\underset{\sim 1}{p} \in IP's$ in T and D be subsets of (T,η). Then

$$D^{*\underset{\sim 1}{p}} = \begin{Bmatrix} \varphi, \ if\ \underset{\sim 1}{p} \notin D \\ cl(\underset{\sim 1}{p}),\ if\ \ \underset{\sim 1}{p} \in D \end{Bmatrix}.$$

Proof. Consider (T,η) as an ITS, $\underset{\sim 1}{p} \in IP's$ in T and D be subsets of (T,η). There araises two cases as follows:

Case (i): If $\underset{\sim 1}{p} \notin D$, to prove $D^{*\underset{\sim 1}{p}} = D$. Let $D^{*\underset{\sim 1}{p}} \neq \varphi_N$, then $\exists$ atleast one element, say $\underset{\sim 2}{p} \in D^{*\underset{\sim 1}{p}}$(by definition $D^{*\underset{\sim 1}{p}}$), we have $C_{\underset{\sim 2}{p}} \cap D \notin I_I(\underset{\sim 1}{p})$. Hence $\underset{\sim 1}{p} \notin D \cap C_{\underset{\sim 2}{p}}$. So $\underset{\sim 1}{p} \in D$ with contradiction then $D^{*\underset{\sim 1}{p}} = \underset{\sim}{\varphi}$.

Case (ii): If $\underset{\sim 1}{p} \in D$, to prove $D^{*\underset{\sim 1}{p}} = cl(\underset{\sim 1}{p})$. Let $\underset{\sim 2}{p} \in D^{\underset{\sim 1}{p}} \Rightarrow \underset{\sim 1}{p} \in D \cap V_{\underset{\sim 2}{p}}$ for each $V_{\underset{\sim 2}{p}} \in \eta_{\underset{\sim 2}{p}} \Rightarrow \underset{\sim 1}{p} \in V_{\underset{\sim 2}{p}}$, for each $V_{\underset{\sim 2}{p}} \in \eta_{\underset{\sim 2}{p}}$ it follows $\underset{\sim 2}{p} \in cl(\underset{\sim 2}{p})$ then $D^{\underset{\sim 1}{p}} \subseteq cl(\underset{\sim 1}{p})$ for each D be subsets of (T,η). Let $\underset{\sim 2}{p} \in cl(\underset{\sim 1}{p})$ and $\underset{\sim 2}{p} \notin D^{*\underset{\sim 1}{p}}$ then $\exists$ an IOS $V_{\underset{\sim 2}{p}}$ containing $\underset{\sim 2}{p}$ s.t $D \cap V_{\underset{\sim 2}{p}} \in I_I(\underset{\sim 1}{p})$, $\Rightarrow \underset{\sim 1}{p} \notin D \cap V_{\underset{\sim 2}{p}}$ then $\underset{\sim 1}{p} \notin D$ or $\underset{\sim 1}{p} \notin V_{\underset{\sim 2}{p}}$ which $\Rightarrow \underset{\sim 1}{p} \in D$ or $\underset{\sim 2}{p} \in cl(\underset{\sim 1}{p})$ which is a contradiction in case(ii). Hence $\underset{\sim 2}{p} \in D^{\underset{\sim 1}{p}} \Rightarrow cl(\underset{\sim 1}{p}) \subseteq D^{\underset{\sim 1}{p}}$.

$\therefore D^{\underset{\sim 1}{p}} = cl(\underset{\sim 1}{p})$ if $\underset{\sim 1}{p} \in D$.

Definition 4.8. Suppose (T,η) , (Y,δ) be ITs. Then $f:(T,\eta) \rightarrow (Y,\delta)$ is said to be I_I^* - map iff for every subset D of (T,η) $\underset{\sim 1}{p} \in IP's$ in T, $f(D^{\underset{\sim 1}{p}}) = (f(D^{\underset{\sim 1}{p}}))^{*\underset{\sim 1}{p}}$.

Example 4.9. Suppose $(T,\eta),(Y,\delta)$ be intuitionistic topological spaces, s.t $T=\{1,2,3\}$, $Y=\{a,b,c\}$, $\eta=\left\{\underset{\sim}{T},\underset{\sim}{\varphi},A_1,A_2\right\}$, where $A_1=\langle T,\{1\},\{2\}\rangle$, $A_2=\langle T,\{1,2\},\varphi\rangle$, $\delta=\left\{\underset{\sim}{T},\underset{\sim}{\varphi},B_1,B_2,B_3\right\}$, where $B_1=\langle Y,\{b\},\{a,c\}\rangle, B_2=\langle Y,\{a\},\{b\}\rangle$ and $B_3=\langle Y,\{a,b\},\varphi\rangle$. Let $f:(T,\eta)\to(Y,\delta)$ be $f(1)=b,f(2)=a,f(3)=c$. Put $D=\{2\}$ subset of (T,η). Then $D^{*2}=A_2=\langle T,\{1,2\},\varphi\rangle$, so $f(D^{*2})=(f(D))^{*f(2)=a}=(\langle T,\{a,b\},\varphi\rangle)^{*a}=\langle T,T,\varphi\rangle$.

Definition 4.10. Let (T,η), (Y,δ) be ITS. Then $f:(T,\eta)\to(Y,\delta)$ is said to be I_I^{**} - map iff for every subset D of (Y,δ), $\underset{\sim}{p}\in IP's$ in Y, $f^{-1}(D^{*\underset{\sim}{p}})=(f^{-1})^{f^{-1}(\underset{\sim}{p})}$.

Example 4.11. Let $(T,\eta),(Y,\delta)$ be intuitionistic topological spaces, s.t $T=\{a,b,c\}$ $Y=\{1,2,3\}$, $\eta=\left\{\underset{\sim}{T},\underset{\sim}{\varphi},A_1,A_2\right\}$, where $A_1=\langle T,\{a\},\{b\}\rangle$, $A_2=\langle T,\{a,b\},\varphi\rangle$, $\delta=\left\{\underset{\sim}{T},\underset{\sim}{\varphi},B_1,B_2,B_3\right\}$ where $B_1=\langle Y,\{2\},\{1,3\}\rangle, B_2=\langle Y,\{1\},\{2\}\rangle$and $B_3=\langle Y,\{1,2\},\varphi\rangle$. Assume $h:(T,\eta)\to(Y,\delta)$ be $h(a)=2,h(b)=1,h(c)=3$. Put $D=\{2\}$ subset of (T,η). Then $D^{*2}=A_2=\langle T,\{2,3\},\varphi\rangle$, so $h^{-1}(D^{*2})=(h^{-1}(D))^{*h^{-1}(2)=a}=(h^{-1}(D))^{*a}=(\langle T,\{a\},\{a,b\}\rangle)^{*a}=\langle T,\{a\},\{b\}\rangle$.

5. CONCLUSION

In this paper intuitionistic turing points and intuitionistic vanishing turing points are introduced, and a new set named intuitionistic Gem set is also introduced. In detail the properties of intuitionistic Gem sets are studied; counterexamples are also given. This set may be used to define the concept of its interior, closure, exterior, and boundary. And it can be extended to study the basic properties of compactness, connectedness, continuity, irresoluteness, etc. More theoretical and practical importance of intuitionistic Gem sets and points can be built by extending the above-discussed concepts. This paper is framed with the expectation that it will stimulate numerous future research studies.

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7. CONFLICT OF INTERESTS

There is no conflict of interest.

8. REFERENCE

Al-Hamido, R. K. 2018. Neutrosophic Crisp Bi-topological spaces. *Neutrosophic Sets and Systems*, **21**: 66 -73.

Al-Nafee, A. B & Al-Swidi, L. A. A. 2018. Seperation Axioms using soft turing point of a soft Ideal in soft topological space. *Journal of New Theory*, **25**: 29-37.

Al-Nafee, A. B & Hamed, A. H. 2019. The concept of Gem Set in Topology. *Journal of University of Babylon for Pure and Applied Sciences*, **27 (3)**: 213-217.

Al-Nafee, A. B., Smarandache, F & Salama, A. A. 2020. New types of Neutrosophic crisp closed sets. *Neutrosophic Sets and Systems*, **36**:175-183.

Al-Swidi, L. A & Al-Sada, D. A. 2012. Turing point of proper ideal. *Archive Des Science*, **65 (7)**: 219-220.

Al-Swidi, L. A & Al-Nafee, A. B. 2013. New Separation Axioms Using the idea of Gem-Set in Topological space. *Mathematical Theory and Modeling*, **3(3)**: 60-66.

Bakhadach, I., Melliani. S., Oukessou, M & Chadli, L.S. 2016. Intuitionistic fuzzy ideal and intuitionistic fuzzy prime ideal in a ring. *International Journal of Notes on Intuitionistic Fuzzy Sets*, **22 (2)**: 59-63.

Coker, D. 1996. A note on intutionistic sets and intutionistic points. *Turkish Journal of Mathematics*, **20(3)**: 343-351.

Coker, D. 2000. An introduction to Intutionistic fuzzy topological spaces. *Fuzzy sets and systems*, **88**: 81-89.

Das, S & Pramanik, S. 2020. Neutrosophic simply soft open set in Neutrosophic soft topological space. *Neutrosophic Sets and Systems*, **38**: 235- 243.

Jun, Y. B & Song, S. Z. 2006. Generalized fuzzy interior ideals in semigroups. *Information Sciences*, **176 (20)**: 3079–3093.

Khan, M & Anis, S. 2013. Semigroups Characterized by Their Generalized Fuzzy Ideals. *Journal of Mathematics*, Article ID 592708: 1-7.

Kroumov, V., Smarandache, F & Salama, A. A. 2014. Neutrosophic crisp sets and Neutrosophic crisp topological spaces. *Neutrosophic Theory and Application*, collected papers-I: 206-212.

Lupianez, L. G. 2005. Quasicoincidence for intuitionistic fuzzy points. *International Journal of Mathematics and Mathematical Sciences*, **10**: 1539 -1542.

Munkers, J. R. 2013. Topology. 2nd edition, ISBN-10: 9353432774, ISBN-13: 978-9353432775, Person Education, 556p.

Rathinam, R & Elango, C. 2014. Gem seperation axioms in topological space. *Annals of pure and Applied Mathematics*, **15**: 133-144.

Rose Venish, A. G., Vidyarani, L & Vigneshwaran, M. 2023. On Binary Neutrosophic Crisp Points and Binary Neutrosophic Neighborhoods. *Journal of Neutrosophic and Fuzzy Systems,* **5(1)**: 15-22.

Salama, A. A., Abdelfattah, M & Alblowi, S. A. 2014. Some intutuinistic topological notations of intuitionistic region, possible application to GIS topological rules. *International Journal of Enhanced Research in Management and Computer Applications*, **3(6)** : 1- 13.

Sardar, S. K., Mandal, M & Majumder, S. K. 2011. Intuitionistic Fuzzy Points in Semigroups. *International Journal of Mathematical and Computational Sciences,* **5(3)**: 505 - 510.

Zadeh, L. A. 1965. Fuzzy sets. *Information and control*, **8**: 338-353.