

Thermal Radiation Effect on Fully Developed Natural Convection Flow in a Vertical Micro-Channel

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ABSTRACT

The effect of thermal radiation on steady fully developed natural convection flow in a vertical micro-channel is presented in this article. Effects of velocity slip and temperature jump conditions are taken into account due to their counter effects on both the volume flow rate and the rate of heat transfer. Due to the presence of thermal radiation, the momentum and energy equations are coupled system of ordinary differential equations. Governing coupled nonlinear equations are solved analytically by employing the perturbation analysis method to obtain an expression for fluid temperature, fluid velocity, rate of heat transfer and skin friction on the microchannel walls. The effect of various parameters controlling the physical situation such as thermal radiation, temperature difference, Knudsen number, and fluid wall interaction are discussed with the aid of line graphs and Tables. Results indicate that both velocity and temperature enhanced with the increase of the thermal radiation parameter.

Keywords: Thermal radiation, Natural convection, Micro-channel, Velocity slip, Temperature jump

INTRODUCTION

Studies related to the interaction of free convection with thermal radiation have increased greatly during the past decade due to its importance in many practical applications (Soundalgekar and Takhar, 1992; Cogley *et al.*, 1968; Hossain *et al.*, 1999; Carey and Mollendorf, 1978; Makinde, 2005; Sparrow and Cess 1962; Clarke and Riley, 1975; Vedhanayagam *et al.*, 1980). Radiation effects on the free convection flow are important in the context of space technology and processes involving high temperature. The role of thermal radiation is of major importance in the design of many advanced energy convection systems operating at high temperature. Due to the increase in science and technology, radiative heat transfer becomes very important in nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles and space vehicles. Ogulu and Makinde (2009) studied the effect of thermal radiation and absorption on unsteady free-convective flow past a vertical plate in the presence of a magnetic field and constant wall heat flux. Makinde and Chinyoka (2010) investigated the transient heat transfer flow of an electrically conducting fluid in the presence of a

magnetic field and thermal radiation and concluded that thermal radiations have a significant impact in controlling the rate of heat transfer in the boundary layer region. Makinde (2012) examined the hydromagnetic mixed convective stagnation point flow towards a vertical plate embedded in a highly porous medium with radiation and internal heat generation. They reported in their work that the local skin-friction and local Nusselt number increase as radiation parameter increases. Hossain and Takhar (1996) analyzed the effect of radiation using the Rossel and diffusion approximation, which leads to a non-similar solution for the forced and free convection of an optically dense viscous incompressible fluid past a heated vertical plate with uniform free stream and uniform surface temperature, while Hossain *et al.*, (1999) studied the effect of radiation on free convection from porous vertical plates. Gupta and Gupta (1974) studied the effect of radiation on the combined free and forced convection of an electrically conducting fluid flowing inside an open-ended vertical channel in the presence of a uniform transverse magnetic field for the case of optically thin limit. They found that radiation tends to increase the rate of heat

transfer of the fluid thereby reducing the effect of natural convection.

The main goal of this work is to model the influence of thermal radiation on steady fully developed natural convection flow in a vertical micro-channel. To the best of the author's knowledge, there is no investigation on fully developed natural convection flow in a vertical parallel plate micro-channel in the presence of thermal radiation. Roles of thermal radiation,

temperature difference, Knudsen number and fluid wall interaction are presented in outputs.

MATHEMATICAL ANALYSIS

A fully developed natural convection flow of viscous incompressible fluid in a vertical parallel plate micro-channel in the presence of thermal radiation is considered. Schematic geometry of the problem under investigation is shown in Figure 1, where x' -axis is taken vertically upward along the walls and the y' -axis normal to it in the presence of a gravitational field g .

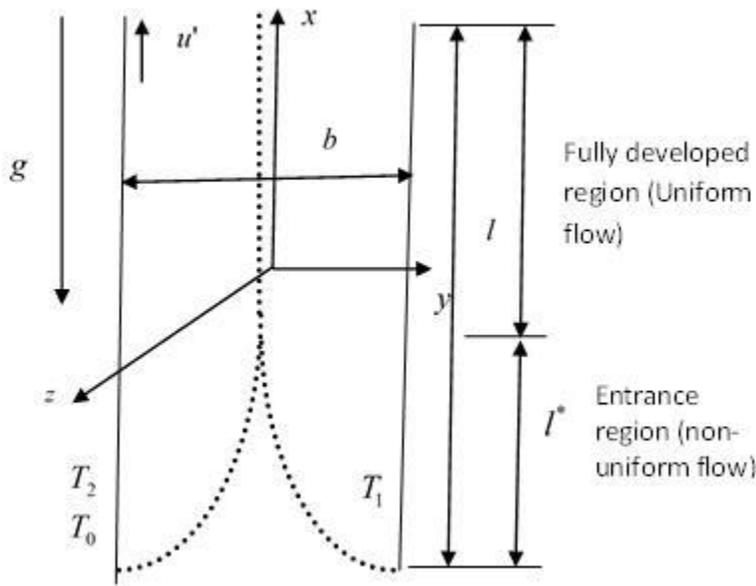


Figure 1: A schematic geometry of natural convection flow in a vertical parallel plate micro-channel

The distance between the microchannel walls is b . The walls are heated asymmetrically with one wall maintained at a temperature T_1 while the other wall at a temperature T_2 , where $T_1 > T_2$. Due to the temperature gradient between the walls of the micro-channel, natural convection flow occurs in the micro-channel. Mathematical model representing natural convection flow in a vertical parallel plate microchannel is discussed in Chen and Weng (2005). They concluded that the temperature jump condition induced by the effects of rarefaction and fluid-wall interaction plays an important role in slip-flow natural

convection. Following Chen and Weng (2005) and considering the effect of thermal radiation, under the usual Boussinesq approximation and following non-dimensional quantities;

$$y = \frac{y'}{b}, U = \frac{\nu u}{g\beta b^2(T_1 - T_0)}, \theta = \frac{T' - T_0}{T_1 - T_0},$$

$$Pr = \frac{\mu c}{k}, q_r = -\frac{4\sigma}{3K} \frac{dT^4}{dy} \quad (1)$$

where β is thermal expansion coefficient, T is the temperature, T_0 is the free stream temperature, g is the acceleration due to gravity,

ν is the kinematic viscosity of the fluid, μ is the dynamic viscosity and k is thermal conductivity. The momentum and energy equations for the fully developed flow can be written in the following dimensionless form respectively:

$$\frac{d^2U}{dY^2} + \theta = 0, \quad (2)$$

$$\left[1 + \frac{4}{3}R(C_T + \theta)^3 \right] \frac{d^2\theta}{dY^2} + 4R[C_T + \theta]^2 \left(\frac{d\theta}{dY} \right)^2 = 0, \quad (3)$$

with the boundary conditions in non-dimensional form as

$$U(Y) = \beta_v Kn \frac{dU}{dY}, \quad \theta(Y) = \xi + \beta_v Kn \ln \frac{d\theta}{dY} \quad \text{at} \\ Y = 0$$

$$U(Y) = -\beta_v Kn \frac{dU}{dY}, \quad \theta(Y) = 1 - \beta_v Kn \ln \frac{d\theta}{dY} \quad \text{at} \\ Y = 1 \quad (4)$$

where:

$$\beta_v = \frac{2 - \sigma_v}{\sigma_v}, \quad \beta_t = \frac{2 - \sigma_t}{\sigma_t} \frac{2\gamma_s}{\gamma_s + 1} \frac{1}{Pr}, \\ Kn = \frac{\lambda}{b}, \quad \ln = \frac{\beta_t}{\beta_v}, \quad \xi = \frac{T_2 - T_0}{T_1 - T_0}$$

Referring to the values of σ_v and σ_t given in Eckert and Drake (1972) and Goniak and Duffa (1995), the value of β_v is near unity, and the value of β_t ranges from near 1 to more than 100 for actual wall surface conditions and is near 1.667 for many engineering applications, corresponding to $\sigma_v = 1$, $\sigma_t = 1$, $\gamma_s = 1.4$ and $Pr = 0.71$ ($\beta_v = 1$, $\beta_t = 1.667$).

The physical quantities used in the above equations are defined in the nomenclature.

Method of Solution

Equations (2) and (3) are coupled non-linear equations due to the presence of thermal radiation and it is difficult, in general, to solve analytically. When neglecting the thermal radiation ($R = 0$), equations (2) and (3) become linear and solutions can easily be obtained. It is

important to mention here that for $R = 0$, the present problem is the same as discussed by Chen and Weng (2005). However, in many practical applications, R cannot be zero ($R \neq 0$), but in many situations, it can take small values. Small value of $R (< 1)$ facilitate finding analytical solutions of equations (2) and (3) by using the perturbation method in the form:

$$U(Y) = U_0(Y) + RU_1(Y) + \dots \quad (5)$$

$$\theta(Y) = \theta_0(Y) + R\theta_1(Y) + \dots \quad (6)$$

where the second and higher terms on the right side give a correction to θ_0 , U_0 accounting for the dissipative effects. Substituting equations (5) - (6) into equations (2) - (3) and equating like powers of R to zero, we obtain, the zero-order equations:

$$\frac{d^2U_0}{dY^2} + \theta_0 = 0 \quad (7)$$

$$\frac{d^2\theta_0}{dY^2} = 0 \quad (8)$$

with the boundary conditions:

$$U_0(Y) = \beta_v Kn \frac{dU_0}{dY}, \quad \theta_0(Y) = \xi + \beta_v Kn \ln \frac{d\theta_0}{dY} \\ \text{at } Y = 0 \quad (9)$$

$$U_0(Y) = -\beta_v Kn \frac{dU_0}{dY}, \quad \theta_0(Y) = 1 - \beta_v Kn \ln \frac{d\theta_0}{dY} \\ \text{at } Y = 1 \quad (10)$$

The first-order equations are obtained by equating like powers of R to one; we obtain the first-order equations:

$$\frac{d^2U_1}{dY^2} + \theta_1 = 0 \quad (11)$$

$$\frac{d^2\theta_1}{dY^2} + \frac{4}{3}[C_T + \theta_0]^3 \frac{d^2\theta_0}{dY^2} + 4[C_T + \theta_0]^2 \left(\frac{d\theta_0}{dY} \right)^2 = 0 \quad (12)$$

with the boundary conditions:

$$U_1(Y) = \beta_v Kn \frac{dU_1}{dY}, \quad \theta_1(Y) = \beta_v Kn \ln \frac{d\theta_1}{dY} \quad \text{at} \\ Y = 0 \quad (13)$$

$$U_1(Y) = -\beta_v Kn \frac{dU_1}{dY}, \theta_1(Y) = -\beta_v Kn \ln \frac{d\theta_1}{dY} \text{ at } Y = 1 \quad (14)$$

By solving equations (7) and (8), under boundary conditions (9) and (10), we obtained $U_0(Y)$ and $\theta_0(Y)$, respectively, as follows:

$$\theta_0(Y) = A_0 + A_1(Y) \quad (15)$$

$$U_0(Y) = B_0 + B_1(Y) - \frac{A_0 Y^2}{2} - \frac{A_1 Y^3}{6} \quad (16)$$

Also, solving equations (11) and (12) under boundary condition (13) and (14), we obtained $U_1(Y)$ and $\theta_1(Y)$, respectively, as follows:

$$\theta_1(Y) = B_3 + B_2(Y) + F_4 Y^2 + F_5 Y^3 - F_6 Y^4 \quad (17)$$

$$U_1(Y) = B_5 + B_4(Y) - \frac{B_3 Y^2}{2} - \frac{B_2 Y^3}{6} - \frac{F_4 Y^4}{12} - \frac{F_5 Y^5}{20} + \frac{F_6 Y^6}{30} \quad (18)$$

The rate of heat transfer on the micro-channel walls are:

$$Nu_0 = \left. \frac{d\theta}{dy} \right|_{y=0} \quad (19)$$

$$Nu_0 = A_1 + RB_2$$

while

$$Nu_1 = \left. \frac{d\theta}{dy} \right|_{y=1} \quad (20)$$

$$Nu_1 = A_1 + R[B_2 + 2F_4 + 3F_5 - 4F_6]$$

The dimensionless skin frictions are:

$$\tau_0 = \frac{dU(0)}{dY} = \frac{dU_0(0)}{dY} + R \frac{dU_1(0)}{dY} \quad (21)$$

$$\tau_0 = B_1 + RB_4 \quad (22)$$

while

$$\tau_1 = \frac{dU(1)}{dY} = \left[\frac{dU_0(1)}{dY} + R \frac{dU_1(1)}{dY} \right] \quad (23)$$

$$\tau_1 = (B_1 - A_0 - 0.5A_1) + R(F_{10} + B_4) \quad (24)$$

where $A_0, A_1, B_1, \dots, B_5$ are all constants given below.

$$A_0 = \xi + \frac{\beta_v Kn \ln(1-\xi)}{1+2\beta_v Kn \ln}, A_1 = \frac{(1-\xi)}{1+2\beta_v Kn \ln},$$

$$B_0 = \beta_v Kn \left[\frac{1}{2} A_0 + \frac{3\beta_v Kn + 1}{6(1+2\beta_v Kn)} A_1 \right],$$

$$B_1 = \frac{1}{2} A_0 + \frac{3\beta_v Kn + 1}{6(1+2\beta_v Kn)} A_1,$$

$$F_1 = -4C_T^2 A_1^2 - 8C_T A_0 A_1^2 - 4A_0^2 A_1^2,$$

$$F_2 = -8C_T A_1^3 - AA_0 A_1^3, F_3 = 4A_1^4,$$

$$F_4 = 0.5F_1, F_5 = \frac{F_2}{6}, F_6 = \frac{F_3}{12},$$

$$F_7 = F_4 + F_5 - F_6,$$

$$F_8 = 4\beta_v Kn \ln F_6 - 2\beta_v Kn \ln F_4 - 3\beta_v Kn \ln F_5,$$

$$F_9 = \frac{F_6}{30} - \frac{B_3}{2} - \frac{B_2}{6} - \frac{F_4}{12} - \frac{F_5}{20},$$

$$F_{10} = \frac{F_6}{5} - B_3 - \frac{B_2}{2} - \frac{F_4}{3} - \frac{F_5}{4},$$

$$B_2 = \frac{(F_8 - F_7)}{1+2\beta_v Kn \ln}, B_3 = \beta_v Kn \ln B_2,$$

$$B_4 = \frac{-(\beta_v Kn F_{10} + F_9)}{1+2\beta_v Kn \ln}, B_5 = \beta_v Kn \ln B_4$$

RESULTS AND DISCUSSION

To study the effects of different flow parameters like thermal radiation (R), temperature difference (C_T), fluid wall interaction (\ln), wall-ambient temperature difference ratio (ξ), and rarefaction ($\beta_v Kn$) on the convective heat transfer, the numerical results of the fluid velocity, temperature, skin friction and rate of heat transfer which is expressed as the Nusselt number are computed. The present parametric study has been performed in the continuum and slips flow regimes ($Kn \leq 0.1$). Also, for air and various surfaces, the values of β_v and β_t range from near 1 to 1.667 and from near 1.64 to more than 10, respectively. Therefore, this study has been performed over the reasonable ranges of $0 \leq \beta_v Kn \leq 0.1$ and $0 \leq \ln \leq 10$. The selected reference values of $\beta_v Kn$ and \ln for the present

analysis are 0.05 and 1.64 respectively as given in Chen and Weng (2005).

Figures 2 and 3 illustrate the impact of thermal radiation parameter (R) on the velocity and temperature distribution, respectively under three cases of the wall-ambient temperature difference ratio ($\xi = -1$: one heating and one cooling; $\xi = 0$: one heating and one not heating, $\xi = 1$: both walls are heated). It is evident for both cases of the wall-ambient temperature difference ratio that, as thermal radiation parameter increases, there is an enhancement in both fluid velocity and temperature. Physically speaking, an increase in thermal radiation adds more heat to the fluid leading to an increased temperature, which enhanced the velocity. Also, the influence of thermal radiation is pronounced as the wall-ambient temperature difference ratio reduces.

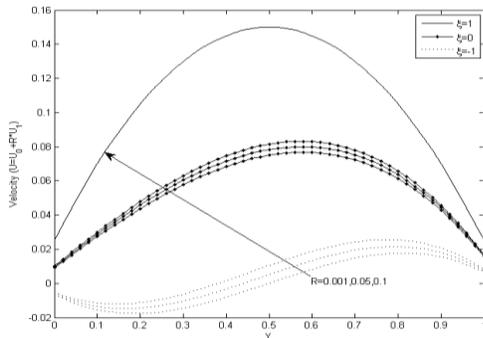


Figure 2: Variation of velocity (U) for different value of R and ξ ($\beta_v Kn = 0.05, In = 1.667$)

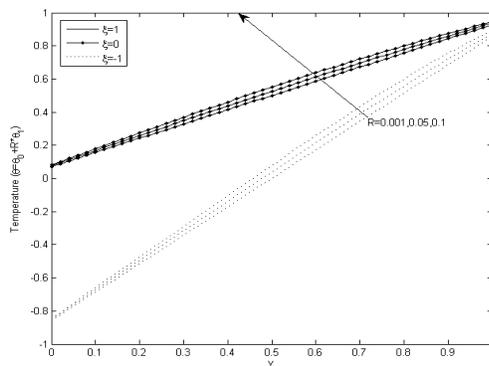


Figure 3: Variation of temperature (θ) for different value of R and ξ ($\beta_v Kn = 0.05, In = 1.667$)

Figures 4 and 5 exhibits the combined effects of rarefaction parameter ($\beta_v Kn$) as well as wall-ambient temperature difference ratio (ξ) on velocity and temperature distribution, respectively. These figures show that increasing the Knudsen number leads to enhancement in the velocity slip and temperature jump. This result yields an observable increase in the fluid velocity. The effect of rarefaction parameter ($\beta_v Kn$) is pronounced in the case of symmetric heating ($\xi = 1$).

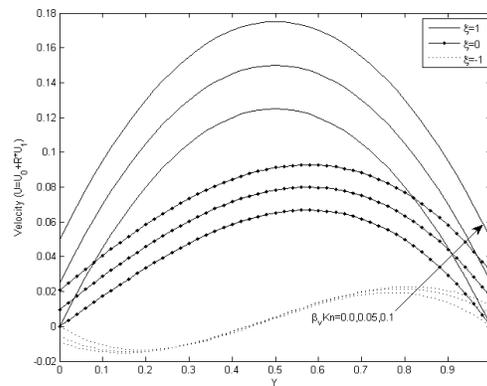


Figure 4: Variation of velocity (U) for different value of $\beta_v Kn$ and ξ ($R = 0.05, In = 1.667$)

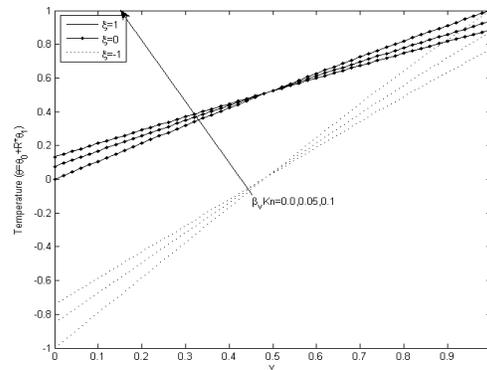


Figure 5: Variation of temperature (θ) for different value of $\beta_v Kn$ ($R = 0.05, In = 1.667$)

Figures 6 and 7 display the effects of fluid wall interaction parameter (In) on velocity and temperature distribution under three cases of the

wall-ambient temperature difference ratio. It is observed that the impact of the fluid wall interaction parameter is to enhance the fluid velocity near micro-channel wall ($Y=0$) while the reverse is the case near the micro-channel wall ($Y=1$). Furthermore, the influences of fluid-wall interaction parameter on the micro-channel slip velocity become significant in the case of symmetric heating ($\xi = 1$).

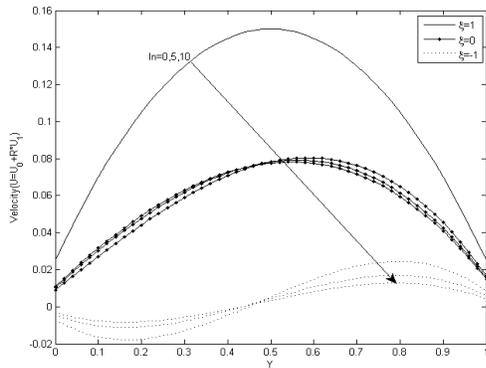


Figure 6: Variation of velocity (U) for different value of In ($R = 0.05, \beta_v Kn = 1.667$)

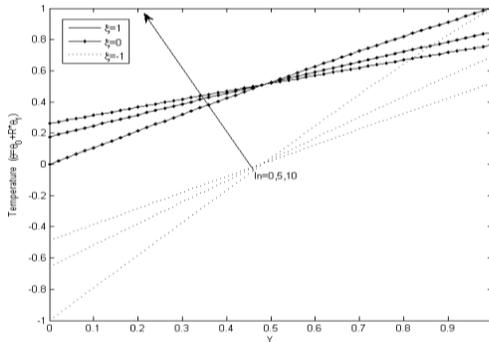


Figure 7: Variation of temperature (θ) for different value of In ($R = 0.05, \beta_v Kn = 1.667$)

Figures 8 and 9 illustrate the effects of temperature difference parameter (C_T) on the temperature and velocity profiles, respectively. It is observed that as the value of temperature difference parameter is increased there is a corresponding increase in the temperature of the fluid, and hence an increase in the velocity for different cases of the wall-ambient temperature difference ratio.

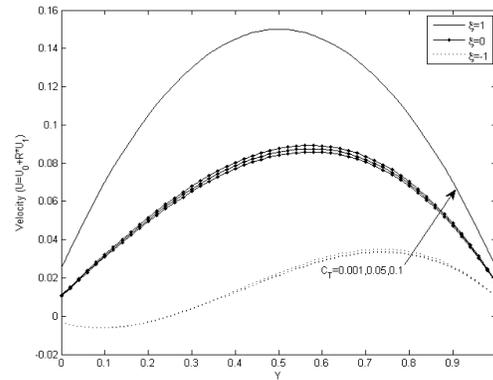


Figure 8: Variation of velocity (U) for different value of C_T ($R = 0.05, \beta_v Kn = 1.667$)

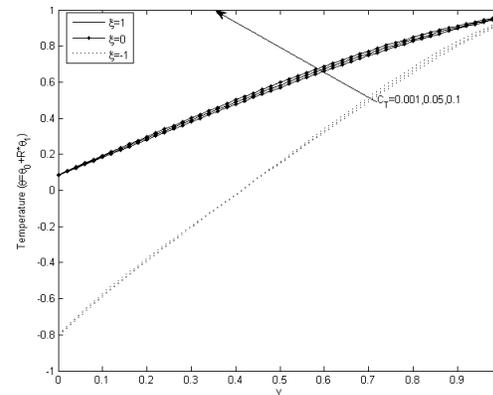


Figure 9: Variation of temperature (θ) for different value of C_T ($R = 0.05, \beta_v Kn = 1.667$)

Table I displays the influence of thermal radiation parameter ($R \neq 0$) as well as rarefaction parameter ($\beta_v Kn$) on the skin friction at the wall ($Y = 0$) and ($Y = 1$), respectively. As it can be seen from Table I, the skin friction increases with an increase in thermal radiation parameter (R) while, as rarefaction parameter increases, the impact of thermal radiation parameter (R) on the skin friction increases. In addition, the effect of thermal radiation parameter ($R \neq 0$) on the skin friction is significant in the case of asymmetric heating.

Table 1: Numerical values of skin friction on the vertical micro-channel walls for different values of $\beta_v Kn$ and R ($\ln = 1.667, C_T = 0.5$) ($\xi = 0, -1$).

$\beta_v Kn$	R	$\xi = 0$		$\xi = -1$	
		τ_0	τ_1	τ_0	τ_1
0.0	0.001	0.1668	0.3335	-0.1664	0.1670
	0.05	0.1749	0.3426	-0.1539	0.1839
	0.1	0.1831	0.3519	-0.1411	0.2011
0.05	0.001	0.1852	0.3151	-0.1296	0.1302
	0.05	0.1941	0.3251	-0.1163	0.1478
	0.1	0.2032	0.3352	-0.1027	0.1658
0.1	0.001	0.1981	0.3023	-0.1039	0.1045
	0.05	0.2072	0.3122	-0.0909	0.1209
	0.1	0.2165	0.3224	-0.0777	0.1377

Table 2 shows the effects of thermal radiation parameter ($R \neq 0$) as well as rarefaction parameter ($\beta_v Kn$) on the rate of heat transfer at the wall ($Y = 0$) and ($Y = 1$), respectively. As it can be seen from Table 2, the rate of heat transfer increases with an increase in thermal radiation parameter (R) while, as rarefaction parameter increases, the impact of thermal radiation parameter (R) on the rate of heat transfer increases. In addition, the effect of thermal radiation parameter ($R \neq 0$) on the rate

of heat transfer is significant in the case of asymmetric heating ($\xi = 0, -1$). Table 3 gives a comparison of the numerical values of the velocity obtained in the present work when $R \rightarrow 0$ with those obtained by Chen and Weng (2005). It can be observed from the table that the numerical values for fluid velocity as $R \rightarrow 0$ (in the absence of radiation parameter) reported in the present work are in excellent agreement with the numerical values for fluid velocity reported in Chen and Weng (2005).

Table 2: Numerical values of Nusselt number on the vertical micro-channel walls for different values of $\beta_v Kn$ and R ($\ln = 1.667, C_T = 0.5$)

$\beta_v Kn$	R	$\xi = 0$		$\xi = -1$	
		Nu_0	Nu_1	Nu_0	Nu_1
0.0	0.001	1.0015	0.9972	2.0020	1.9927
	0.05	1.0750	0.8583	2.1000	1.6333
	0.1	1.1500	0.7167	2.2000	1.2667
0.05	0.001	0.8583	0.8552	1.7157	1.7099
	0.05	0.9171	0.7612	1.7877	1.4968
	0.1	0.9771	0.6652	1.8612	1.2795
0.1	0.001	0.7509	0.7486	1.5010	1.4971
	0.05	0.7983	0.6805	1.5562	1.3593
	0.1	0.8466	0.6111	1.6124	1.2187

Table 3: Numerical Comparison of the values of velocity (u) obtained in the present work with those of Chen and Weng (2005)

Velocity ($ln = 1.667, \beta_v Kn = 0.05$)			
ξ	Y	Chen and Weng (2005)	Present work (when $R \rightarrow 0$)
1.0	0.2	0.1050	0.1053
	0.4	0.1450	0.1452
	0.6	0.1450	0.1448
	0.8	0.1050	0.1045
	1.0	0.0250	0.0248
0.0	0.2	0.0437	0.0445
	0.4	0.0684	0.0693
	0.6	0.0766	0.0772
	0.8	0.0613	0.0616
	1.0	0.0157	0.0158
-1	0.2	-0.0168	-0.0166
	0.4	-0.0082	-0.0081
	0.6	0.0081	0.0081
	0.8	0.0185	0.0186
	1.0	0.0065	0.0067

CONCLUSIONS

The exact solution for the fully developed natural convection flow of a viscous and incompressible in a vertical microchannel taking into account the influence of thermal radiation is obtained in this present work. Due to the presence of thermal radiation, the momentum and energy equations are a coupled system of ordinary differential equations. Governing coupled nonlinear equations are solved analytically by employing the perturbation analysis method. The study revealed that an increase in thermal radiation parameter leads to a reduction in fluid temperature. Furthermore, fluid velocity within the microchannel could be enhanced with the increase in the rarefaction parameter

Nomenclature

b = gap between the walls
 C_p = specific heat of the fluid at constant pressure
 g = gravitational acceleration
 ln = fluid wall interaction parameter,

$\beta_v Kn$ Knudsen number λ/b

Pr = Prandtl number

T = temperature of the fluid

T_0 = temperature of the fluid and walls in reference state

R = thermal radiation parameter

u = dimensional velocity of the fluid

U = dimensionless velocity of the fluid

y = dimensional coordinate perpendicular to the walls

Y = dimensionless coordinate perpendicular to the walls

Greek Letters

α = thermal diffusivity

β = coefficient of thermal expansion

β_t, β_v = dimensionless variables

γ = ratio of specific heats (C_p/C_v)

θ = dimensionless temperature

ρ = density

μ_0 = fluid dynamic viscosity

ν = fluid kinematic viscosity

λ = molecular mean free path

k = thermal conductivity

σ_t, σ_v = thermal and tangential momentum accommodation coefficients, respectively

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