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Coexistence of Superconductivity and Antiferromagnetic Spin Density Wave in Two-Band Model for the Iron-Based Superconductor $Ba_{1-x}K_xFe_2As_2$

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ABSTRACT

The current research work focuses on the theoretical investigation of the coexistence of superconductivity and antiferromagnetic spin density wave (SDW) in a two-band model for the iron-based superconductor. In this study, we considered the electron and hole intra-band interactions, the inter-band and magnetic interactions between the two bands. By employing a system Hamiltonian in a two-band model for the given superconductor and by using the double time temperature dependent Green's function formalism at the Bardeen-Cooper Schrieffer (BCS) mean field approximation (MFA) level, we obtained mathematical expressions for the dependence of superconducting transition temperature () on antiferromagnetic SDW order parameter () in the electron and hole intra-band interactions and the inter-band interaction for the superconductor. Similarly, we have obtained an expression for the dependence of antiferromagnetic SDW transition temperature () on antiferromagnetic SDW order parameter () for. By using experimental values and considering some plausible approximations of the parameters in the obtained expressions, phase diagrams of versus in the electron and hole intra-band interactions and the inter-band interaction are plotted for the superconductor. Similarly, a phase diagram of versus is plotted for the same superconductor. By merging the two-phase diagrams, we have demonstrated the possible coexistence of superconductivity and antiferromagnetic SDW in the electron and hole intra-band interactions and the inter-band interaction. Furthermore, we have investigated the dependence of on temperature in the pure antiferromagnetic SDW region, and the phase diagram is plotted for the variation of with temperature for the superconductor. Our findings are in agreement with previous experimental findings.

Keywords: Superconductivity, Antiferromagnetic spin density wave, Two-band model, Coexistence, Green's function, $Ba_{1-x}K_xFe_2As_2$.

1 INTRODUCTION

The discovery of high-temperature superconductivity in the iron-based superconductors has been one of the major phenomena in the field of superconductivity since the discovery of cuprate superconductors in the late eighties and has stimulated enormous interests in the field of condensed matter physics (Kamihara et al., 2008). This recent discovery of the iron-based superconductors

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had opened a new frontier for both experimental and theoretical researches and rekindled a global interest in the field. It was an unexpected to find superconductivity in the iron-based superconducting materials because of the presence of large concentration of Fe with a large magnetic spin moment which makes their parent compounds strongly magnetic. Historically, the presence of iron had been believed to be one of the most harmful elements for the emergence of superconductivity. However, the situation was totally changed since the discovery of iron-based superconductor $LaO_{1-x}F_xFe_2As_2$ by the Hosono's group in 2008 (Kamihara et al., 2008). The presence of Fe in the iron-based superconductors indicates the role of magnetism in the progress of high-temperature superconductivity. The discovery of high-temperature superconductivity in these materials had ended the hegemony of cuprates as the only high-temperature superconductors and, this has sparked wider research activities on the iron-based superconductors at ambient pressure (Wang et al., 2008; Nuwal et al., 2014).

The first discovered and one of the mostly studied class of iron-based high-temperature superconductors is the 1111-family superconductors which crystallizes in the ZrCuSiAs-type structure with a tetragonal P4/nmm space group (Hosono et al., 2015). Superconductivity was also obtained in the 122- family of the iron-based high-temperature superconductors such as, AFe_2As_2 (A=Ba, Ca, Sr, Eu) which have $ThCr_2Si_2$ crystal structure with a tetragonal I4/mmm space group structure (Lumsden and Christianson, 2010). Currently, the high-temperature superconductivity in the 122-family of iron-based superconductors such as $BaFe_2As_2$, $CaFe_2As_2$ and $SrFe_2As_2$ are becoming preferable to deal with. The 122-family iron-based parent compounds become superconducting by the introduction of additional hole or electron dopants and by the application of chemical or physical pressure. $BaFe_2As_2$ becomes superconducting by hole doping upon replacement of Ba^{2+} by K^+ (Rotter et al., 2008), by electron doping upon replacement of Fe^{2+} by Co^{3+} or Ni^{4+} (Sefat et al, 2008; Li et al., 2009), by inducing pressure upon replacement of arsenic (As) by the isovalent but smaller atom phosphorus (P) (Kasahara et al., 2010) and by application of physical pressure (Alireza et al., 2008). Great efforts had been made to raise the transition temperature in the 122-family of the iron-based superconductors and an optimal superconductivity was detected in the hole-doped iron-based superconductor $Ba_{1-x}K_xFe_2As_2$ for x=0.4 (x being the carrier concentration) at a superconducting transition temperature of $T_C = 38$ K (Rotter et al., 2008).

Multi-band superconductivity is a novel quantum phenomenon in many different superconducting materials in which different superconducting gaps open in different Fermi surfaces or contains different carriers which participate in the formation of the superconducting condensate in the material. The study of multi-band superconductivity which consist of hole and electron pockets in their Fermi surface is relevant for a variety of systems in condensed matter physics. Theoretical and experimental investigations have emphasized the importance of multi-band superconductivity as a possible explanation for the appearance of materials with high T_C and the multi-band effects can strongly enhance superconductivity (Suhl et al., 1959; Bersier et al., 2009; Stanev et al., 2008; Mazin et al., 2008). The multi-band superconductivity in the iron-based superconductors arises due to the appearance of multiple Fermi-surface pockets with different values of superconducting energy gap amplitudes on different sheets of the Fermi surfaces and several intra-band and inter-band interaction terms are also present. The importance of inter-band interaction term in multi-band models has been emphasized by De Menezes et al. (1986). A minimal two-band model superconductivity was investigated in the iron-based high-temperature superconductors (Si and Abrahams, 2008; Daghofer et al., 2008).

The main features of the minimal two-band superconductors is the presence of intra-band and inter-band interaction terms and two different superconducting energy gaps which vanish at the same superconducting transition temperature. The presence of inter-band interaction potential enhances the pairing strength in the system and also leads the electron and hole intra-band superconducting order parameters to vanish at the same superconducting transition temperature. It was reported that, a two-band model superconductivity was found in the high-temperature iron-based superconductors $\mathrm{Sm_{0.95}La_{0.005}O_{0.85}F_{0.15}FeAs}$ and $\mathrm{SmO_{1-x}F_xFeAs}$ with two different superconducting order parameters in each band which vanish at the same T_C of 50 K and 57.3 K respectively as detected by resistivity and susceptibility measurements (Nuwal et al., 2014). The data obtained by angle resolved photoemission spectroscopy (ARPES) on the hole doped crystal $\mathrm{Ba_{0.6}K_{0.4}Fe_2As_2}$ have shown that, the experimental values for the zero temperature superconducting order parameter in the electron Fermi surface is, $\Delta_e = 12\ \mathrm{meV}$ and the hole Fermi surface has a value of $\Delta_h = 6\ \mathrm{meV}$ (Ding et al., 2008).

Most of the iron-based high-temperature superconductors exhibit a superconducting phase emerging from an antiferromagnetically ordered spin density wave state (Wang and Lee, 2011; Basov et al., 2011; De la Cruz et al., 2008). The iron-based superconducting materials undergo a magnetic transition at TSDW leading to the formation of a SDW in the system. Since the AFM and superconducting phases lie adjacent to each other, the boundary of the two phases may provide useful information on the nature of the superconductivity properties. Muon spin relaxation (μSR) and M¨ussbauer spectroscopy observations made on $\mathrm{LaFeAsO_{1-x}F_x}$ suggest that, superconductivity and magnetic ordering are mutually exclusive and a first-order abrupt transition from SDW to superconducting state is observed without coexistence between the two phases (Luetkens et al., 2009). The coexistence of superconductivity and magnetism in the iron-based superconductors has been the focus of numerous experimental and theoretical studies. As demonstrated by muon spin relaxation, neutron scattering and ARPES studies, there is a unique superconducting regime in the phase diagram of iron-based superconductors, where antiferromagnetically ordered SDW and superconductivity coexist in a certain interval of temperature and doping concentration (x) (Drew et al., 2009; Fernandes et al., 2010; Chen et al., 2009; Ge et al., 2013; Zhang et al., 2009). The resistivity measurements show that the coexistence of SDW and superconductivity was observed in the hole doped iron-based superconductor $\mathrm{Ba_{1-x}K_xFe_2As_2}$ for the interval $0.2 \leq x < 0.4$ (Chen et al., 2009).

2. FORMULATION OF A SYSTEM HAMILTONIAN IN TWO-BAND MODEL

To investigate the coexistence of superconductivity and antiferromagnetic spin density wave, we considered a minimal two-band model in the presence of the electron and hole intra-band interactions and the inter-band interaction between the two bands with similar spin density wave instabilities in both bands nested by the nesting vector, q (Mohapatra and Rout, 2014).

The decoupled mean field system Hamiltonian of the superconducting state by including the effect of the magnetic interaction in a two-band model for our system can be described as (Fernandes et al.,2010),

$$\widehat{\mathrm{H}} = \widehat{\mathrm{H}}_{\mathrm{e}} + \widehat{\mathrm{H}}_{\mathrm{h}} + \widehat{\mathrm{H}}_{\mathrm{eh}} + \widehat{\mathrm{H}}_{\mathrm{SDW}} \tag{1}$$

Where, $\widehat{\mathrm{H}}_{\mathrm{e}}$ is the mean field Hamiltonian in the electron intra-band interaction and is expressed as,

$$\widehat{\mathrm{H}}_{\mathrm{e}} = \sum_{\mathrm{k},\sigma} \epsilon_{\mathrm{e}}(\mathrm{k}) \hat{\mathrm{a}}^{\dagger}_{\mathrm{k},\sigma} \hat{\mathrm{a}}_{-\mathrm{k},\sigma} - \Delta_{\mathrm{e}} \sum_{\mathrm{k}} \left(\hat{\mathrm{a}}^{\dagger}_{\mathrm{k}\uparrow} \hat{\mathrm{a}}^{\dagger}_{-\mathrm{k}\downarrow} + \hat{\mathrm{a}}_{-\mathrm{k}\downarrow} \hat{\mathrm{a}}_{\mathrm{k}\uparrow} \right) \tag{2}$$

Where, the first and second terms are the energy of conduction electrons and the term involving superconductivity due to the intra-band interaction in the electron Fermi surface respectively. $\epsilon_{\mathrm{e}}(\mathrm{k})$ is the single particle energy measured with respect to the Fermi energy in the electron Fermi surface.

$\hat{a}^{\dagger}_{k\uparrow}(\hat{a}_{-k\downarrow})$ are the creation (annihilation) operators in the electron intra-band interaction having the wave number k(-k) and spin ↑ (↓), respectively. Δ_e is the superconducting order parameter due to the intra-band interaction within the electron band and is a real quantity such that $\Delta_e = \Delta_e^*$ and is given by,

$$\Delta_e = \sum_k V_e \ll \hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg = \sum_k V_e \ll \hat{a}_{-k\downarrow}, \hat{a}_{k\uparrow} \gg \tag{3}$$

Where, V_e being the interaction potential in the electron intra-band interaction.

$\hat{H}_h$ is the mean field system Hamiltonian in the hole intra-band interaction and is expressed as,

$$\hat{H}_h = \sum_{k,\sigma} \epsilon_h(k)\hat{c}^{\dagger}_{k,\sigma}\hat{c}_{-k,\sigma} - \Delta_h \sum_k \left(\hat{c}^{\dagger}_{k\uparrow}\hat{c}^{\dagger}_{-k\downarrow} + \hat{c}_{-k\downarrow}\hat{c}_{k\uparrow}\right) \tag{4}$$

Where, the first and second terms are the energy of conduction electrons and the term involving superconductivity due to the intra-band interaction in the hole Fermi surface respectively. $\epsilon_h(k)$ is the single particle energy measured with respect to the Fermi energy in the hole Fermi surface.

$\hat{c}^{\dagger}_{k\uparrow}(\hat{c}_{-k\downarrow})$ are the creation (annihilation) operators in the hole intra-band interaction having the wave number k(-k) and spin ↑ (↓), respectively. Δ_h is the superconducting order parameter due to the intra-band interaction within the hole band and is a real quantity such that $\Delta_h = \Delta_h^*$ and is given by,

$$\Delta_h = \sum_k V_h \ll \hat{c}^{\dagger}_{k\uparrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg = \sum_k V_h(k) \ll \hat{c}_{-k\downarrow}, \hat{c}_{k\uparrow} \gg \tag{5}$$

Where, V_h is the interaction potential in the hole intra-band interaction.

Besides, $\hat{H}_{eh}$ in equation (1) is the mean field system Hamiltonian due to the inter-band interaction between the two bands and is expressed as,

$$\hat{H}_{eh} = -\Delta_{eh} \sum_k \left(a^{\dagger}_{k\uparrow}\hat{a}^{\dagger}_{-k\downarrow} + \hat{c}_{-k\downarrow}\hat{c}_{k\uparrow}\right) - \Delta_{eh} \sum_k \left(c^{\dagger}_{k\uparrow}\hat{c}^{\dagger}_{-k\downarrow} + \hat{a}_{-k\downarrow}\hat{a}_{k\uparrow}\right) \tag{6}$$

Where, the first and second terms are the terms involving superconductivity due to the inter-band interaction between the two bands. Δ_{eh} is the superconducting order parameter due to inter-band interaction between the bands and is given by,

$$\Delta_{eh} = \sum_k V_{eh} \ll \hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg = \sum_k V_{eh} \ll \hat{c}^{\dagger}_{k\uparrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg = \sum_k V_{eh} \ll \hat{a}_{-k\downarrow}, \hat{a}_{k\uparrow} \gg = \sum_k V_{eh} \ll \hat{c}_{-k\downarrow}, \hat{c}_{k\uparrow} \gg \tag{7}$$

Where, V_{eh} is the inter-band interaction potential between the two bands.

$\hat{H}_{SDW}$ in equation (1) is the mean field Hamiltonian which describes the magnetic interactions which drive the antiferromagnetic SDW state and is expressed as,

$$\hat{H}_{SDW} = -\Delta_{SDW} \sum_k \left(\hat{a}^{\dagger}_{(k+q)\uparrow}\hat{c}_{k\uparrow} + \hat{c}^{\dagger}_{(k+q)\uparrow}\hat{a}_{k\uparrow}\right) \tag{8}$$

Where, $\hat{a}^{\dagger}_{(k+q)\uparrow}\left(\hat{c}^{\dagger}_{(k+q)\uparrow}\right)$ is the operator which creates fermions with momentum (k+q) in the electron (hole) band interaction. Here, Δ_{SDW} represents the antiferromagnetic SDW order parameter which governs the presence of magnetic ordering and is a real quantity such that $\Delta_{SDW}= \Delta_{SWD}{}^{*}$ and is given by,

$$\Delta_{SDW}=\sum_k V_{SDW} \ll \hat{c}^{\dagger}_{(k+q)\uparrow}, \hat{a}_{k\uparrow} \gg = \sum_k V_{SDW} \ll \hat{a}^{\dagger}_{(k+q)\uparrow}, \hat{c}_{k\uparrow} \gg \tag{9}$$

Now, substituting equations (2), (4), (6) and (8) into equation (1), the expression for the mean field system Hamiltonian in the two-band model becomes,

$$\begin{aligned}\widehat{H} = {} & \sum_{k,\sigma} \epsilon_e(k)\hat{a}^{\dagger}_{k,\sigma}\,\hat{a}_{-k,\sigma} + \sum_{k,\sigma} \epsilon_h(k)\hat{c}^{\dagger}_{k,\sigma}\,\hat{c}_{-k,\sigma} - \Delta_e \sum_k \left(\hat{a}^{\dagger}_{k\uparrow}\hat{a}^{\dagger}_{-k\downarrow} + \hat{a}_{-k\downarrow}\hat{a}_{k\uparrow}\right) - \Delta_h \sum_k\left(\hat{c}^{\dagger}_{k\uparrow}\hat{c}^{\dagger}_{-k\downarrow} + \right.\\ & \left.\hat{c}_{-k\downarrow}\hat{c}_{k\uparrow}\right) - \Delta_{eh} \sum_k\left(a^{\dagger}_{k\uparrow}\hat{a}^{\dagger}_{-k\downarrow} + \hat{c}_{-k\downarrow}\hat{c}_{k\uparrow}\right) - \Delta_{eh} \sum_k\left(c^{\dagger}_{k\uparrow}\hat{c}^{\dagger}_{-k\downarrow} + \hat{a}_{-k\downarrow}\hat{a}_{k\uparrow}\right) - \\ & \Delta_{SDW} \sum_k\left(\hat{a}^{\dagger}_{(k+q)\uparrow}\hat{c}_{k\uparrow} + \hat{c}^{\dagger}_{(k+q)\uparrow}\hat{a}_{k\uparrow}\right)\end{aligned} \tag{10}$$

2.1. Equations of motion

Applying the double time temperature dependent Green's function formalism (Zubarev, 1960), the equation of motion for the superconducting correlation function $\ll \hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg$ in the electron intra-band interaction is expressed as,

$$\omega \ll \hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg = \ll \left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}\right], \hat{a}^{\dagger}_{-k\downarrow} \gg$$

$$\omega \ll \hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg = \ll \left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_e\right] + \left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_h\right] + \left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_{eh}\right] + \left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_{SDW}\right], \hat{a}^{\dagger}_{-k\downarrow} \gg \tag{11}$$

Now, evaluating the commutation relation $\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_e\right]$ in equation (11), we obtain,

$$\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_e\right] = \sum_{k,\sigma} \epsilon_e(k)\left[\hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{k,\sigma}\hat{a}_{-k,\sigma}\right] - \Delta_e \sum_k \left[\hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{k\uparrow}\hat{a}^{\dagger}_{-k\downarrow}\right] - \Delta_e \sum_k \left[\hat{a}^{\dagger}_{k\uparrow}, \hat{a}_{-k\downarrow}\hat{a}_{k\uparrow}\right]$$

Applying the commutation properties of the operators we have,

$$\begin{aligned}\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_e\right] = {} & \sum_{k,\sigma} \epsilon_e(k)\left(\left[\hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{k,\sigma}\right]\hat{a}_{-k,\sigma} - \hat{a}^{\dagger}_{k,\sigma}\left[\hat{a}^{\dagger}_{k\uparrow}, \hat{a}_{-k,\sigma}\right]\right) \\ & -\Delta_e \sum_k \left(\left[\hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{k\uparrow}\right]\hat{a}^{\dagger}_{-k\downarrow} - \hat{a}^{\dagger}_{k\uparrow}\left[\hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{-k\downarrow}\right]\right) \\ & -\Delta_e \sum_k \left(\left[\hat{a}^{\dagger}_{k\uparrow}, \hat{a}_{-k\downarrow}\right]\hat{a}_{k\uparrow} - \hat{a}_{-k\downarrow}\left[\hat{a}^{\dagger}_{k\uparrow}, \hat{a}_{k\uparrow}\right]\right)\end{aligned}$$

from which we obtain,

$$\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_e\right] = -\epsilon_e(k)\hat{a}^{\dagger}_{k\uparrow} + \Delta_e\hat{a}_{-k\downarrow}. \tag{12}$$

Similarly, the commutation relation for $\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_h\right]$ in equation (11) becomes,

$\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_{h}\right] = \left[\hat{a}^{\dagger}_{k\uparrow}, \sum_{k,\sigma} \epsilon_{h}(k)\hat{c}^{\dagger}_{k,\sigma}\,\hat{c}_{-k,\sigma} - \Delta_{h}\sum_{k}\left(\hat{c}^{\dagger}_{k\uparrow}\hat{c}^{\dagger}_{-k\downarrow} + \hat{c}_{-k\downarrow}\hat{c}_{k\uparrow}\right)\right],$

which leads to,

$$\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_{h}\right] = 0. \tag{13}$$

Furthermore, the commutation relation for $\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_{eh}\right]$ in equation (11) yields,

$\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_{eh}\right] = \left[\hat{a}^{\dagger}_{k\uparrow}, -\Delta_{eh}\sum_{k}\left(a^{\dagger}_{k\uparrow}\hat{a}^{\dagger}_{-k\downarrow} + \hat{c}_{-k\downarrow}\hat{c}_{k\uparrow}\right) - \Delta_{eh}\sum_{k}\left(c^{\dagger}_{k\uparrow}\hat{c}^{\dagger}_{-k\downarrow} + \hat{a}_{-k\downarrow}\hat{a}_{k\uparrow}\right)\right].$

Thus, we get,

$$\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_{eh}\right] = \Delta_{eh}\hat{a}_{-k\downarrow}. \tag{14}$$

Also, the commutation relation for $\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_{SDW}\right]$ in equation (11) becomes,

$$\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_{SDW}\right] = \left[\hat{a}^{\dagger}_{k\uparrow}, -\Delta_{SDW}\sum_{k}\left(\hat{a}^{\dagger}_{(k+q)\uparrow}\hat{c}_{k\uparrow} + \hat{c}^{\dagger}_{(k+q)\uparrow}\hat{a}_{k\uparrow}\right)\right]$$

$$\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_{SDW}\right] = \left[\hat{a}^{\dagger}_{k\uparrow}, -\Delta_{SDW}\sum_{k}\hat{a}^{\dagger}_{(k+q)\uparrow}\hat{c}_{k\uparrow}\right] + \left[\hat{a}^{\dagger}_{k\uparrow}, -\Delta_{SDW}\sum_{k}\hat{c}^{\dagger}_{(k+q)\uparrow}\hat{a}_{k\uparrow}\right]$$

which yields,

$$\begin{aligned}\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_{SDW}\right] = &-\Delta_{SDW}\sum_{k}\left(\left[\hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{(k+q)\uparrow}\right]\hat{c}_{k\uparrow} - \hat{a}^{\dagger}_{(k+q)\uparrow}\left[\hat{a}^{\dagger}_{k\uparrow}, \hat{c}_{k\uparrow}\right]\right)\\ &-\Delta_{SDW}\sum_{k}\left(\left[\hat{a}^{\dagger}_{k\uparrow}, \hat{c}^{\dagger}_{(k+q)\uparrow}\right]\hat{a}_{k\uparrow} - \hat{c}^{\dagger}_{(k+q)\uparrow}\left[\hat{a}^{\dagger}_{k\uparrow}, \hat{a}_{k\uparrow}\right]\right)\end{aligned}$$

From which we get,

$$\left[\hat{a}^{\dagger}_{k\uparrow}, \widehat{H}_{SDW}\right] = \Delta_{SDW}\hat{a}^{\dagger}_{k\uparrow} \tag{15}$$

Now, substituting equations (12-15) into equation (11), the equation of motion for $\ll \hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg$ becomes,

$$\ll \hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg = \frac{\Delta_{e}+\Delta_{eh}}{\omega+\epsilon_{e}(k)-\Delta_{SDW}} \ll \hat{a}_{-k\downarrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg \tag{16}$$

Furthermore, applying the double time temperature dependent Green's function formalism, the equation of motion for the superconducting correlation relation $\ll \hat{a}_{-k\downarrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg$ in the electron intra-band interaction can be expressed as,

$$\omega \ll \hat{a}_{-k\downarrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg = 1 + \ll \left[\hat{a}_{-k\downarrow}, \widehat{H}_{e}\right] + \left[\hat{a}_{-k\downarrow}, \widehat{H}_{h}\right] + \left[\hat{a}_{-k\downarrow}, \widehat{H}_{eh}\right] + \left[\hat{a}_{-k\downarrow}, \widehat{H}_{SDW}\right], \hat{a}^{\dagger}_{-k\downarrow} \gg \tag{17}$$

Now, evaluating the commutation relation $\left[\hat{a}_{-k\downarrow}, \widehat{H}_{e}\right]$ in equation (17), we obtain,

$$\left[\hat{a}_{-k\downarrow}, \widehat{H}_{e}\right] = \left[\hat{a}_{-k\downarrow}, \sum_{k,\sigma}\epsilon_{e}(k)\hat{a}^{\dagger}_{k,\sigma}\,\hat{a}_{-k,\sigma} - \Delta_{e}\sum_{k}\left(\hat{a}^{\dagger}_{k\uparrow}\hat{a}^{\dagger}_{-k\downarrow} + \hat{a}_{-k\downarrow}\hat{a}_{k\uparrow}\right)\right]$$

which gives,

$$\left[\hat{a}_{-k\downarrow}, \widehat{H}_e\right] = \epsilon_e(k)\hat{a}_{-k\downarrow} + \Delta_e \hat{a}^{\dagger}_{k\uparrow}. \qquad (18)$$

The commutation relation for $\left[\hat{a}_{-k\downarrow}, \widehat{H}_h\right]$ in equation (17) yields,

$$\left[\hat{a}_{-k\downarrow}, \widehat{H}_h\right] = \left[\hat{a}_{-k\downarrow}, \sum_{k,\sigma} \epsilon_h(k)\hat{c}^{\dagger}_{k,\sigma}\hat{c}_{-k,\sigma} - \Delta_h \sum_k \left(\hat{c}^{\dagger}_{k\uparrow}\hat{c}^{\dagger}_{-k\downarrow} + \hat{c}_{-k\downarrow}\hat{c}_{k\uparrow}\right)\right].$$

Hence we get,

$$\left[\hat{a}_{-k\downarrow}, \widehat{H}_h\right] = 0 \qquad (19)$$

Similarly, the commutation relation for $\left[\hat{a}_{-k\downarrow}, \widehat{H}_{eh}\right]$ in equation (17) becomes,

$$\left[\hat{a}_{-k\downarrow}, \widehat{H}_{eh}\right] = \left[\hat{a}_{-k\downarrow}, -\Delta_{eh} \sum_k \left(a^{\dagger}_{k\uparrow}\hat{a}^{\dagger}_{-k\downarrow} + \hat{c}_{-k\downarrow}\hat{c}_{k\uparrow}\right) - \Delta_{eh} \sum_k \left(c^{\dagger}_{k\uparrow}\hat{c}^{\dagger}_{-k\downarrow} + \hat{a}_{-k\downarrow}\hat{a}_{k\uparrow}\right)\right]$$

From which we get,

$$\left[\hat{a}_{-k\downarrow}, \widehat{H}_{eh}\right] = \Delta_{eh}\hat{a}^{\dagger}_{k\uparrow}. \qquad (20)$$

Furthermore, the commutation relation for $\left[\hat{a}_{-k\downarrow}, \widehat{H}_{SDW}\right]$ in equation (17) becomes,

$$\left[\hat{a}_{-k\downarrow}, \widehat{H}_{SDW}\right] = \left[\hat{a}_{-k\downarrow}, -\Delta_{SDW} \sum_k \left(\hat{a}^{\dagger}_{(k+q)\uparrow}\hat{c}_{k\uparrow} + \hat{c}^{\dagger}_{(k+q)\uparrow}\hat{a}_{k\uparrow}\right)\right]$$

which leads to,

$$\left[\hat{a}_{-k\downarrow}, \widehat{H}_{SDW}\right] = -\Delta_{SDW}\hat{a}_{-k\downarrow} \qquad (21)$$

Now, substituting equations (18-21) into equation (17), the equation of motion for $\ll \hat{a}_{-k\downarrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg$ becomes,

$$\ll \hat{a}_{-k\downarrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg = \frac{1}{\omega - \epsilon_e(k) + \Delta_{SDW}} + \frac{\Delta_e + \Delta_{eh}}{\omega - \epsilon_e(k) + \Delta_{SDW}} \ll \hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg \qquad (22)$$

Also applying the same technique as above, the equation of motion for the antiferromagnetic SDW correlation function $\ll \hat{a}^{\dagger}_{(k+q)\uparrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg$ is written as,

$$\omega \ll \hat{a}^{\dagger}_{(k+q)\uparrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg = \left\langle \left[\hat{a}^{\dagger}_{(k+q)\uparrow}, \hat{c}^{\dagger}_{-k\downarrow}\right]\right\rangle + \ll \left[\hat{a}^{\dagger}_{(k+q)\uparrow}, \widehat{H}\right], \hat{c}^{\dagger}_{-k\downarrow} \gg$$

$$\omega \ll \hat{a}^{\dagger}_{(k+q)\uparrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg = \ll \left[\hat{a}^{\dagger}_{(k+q)\uparrow}, \widehat{H}_e\right] + \left[\hat{a}^{\dagger}_{(k+q)\uparrow}, \widehat{H}_h\right] + \left[\hat{a}^{\dagger}_{(k+q)\uparrow}, \widehat{H}_{eh}\right] + \left[\hat{a}^{\dagger}_{(k+q)\uparrow}, \widehat{H}_{SDW}\right], \hat{c}^{\dagger}_{-k\downarrow} \gg \qquad (23)$$

Now, evaluating the commutation relations in equation (23), we obtain,

$$\left[\hat{a}^{\dagger}_{(k+q)\uparrow}, \widehat{H}_e\right] = -\epsilon_e(k)\hat{a}^{\dagger}_{(k+q)\uparrow} + \Delta_e \hat{a}_{-(k+q)\downarrow} \qquad (24)$$

$$\left[\hat{a}^{\dagger}_{(k+q)\uparrow}, \widehat{H}_h\right] = 0 \qquad (25)$$

$$\left[\hat{a}^{\dagger}_{(k+q)\uparrow}, \hat{H}_{eh}\right] = \Delta_{eh}\hat{a}_{-(k+q)\downarrow} \quad (26)$$

$$\left[\hat{a}^{\dagger}_{(k+q)\uparrow}, \hat{H}_{SDW}\right] = \Delta_{SDW}\hat{a}^{\dagger}_{(k+q)\uparrow}. \quad (27)$$

Now, substituting equations (24-27) into equation (23), the equation of motion for $\ll \hat{a}^{\dagger}_{(k+q)\uparrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg$ is expressed as,

$$\ll \hat{a}^{\dagger}_{(k+q)\uparrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg = \frac{\Delta_e+\Delta_{eh}}{\omega+\epsilon_e(k)-\Delta_{SDW}} \ll \hat{a}_{-(k+q)\downarrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg. \quad (28)$$

Following the same procedure as above, the equation of motion for the antiferromagnetic SDW correlation relation $\ll \hat{a}_{-(k+q)\downarrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg$ is expressed as,

$$\omega \ll \hat{a}_{-(k+q)\downarrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg = \langle\left[\hat{a}_{-(k+q)\downarrow}, \hat{c}^{\dagger}_{-k\downarrow}\right]\rangle + \ll \left[\hat{a}_{-(k+q)\downarrow}, \hat{H}\right], \hat{c}^{\dagger}_{-k\downarrow} \gg$$

$$\omega \ll \hat{a}_{-(k+q)\downarrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg = 1 + \ll \left[\hat{a}_{-(k+q)\downarrow}, \hat{H}_e\right] + \left[\hat{a}_{-(k+q)\downarrow}, \hat{H}_h\right] + \left[\hat{a}_{-(k+q)\downarrow}, \hat{H}_{eh}\right] + \left[\hat{a}_{-(k+q)\downarrow}, \hat{H}_{SDW}\right], \hat{c}^{\dagger}_{-k\downarrow} \gg \quad (29)$$

Now, evaluating the commutation relations in equation (29) we obtain,

$$\left[\hat{a}_{-(k+q)\downarrow}, \hat{H}_e\right] = \epsilon_e(k)\hat{a}_{-(k+q)\downarrow} + \Delta_e\hat{a}^{\dagger}_{(k+q)\uparrow} \quad (30)$$

$$\left[\hat{a}_{-(k+q)\downarrow}, \hat{H}_h\right] = 0 \quad (31)$$

$$\left[\hat{a}_{-(k+q)\downarrow}, \hat{H}_{eh}\right] = \Delta_{eh}\hat{a}^{\dagger}_{(k+q)\uparrow} \quad (32)$$

$$\left[\hat{a}_{-(k+q)\downarrow}, \hat{H}_{SDW}\right] = -\Delta_{SDW}\hat{a}_{-(k+q)\downarrow} \quad (33)$$

Substituting equations (30-33) into equation (29), the equation of motion for $\ll \hat{a}_{(k+q)\uparrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg$ becomes,

$$\ll \hat{a}_{-(k+q)\downarrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg = \frac{1}{\omega-\epsilon_e(k)+\Delta_{SDW}} + \frac{\Delta_e+\Delta_{eh}}{\omega-\epsilon_e(k)+\Delta_{SDW}} \ll \hat{a}^{\dagger}_{(k+q)\uparrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg \quad (34)$$

2.2. Dependence of superconducting transition temperature (T_C) on antiferromagnetic SDW order parameter (Δ_{SDW}) in the electron intra-band interaction

In this subsection, we obtained the expression for the dependence of superconducting transition temperature (T_C) on antiferromagnetic SDW order parameter (Δ_{SDW}) by considering the effect of the electron intra-band interaction only. Now, substituting equation (22) into equation (16), we get,

$$\ll \hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg = \frac{\Delta_e+\Delta_{eh}}{\omega^2-\epsilon_e^2(k)+2\epsilon_e(k)\Delta_{SDW}-\Delta_{SDW}^2-(\Delta_e+\Delta_{eh})^2} \quad (35)$$

Now, by decoupling equation (35) using the partial fraction decomposition method and ignoring the inter-band superconducting order parameter term ($\Delta_{eh}= 0$), the expression for $\ll \hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg$ becomes,

$$\ll \hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg = -\frac{1}{2}\left(\frac{\Delta_e+\Delta_{SDW}}{\omega^2-\epsilon_e^2(k)-(\Delta_e+\Delta_{SDW})^2}+\frac{\Delta_e-\Delta_{SDW}}{\omega^2-\epsilon_e^2(k)-(\Delta_e-\Delta_{SDW})^2}\right). \quad (36)$$

The superconducting order parameter in the electron intra-band interaction is related to the Green's function as,

$$\Delta_e = \frac{V_e}{2\beta}\sum_k \ll \hat{a}^{\dagger}_{k\uparrow}, \hat{a}^{\dagger}_{-k\downarrow} \gg \quad (37)$$

where $\beta = \frac{1}{k_B T}$ and k_B is the Boltzmann constant.

Substituting equation (36) into equation (37), we get,

$$\Delta_{SC}^{e} = -\frac{V_e}{4\beta}\sum_{k,n}\left(\frac{\Delta_e+\Delta_{SDW}}{\omega_n^2-\epsilon_e^2(k)-(\Delta_e+\Delta_{SDW})^2}+\frac{\Delta_e-\Delta_{SDW}}{\omega_n^2-\epsilon_e^2(k)-(\Delta_e-\Delta_{SDW})^2}\right). \quad (38)$$

Applying Matsubara's frequency and changing $\omega \longrightarrow i\omega_n$, equation (38) becomes,

$$\Delta_{SC}^{e} = \frac{V_e\beta}{4}\sum_{k,n}\left(\frac{\Delta_e+\Delta_{SDW}}{((2n+1)\pi)^2+\beta^2\left(\epsilon_e^2(k)+(\Delta_e+\Delta_{SDW})^2\right)}+\frac{\Delta_e-\Delta_{SDW}}{((2n+1)\pi)^2+\beta^2\left(\epsilon_e^2(k)+(\Delta_e-\Delta_{SDW})^2\right)}\right) \quad (39)$$

Changing summation into integration and introducing the density of state at the Fermi level of the electron intra-band interaction, $N_e(0)$, we get,

$$\Delta_{SC}^{e} = \frac{V_e\beta}{4}\int_{-\hbar\omega_D}^{\hbar\omega_D} N_e(0)\sum_n\left(\frac{\Delta_e+\Delta_{SDW}}{((2n+1)\pi)^2+\beta^2(\epsilon_e^2(k)+(\Delta_e+\Delta_{SDW})^2)}\right)d\epsilon_e(k)$$
$$+\frac{V_e\beta}{4}\int_{-\hbar\omega_D}^{\hbar\omega_D} N_e(0)\sum_n\left(\frac{\Delta_e-\Delta_{SDW}}{((2n+1)\pi)^2+\beta^2(\epsilon_e^2(k)+(\Delta_e-\Delta_{SDW})^2)}\right)d\epsilon_e(k)$$

From which we get,

$$\frac{2}{N_e(0)V_e} = \int_0^{\hbar\omega_D}\left(1+\frac{\Delta_{SDW}}{\Delta_e}\right)\frac{\tanh\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+(\Delta_e+\Delta_{SDW})^2}\right)}{\sqrt{\epsilon_e^2(k)+(\Delta_e+\Delta_{SDW})^2}}d\epsilon_e(k)$$
$$+\int_0^{\hbar\omega_D}\left(1-\frac{\Delta_{SDW}}{\Delta_e}\right)\frac{\tanh\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+(\Delta_e-\Delta_{SDW})^2}\right)}{\sqrt{\epsilon_e^2(k)+(\Delta_e-\Delta_{SDW})^2}}d\epsilon_e(k)d\epsilon_e(k) \quad (40)$$

Where, $N_e(0)$ is the density of states at the Fermi level of the electron intra-band interaction.

We know that, as $T \longrightarrow T_C$, $\Delta_e \longrightarrow 0$, hence equation (40), becomes,

$$\frac{2}{N_e(0)V_e} = I_1 + I_2 \quad (41)$$

where I_1 and I_2 are the integrals defined by,

$$I_1 = \int_0^{\hbar\omega_D} \frac{\tanh\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}\right)}{\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}} d\epsilon_e(k) + \int_0^{\hbar\omega_D} \frac{\Delta_{SDW}}{\Delta_e} \frac{\tanh\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}\right)}{\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}} d\epsilon_e(k) \quad (42)$$

and

$$I_2 = \int_0^{\hbar\omega_F} \frac{\tanh\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}\right)}{\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}} d\epsilon_e(k) - \int_0^{\hbar\omega_F} \frac{\Delta_{SDW}}{\Delta_e} \frac{\tanh\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}\right)}{\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}} d\epsilon_e(k) \quad (43)$$

Now, applying the Laplaces Transform and employing a couple of steps, the first integral in equation (42) gives,

$$\int_0^{\hbar\omega_D} \frac{\tanh\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}\right)}{\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}} d\epsilon_e(k) = \ln\left(1.14\frac{\hbar\omega_D}{k_B T_C}\right) - \frac{7\Delta_{SDW}^2\beta^2\zeta(3)}{8\pi^2} \quad (44)$$

Where, ζ is the Zeta function and $\zeta(3) = 1.202$.

Applying L'Hospital's rule and using a couple of steps, the second integral in equation (42) yields,

$$\int_0^{\hbar\omega_D} \lim_{\Delta_{SC}^e\to 0} \frac{\Delta_{SDW}}{\Delta_e} \frac{\tanh\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}\right)}{\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}} d\epsilon_e(k) = -\frac{\beta\Delta_{SDW}}{4}\ln\left(\frac{\hbar\omega_D+\Delta_{SDW}}{\hbar\omega_D-\Delta_{SDW}}\right) - \quad (45)$$

$$\int_0^{\hbar\omega_D} \frac{\beta\Delta_{SDW}^2\tanh^2\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}\right)}{2\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}} d\epsilon_e(k)$$

Similarly, the integral in equation (43) gives,

$$\int_0^{\hbar\omega_D} \frac{\tanh\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}\right)}{\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}} d\epsilon_e(k) = \ln\left(1.14\frac{\hbar\omega_D}{k_B T_C}\right) - \frac{7\Delta_{SDW}^2\beta^2\zeta(3)}{8\pi^2} \quad (46)$$

and

$$\int_0^{\hbar\omega_D} \lim_{\Delta_e\to 0} \frac{\Delta_{SDW}}{\Delta_e} \frac{\tanh\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}\right)}{\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}} d\epsilon_e(k) = -\frac{\beta\Delta_{SDW}}{4}\ln\left(\frac{\hbar\omega_D+\Delta_{SDW}}{\hbar\omega_D-\Delta_{SDW}}\right) +$$

$$\int_0^{\hbar\omega_D} \frac{\beta\Delta_{SDW}^2\tanh^2\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}\right)}{2\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}} d\epsilon_e(k) \quad (47)$$

Now, substituting equations (44-47) into equation (41), we get,

$$\frac{1}{N_e(0)V_e} = \ln\left(1.14\frac{\hbar\omega_D}{k_B T_C}\right) - \frac{7\Delta_{SDW}^2\beta^2\zeta(3)}{8\pi^2} - \frac{\beta\Delta_{SDW}}{4}\ln\left(\frac{\hbar\omega_D+\Delta_{SDW}}{\hbar\omega_D-\Delta_{SDW}}\right). \quad (48)$$

For small Δ_{SDW} we can ignore the Δ_{SDW}^2 term. Thus, equation (48) reduces to,

$$\frac{1}{N_e(0)V_e} = \ln\left(1.14\frac{\hbar\omega_D}{k_B T_C}\right) - \frac{\Delta_{SDW}}{4k_B T_C}\ln\left(\frac{\hbar\omega_D+\Delta_{SDW}}{\hbar\omega_D-\Delta_{SDW}}\right). \quad (49)$$

Rearranging equation (49), the expression for the dependence of T_C on Δ_{SDW} in the electron intra-band interaction becomes,

$$T_C = 1.14\frac{\hbar\omega_D}{k_B}\exp\left(-\frac{1}{N_e(0)V_e} - \alpha\Delta_{SDW}\right) \quad (50)$$

Where, $\alpha = \frac{1}{4k_B T_C}\ln\left(\frac{\hbar\omega_D+\Delta_{SDW}(0)}{\hbar\omega_D-\Delta_{SDW}(0)}\right)$, $N_e(0) = 0.1671\ (\text{meV})^{-1}$, $V_e = 3.4$ meV and the Debye energy, $\hbar\omega_D = 20$ meV meV (Kuzmicheva et al., 2017).

2.3. Dependence of superconducting transition temperature (T_C) on antiferromagnetic SDW order parameter (Δ_{SDW}) in the hole intra-band interaction

Applying the same procedure as for the electron intra-band interaction above, the expression for the dependence of T_C on Δ_{SDW} at zero temperature in the hole intra-band interaction is given by,

$$T_C = 1.14\frac{\hbar\omega_D}{k_B}\exp\left(-\frac{1}{N_h(0)V_h} - \alpha\Delta_{SDW}\right) \quad (51)$$

where $N_h(0) = 0.225\ (\text{meV})^{-1}$, $V_h = 1.8$ meV (Kuzmicheva et al., 2017).

2.4. Dependence of superconducting transition temperature (T_C) on antiferromagnetic SDW order parameter (Δ_{SDW}) in the inter-band interaction

The superconducting order parameter due to the inter-band interaction between electron and hole bands can be related to the Green's function as,

$$\Delta_{eh} = \frac{V_{eh}}{4\beta}\sum_k\left(\ll \hat{a}_{k\uparrow}^{\dagger}, \hat{a}_{-k\downarrow}^{\dagger} \gg + \ll \hat{c}_{k\uparrow}^{\dagger}, \hat{c}_{-k\downarrow}^{\dagger} \gg\right). \quad (52)$$

Applying the same procedure as above, the expression for the dependence of T_C on Δ_{SDW} at zero temperature in the inter-band interaction is given by,

$$T_C = 1.14\frac{\hbar\omega_D}{k_B}\exp\left(-\frac{1}{V_{eh}\sqrt{N_e(0)N_h(0)}} - \alpha\Delta_{SDW}\right) \quad (53)$$

Here, we calculated, $V_{eh} = 1.4331$ meV and is the inter-band interaction potential between the electron and the hole bands.

2.5. Dependence of antiferromagnetic SDW transition temperature (T_{SDW}) on antiferromagnetic SDW order parameter (Δ_{SDW})

Using equation (28) into equation (34) and considering similar SDW instabilities in both bands, the expression for $\ll \hat{a}_{-(k+q)\downarrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg$ becomes,

$$\ll \hat{a}_{-(k+q)\downarrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg = -\frac{1}{2}\left(\frac{\Delta_{SC}+\Delta_{SDW}}{\omega^2-\epsilon^2(k)-(\Delta_{SC}+\Delta_{SDW})^2}+\frac{\Delta_{SDW}-\Delta_{SC}}{\omega^2-\epsilon^2(k)-(\Delta_{SDW}-\Delta_{SC})^2}\right) \quad (54)$$

The antiferromagnetic SDW order parameter is related to the Green's function as,

$$\Delta_{SDW} = \frac{V_{SDW}}{2\beta}\sum_k \ll \hat{a}_{-(k+q)\downarrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg \quad (55)$$

where V_{SDW} is the effective Coulomb interaction term.

Now, substituting equation (54) into equation (55), we obtain,

$$\Delta_{SDW} = -\frac{V_{SDW}}{4\beta}\sum_k\left(\frac{\Delta_{SC}+\Delta_{SDW}}{\omega^2-\epsilon^2(k)-(\Delta_{SC}+\Delta_{SDW})^2}+\frac{\Delta_{SDW}-\Delta_{SC}}{\omega^2-\epsilon^2(k)-(\Delta_{SDW}-\Delta_{SC})^2}\right). \quad (56)$$

Applying Matsubara's frequency and changing $\omega \longrightarrow i\omega_n$, equation (56) becomes,

$$\Delta_{SDW} = \frac{V_{SDW}\beta}{4}\sum_{k,n}\left(\frac{\Delta_{SC}+\Delta_{SDW}}{((2n+1)\pi)^2+\beta^2(\epsilon^2(k)+(\Delta_{SC}+\Delta_{SDW})^2)}\right)+$$

$$\frac{V_{SDW}\beta}{4}\sum_{k,n}\left(\frac{\Delta_{SDW}-\Delta_{SC}}{((2n+1)\pi)^2+\beta^2(\epsilon^2(k)+(\Delta_{SDW}-\Delta_{SC})^2)}\right) \quad (57)$$

Changing summation into integration and introducing the density of state at the Fermi level of the electron intra-band interaction, $N(0)$, we get,

$$\Delta_{SDW} = \frac{V_{SDW}\beta N(0)}{4}\int_0^{\hbar\omega_D}\sum_n\left(\frac{\Delta_{SC}+\Delta_{SDW}}{((2n+1)\pi)^2+\beta^2(\epsilon^2(k)+(\Delta_{SC}+\Delta_{SDW})^2)}\right)d\epsilon(k)$$

$$+\frac{V_{SDW}\beta N(0)}{4}\int_0^{\hbar\omega_D}\sum_n\left(\frac{\Delta_{SDW}-\Delta_{SC}}{((2n+1)\pi)^2+\beta^2(\epsilon^2(k)+(\Delta_{SDW}-\Delta_{SC})^2)}\right)d\epsilon(k) \quad (58)$$

Applying the same procedure as above, we get,

$$\frac{1}{\lambda_{SDW}} = \ln\left(1.14\frac{\hbar\omega_D}{k_B T_{SDW}}\right)-\frac{7\Delta_{SDW}^2\beta^2\zeta(3)}{8\pi^2}+\frac{\beta\Delta_{SDW}}{4}\ln\left(\frac{\hbar\omega_D+\Delta_{SDW}}{\hbar\omega_D-\Delta_{SDW}}\right) \quad (59)$$

Where, for small Δ_{SDW}, the Δ_{SDW}^2 term can be ignored.

Thus, equation (59) becomes,

$$\frac{1}{\lambda_{SDW}} = \ln\left(1.14\frac{\hbar\omega_D}{k_B T_{SDW}}\right)+\frac{\Delta_{SDW}}{4k_B T_C}\ln\left(\frac{\hbar\omega_D+\Delta_{SDW}}{\hbar\omega_D-\Delta_{SDW}}\right) \quad (60)$$

Rearranging equation (60), the expression for the dependence of T_{SDW} on Δ_{SDW} is given by,

$$T_{SDW} = 1.14\frac{\hbar\omega_D}{k_B}\exp\left(-\frac{1}{\lambda_{SDW}}+\gamma\Delta_{SDW}\right) \quad (61)$$

Where, $\gamma = \frac{1}{4k_B T_{SDW}}\ln\left(\frac{\hbar\omega_D+\Delta_{SDW}(0)}{\hbar\omega_D-\Delta_{SDW}(0)}\right)$

2.6. Dependence of antiferromagnetic SDW order parameter on temperature in the pure antiferromagnetic SDW region

For pure antiferromagnetic SDW region, the superconducting state does not appear and describes magnetic state only. In this case, $\Delta_{SC} = 0$ and $\Delta_{SDW} \neq 0$.

Thus, the expression for the antiferromagnetic SDW coupling parameter becomes,

$$\frac{1}{\lambda_{SDW}} = \int_0^{\hbar\omega_D} \frac{\tanh\left(\frac{\beta}{2}\sqrt{\epsilon^2(k)+\Delta_{SDW}^2}\right)}{\sqrt{\epsilon^2(k)+\Delta_{SDW}^2}} d\epsilon_k\ . \tag{62}$$

After a couple of steps we get,

$$\frac{1}{\lambda_{SDW}} = \ln\left(1.14\frac{\hbar\omega_D}{k_B T}\right) - \Delta_{SDW}^2\left(\frac{1}{\pi k_B T_{SDW}}\right)^2 (0.1065) \tag{63}$$

$\Delta_{SDW} \to 0$ as $T \to T_{SDW}$ in pure antiferromagnetic SDW region.

Thus, equation (63) becomes,

$$\frac{1}{\lambda_{SDW}} = \ln\left(1.14\frac{\hbar\omega_D}{k_B T_{SDW}}\right). \tag{64}$$

Substituting equation (64) into equation (63), we get,

$$\ln\left(1.14\frac{\hbar\omega_D}{k_B T_{SDW}}\right) = \ln\left(1.14\frac{\hbar\omega_D}{k_B T}\right) - \Delta_{SDW}^2\left(\frac{1}{\pi k_B T_{SDW}}\right)^2 (0.1065)$$

Thus, the expression for the temperature dependence of ΔSDW becomes,

$$\Delta_{SDW}(T) = 3.06 k_B T_{SDW}\left(1 - \frac{T}{T_{SDW}}\right)^{1/2} \tag{65}$$

Where, $T_{SDW} = 140$ K (Yi et al., 2014) and is antiferromagnetic SDW transition temperature.

3. RESULTS AND DISCUSSION

In this section, we present the results we obtained in the previous sections and the analysis we made for the theoretical investigation of the coexistence of superconductivity and antiferromagnetic spin density waves (SDW) in two-band model for the iron-based superconductor $Ba_{1-x}K_xFe_2As_2$. By employing a system Hamiltonian for the two-band model and by using the double time temperature dependent Green's function formalism, we obtained the expressions for the dependence of superconducting transition temperature (T_C) and antiferromagnetic SDW transition temperature (T_{SDW}) on antiferromagnetic SDW order parameter (Δ_{SDW}) for superconductor $Ba_{1-x}K_xFe_2As_2$. Using equations (50), (51) and (53) and by considering some plausible approximations, the phase diagrams of T_C versus Δ_{SDW} in the electron and hole intra-band interactions and the inter-band interaction are plotted respectively for $Ba_{1-x}K_xFe_2As_2$, as shown in figure 1.

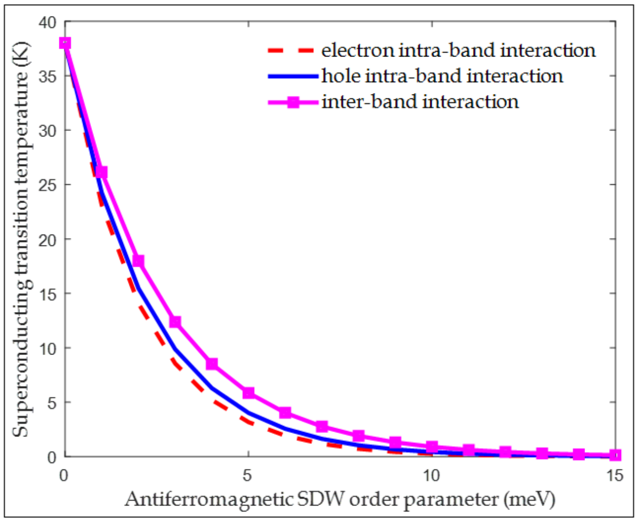

Figure 1. Superconducting transition temperature (T_C) versus antiferromagnetic SDW order parameter (Δ_{SDW}) in the electron intra-band interaction (dashed line), hole intra-band interaction (solid line) and inter-band interaction (solid-square) for superconductor $Ba_{1-x}K_xFe_2As_2$.

As can be seen from the figure, the superconducting transition temperature is suppressed as the value of the antiferromagnetic SDW order parameter increases in each band for $\mathrm{Ba_{1-x}K_xFe_2As_2}$. The presence of the magnetic ordering is found to suppress the superconducting transition temperature due to the strong spin scattering that flips the spins of the charge carriers and destroys the singlet correlations which is responsible for the Cooper pairing interactions in $\mathrm{Ba_{1-x}K_xFe_2As_2}$.

Furthermore, using equation (61), the phase diagram of antiferromagnetic SDW transition temperature ($\mathrm{T_{SDW}}$) versus antiferromagnetic SDW order parameter (Δ_{SDW}) is plotted for $\mathrm{Ba_{1-x}K_xFe_2As_2}$, as shown in figure 2. As can be seen from the figure, $\mathrm{T_{SDW}}$ increases with increasing Δ_{SDW} in each intra and inter-band interactions for $\mathrm{Ba_{1-x}K_xFe_2As_2}$.

As can be seen from figures 1 and 2, the effect of magnetic order parameter suppresses the superconductivity and promotes the magnetic nature in the system. Now, by merging figures 1 (in the electron intra-band interaction) and 2, we demonstrated a region where both superconductivity and antiferromagnetic SDW coexist in the electron intra-band interaction as shown in figure 3.

Similarly, by merging figures 1 (in the hole intra-band interaction) and 2, we demonstrated a region where both superconductivity and antiferromagnetic SDW coexist in the hole intra-band interaction as shown in figure 4.

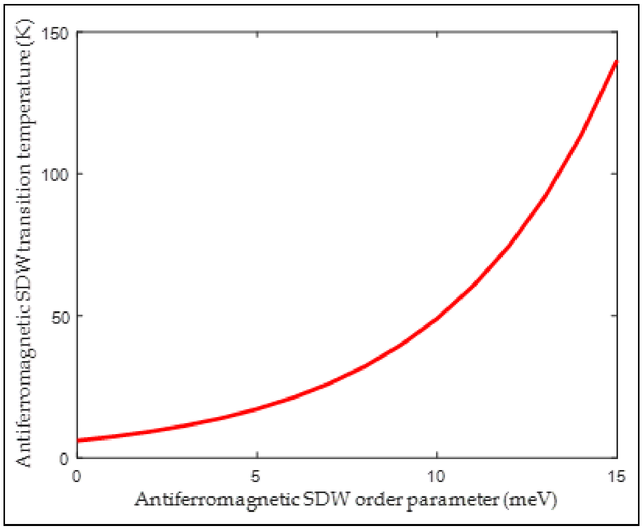

Figure 2. Antiferromagnetic SDW transition temperature (T_{SDW}) versus antiferromagnetic SDW order parameter (Δ_{SDW}) for superconductor $Ba_{1-x}K_xFe_2As_2$.

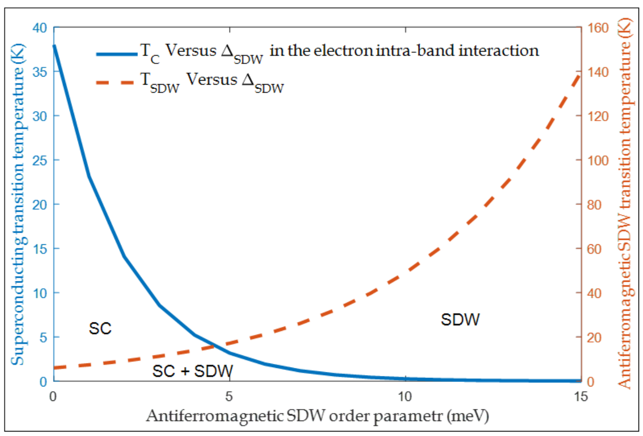

Figure 3. Coexistence of superconductivity and antiferromagnetic SDW in the electron intra-band interaction for superconductor $Ba_{1-x}K_xFe_2As_2$.

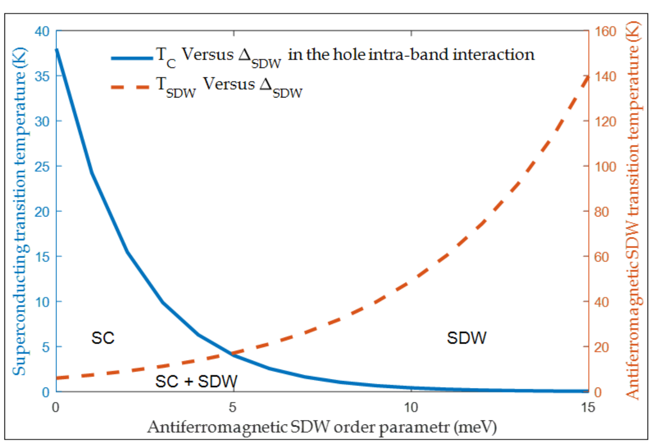

Figure 4. Coexistence of superconductivity and antiferromagnetic SDW in the hole intra-band interaction for superconductor $Ba_{1-x}K_xFe_2As_2$.

Furthermore, by merging figures 1 (in the inter-band interaction) and 2, we depicted a region in which both superconductivity and antiferromagnetic SDW coexist in the inter-band interaction as shown in figure 5.

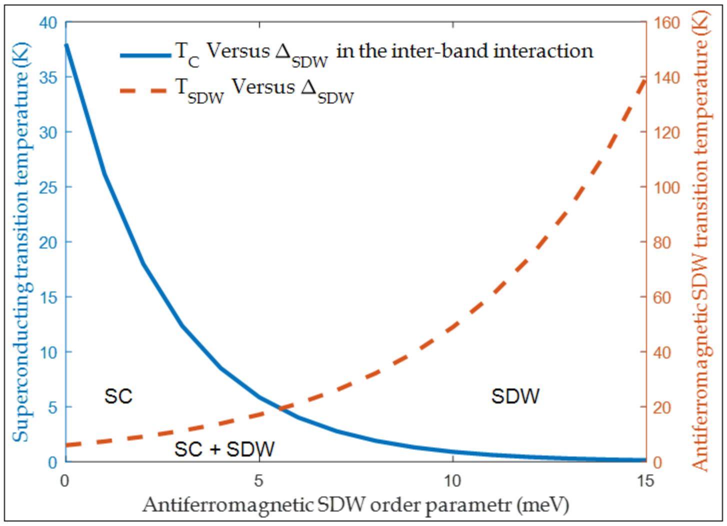

Figure 5. Coexistence of superconductivity and antiferromagnetic SDW in the inter-band interaction for superconductor $Ba_{1-x}K_xFe_2As_2$.

Moreover, using equation (65), the phase diagram for the dependence of the antiferromagnetic SDW order parameter (Δ_{SDW}) on temperature in the pure antiferromagnetic SDW region is plotted as shown in figure 6. As can be seen from the figure, Δ_{SDW} decreases as the temperature increases and vanishes at T_{SDW}.

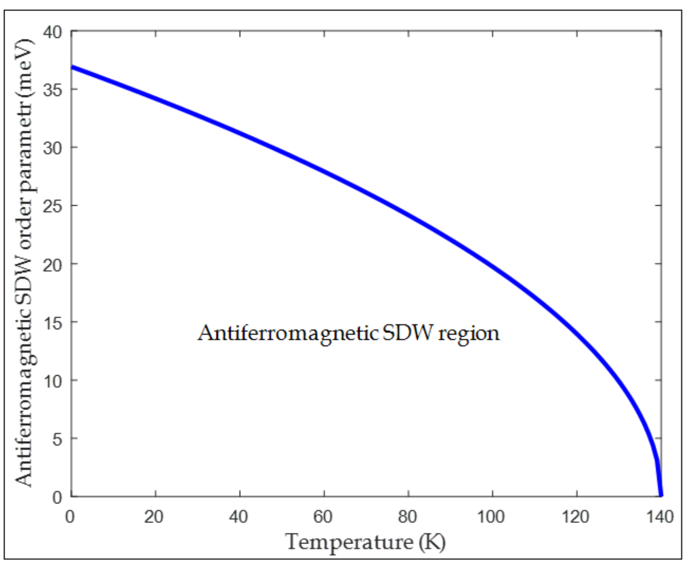

Figure 6. Antiferromagnetic SDW order parameter (Δ_{SDW}) versus temperature in the pure antiferromagnetic SDW region for superconductor $Ba_{1-x}K_xFe_2As_2$.

4. CONCLUSIONS

In this research work, we have demonstrated the possible coexistence of superconductivity and antiferromagnetic spin density wave (SDW) in the electron and hole intra-band interactions and the inter-band interaction in two-band model for the iron-based superconductor $Ba_{1-x}K_xFe_2As_2$. By formulating a model Hamiltonian in the two-band model for the given system and by using the double time temperature dependent Green's function formalism, we obtained mathematical expressions for the dependence of superconducting transition temperature (T_C) on antiferromagnetic SDW order parameter (Δ_{SDW}) in the electron and hole intra-band interactions and the inter-band interaction and phase diagrams are plotted as shown in figure 1. As can be seen from the figure, the superconducting transition temperature is suppressed as the magnitude of the antiferromagnetic SDW order parameter increases for $Ba_{1-x}K_xFe_2As_2$. Furthermore, we obtained an expression for the dependence of antiferromagnetic SDW transition temperature (T_{SDW}) on antiferromagnetic SDW order parameter (Δ_{SDW}) and a phase diagram is plotted as shown in figure 2. As can be seen from the figure, the antiferromagnetic SDW transition temperature increases with increasing antiferromagnetic SDW order parameter for $Ba_{1-x}K_xFe_2As_2$. Finally, by merging figures 1 and 2, we demonstrated the possible coexistence of superconductivity and antiferromagnetic SDW in the electron and hole intra-band interactions and the inter-band interaction for $Ba_{1-x}K_xFe_2As_2$ as shown in figures 3, 4 and 5 respectively. This coexistence implies that there is a weak exchange coupling between the itinerant superconducting electrons and the localized ordered spins in $Ba_{1-x}K_xFe_2As_2$. We also investigated the dependence of antiferromagnetic SDW order parameter (Δ_{SDW}) on temperature in the pure antiferromagnetic SDW region for the iron-based superconductor $Ba_{1-x}K_xFe_2As_2$. The SDW order parameter is suppressed when temperature increases and vanishes at T_{SDW} for $Ba_{1-x}K_xFe_2As_2$ as shown in figure 6. The possible coexistence of superconductivity and antiferromagnetic SDW in $Ba_{1-x}K_xFe_2As_2$ opens some interests in the field of condensed matter physics for further studies on spintronics. The results we obtained in this research study are in agreement with the previous experimental findings of Desta et al. (2016); Chen et al. (2009); Paglione et al. (2010); Zhang et al. (2011); Wiesenmayer et al. (2011).

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6. CONFLICT OF INTEREST

The authors declare that there are no conflicts of interests.

7. REFERENCE

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