Mean Time to System Failure and Availability Modeling of a Repairable non Identical Three Unit Warm Standby Redundant System

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ABSTRACT: Most of the studies on reliability characteristics of redundant systems deal with cold standby systems only. Little attention is paid on 2-out-of-3 warm standby system involving three types of failures. In this study models for mean time to system failure and availability have been developed to study the effect of failure rate on some measures of system effectiveness. Using Chapman Kolmogorov's forward equations methods, explicit expressions for measures of system effectiveness like mean time to system failure (MTSF) and availability have been obtained. Graphs were plotted to see the behavior of MTSF and availability of system. Also a special case like linear consecutive 2-out-of-3 system is considered to see the effect of failure rate on system design. Results show that three units warm standby is more effective that linear consecutive 2-out-of-3 system.
Keywords: MTSF, availability, 2-out-of-3 system, warm standby, Chapman Kolmogorov's equations methods

INTRODUCTION

Redundancy is a technique used to improve system reliability and availability. Reliability is vital for proper utilization and maintenance of any system. It involves technique for increasing system effectiveness through reducing failure frequency and maintenance cost minimization. One of the forms of redundancy is the *k*-out-of-*n* system which finds wide application in industrial system. There are systems of three/four units in which two/three units are sufficient to perform the entire function of the system. Examples of such systems are 2-out-of-3, 2-out-of-4, or 3-out-of-4 redundant systems. These systems have wide application in the real world especially in industries. Furthermore, a communication system with three transmitters can be sited as a good example of 2-out-of-3 redundant system. Many researchers have reported on reliability of 2-out-of-3 redundant systems. For example, Chander and Bhardwaj (2007), analyzed reliability models for 2-out-of-3 redundant system subject to conditional arrival time of the server. Chander and Bhardwaj (2009) present reliability and economic analysis of 2-out-of-3 redundant system with priority to repair. Bhardwaj and Malik(2010) studied MTSF and cost effectiveness of 2-out-of-3 cold standby system with probability of repair and inspection. Yusuf and Hussaini (2012) studied the reliability characteristics of 2-out-of-3 cold standby system with perfect repair options. Yusuf (2012) deal with the analysis of 3-out-of-4 cold standby system in the presence of preventive maintenance. Mokaddis et al (2009) investigated the probabilistic of a two unit warm standby system with constant failure time and two types of repairmen and patience time using regenerative point technique.

In this paper, we construct a redundant 2-out-of-3 system and derived its corresponding mathematical models. Furthermore, we studied reliability characteristics of the system model involving three types of failures using Chapman-Kolmogorov's differential equation.

The objectives of this paper are:
1. To obtain explicit expression for (a) MTSF when the system viewed as (i) random 2-out-of-3 warm standby (ii) linear consecutive 2-out-of-3 warm standby system, (b) system availability.
2. To capture the effect of both failure and repair rates on the measures of system effectiveness like MTSF and availability based on assumed numerical values given to the system parameters.
3. To compare the two configurations for the MTSF.

METHODOLOGY
Notations

β_i = Constant type i failure rate,   S = Unit in Standby

α_i = Constant type i repair rate, O = Unit in operation

F_{Ri} = Failed unit under type i repair

F_{Wi} = Failed unit waiting for type i repair $i = 1, 2, 3$

Assumptions
(a) The system consist of 3 components/units
(b) Initially two units are in operable condition of full capacity
(c) The system is failed when the number of working

 component goes down below 2
(d) Failure and repair time follow exponential distribution
(e) Repair is as good as new(Perfect repair)
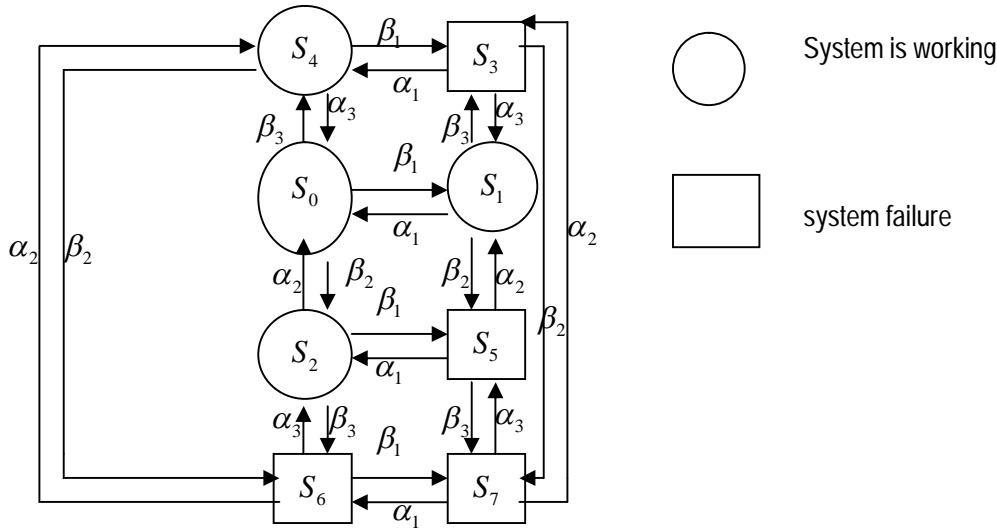(f) The system is attended by one repairman

Figure 1: States of the System

$$S_0(O_1,O_2,O_S)\text{, } S_1(F_{R1},O_2,O_3)\text{, }S_2(O_1,F_{R2},O_3)\text{, } S_3(F_{R1},\overline{O}_{2S},F_{W3})\text{, } S_4(O_1,O_2,F_{R3})\text{,}$$
$$S_5(F_{R1},F_{W2},\overline{O}_{3S})\text{, } S_6(\overline{O}_{1S},F_{R2},F_{W3})\text{, } S_7(F_{R1},F_{W2},F_{R3})$$

Model Formulation
Mean time to system failure analysis for a random 2-out-of-3 warm standby system:
From Figure 1 above, the up state of the system are: $S_0(O_1,O_2,O_S)$, $S_1(F_{R1},O_2,O_3)$, $S_2(O_1,F_{R2},O_3)$,

$S_4(O_1,O_2,F_{R3})$ and down states are $S_3(F_{R1},\overline{O}_{2S},F_{W3})$, $S_5(F_{R1},F_{W2},\overline{O}_{3S})$, $S_6(\overline{O}_{1S},F_{R2},F_{W3})$,

$S_7(F_{R1},F_{W2},F_{R3})$.

Define $P_i(t)$ to be the probability that the system at time $t,(t\geq 0)$ is in state S_i. Let $P(t)$ be the probability row vector at time t, the initial condition for this paper are
$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0)] = [1,0,0,0,0,0,0,0]$$
We obtain the following differential equations:

$$\frac{dP_0(t)}{dt} = -(\beta_1 + \beta_2 + \beta_3)P_0(t) + \alpha_1 P_1(t) + \alpha_2 P_2(t) + \alpha_3 P_4(t)$$

$$\frac{dP_1(t)}{dt} = -(\alpha_1 + \beta_2 + \beta_3)P_1(t) + \beta_1 P_0(t) + \alpha_3 P_3(t) + \alpha_2 P_5(t)$$

$$\frac{dP_2(t)}{dt} = -(\alpha_2 + \beta_1 + \beta_3)P_2(t) + \beta_2 P_0(t) + \alpha_1 P_5(t) + \beta_3 P_6(t)$$

$$\frac{dP_3(t)}{dt} = -(\alpha_1 + \alpha_3 + \beta_2)P_3(t) + \beta_3 P_1(t) + \beta_1 P_4(t) + \alpha_2 P_7(t)$$

$$\frac{dP_4(t)}{dt} = -(\alpha_3 + \beta_1 + \beta_2)P_4(t) + \beta_3 P_0(t) + \alpha_1 P_3(t) + \alpha_2 P_6(t)$$

$$\frac{dP_5(t)}{dt} = -(\alpha_1 + \alpha_2 + \beta_3)P_5(t) + \beta_2 P_1(t) + \beta_1 P_2(t) + \alpha_3 P_7(t)$$

$$\frac{dP_6(t)}{dt} = -(\alpha_2 + \alpha_3 + \beta_1)P_6(t) + \beta_3 P_2(t) + \beta_2 P_4(t) + \alpha_1 P_7(t)$$

$$\frac{dP_7(t)}{dt} = -(\alpha_1 + \alpha_2 + \alpha_3)P_7(t) + \beta_2 P_3(t) + \beta_3 P_5(t) + \beta_1 P_6(t) \tag{1}$$

The differential equations (1) above can be written in matrix form as

$$\dot{P} = AP \tag{2}$$

Where

$$\dot{P} = \begin{bmatrix} \dfrac{dP_0(t)}{dt} \\ \dfrac{dP_1(t)}{dt} \\ \dfrac{dP_2(t)}{dt} \\ \dfrac{dP_3(t)}{dt} \\ \dfrac{dP_4(t)}{dt} \\ \dfrac{dP_5(t)}{dt} \\ \dfrac{dP_6(t)}{dt} \\ \dfrac{dP_7(t)}{dt} \end{bmatrix} \quad A = \begin{bmatrix} -a_{11} & \alpha_1 & \alpha_2 & 0 & \alpha_3 & 0 & 0 & 0 \\ \beta_1 & -a_{22} & 0 & \alpha_3 & 0 & \alpha_2 & 0 & 0 \\ \beta_2 & 0 & -a_{33} & 0 & 0 & \alpha_1 & \beta_3 & 0 \\ 0 & \beta_3 & 0 & -a_{44} & \beta_1 & 0 & 0 & \alpha_2 \\ \beta_3 & 0 & 0 & \alpha_1 & -a_{55} & 0 & \alpha_2 & 0 \\ 0 & \beta_2 & \beta_1 & 0 & 0 & -a_{66} & 0 & \alpha_3 \\ 0 & 0 & \beta_3 & 0 & \beta_2 & 0 & -a_{77} & \alpha_1 \\ 0 & 0 & 0 & \beta_2 & 0 & \beta_3 & \beta_1 & -a_{88} \end{bmatrix} \quad P = \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \\ P_6(t) \\ P_7(t) \end{bmatrix}$$

$$a_{11} = (\beta_1 + \beta_2 + \beta_3), a_{22} = (\alpha_1 + \beta_2 + \beta_3), a_{33} = (\alpha_2 + \beta_1 + \beta_3),$$
$$a_{44} = (\alpha_1 + \alpha_3 + \beta_2), a_{55} = (\alpha_3 + \beta_1 + \beta_2), a_{66} = (\alpha_1 + \alpha_2 + \beta_3),$$
$$a_{77} = (\alpha_2 + \alpha_3 + \beta_1), a_{88} = (\alpha_1 + \alpha_2 + \alpha_3)$$

It is difficult to evaluate the transient solutions hence we delete the rows and columns of absorbing state of matrix A and take the transpose to produce a new matrix, say Q_1 (El said, 2008; Haggag, 2009).

The expected time to reach an absorbing state is obtained from the following:

$$E\left[T_{P(0)\rightarrow P(absorbing)}\right] = P(0)\int_0^\infty e^{Q_1 t} dt \tag{3}$$

and

$$\int_0^\infty e^{Qt} dt = Q_1^{-1} \text{, since } Q_1^{-1} < 0 \tag{4}$$

explicit expression for the $MTSF_1$ is given by

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTSF_1 = P(0)(-Q_1^{-1})\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix} = \frac{N_1}{D_1} \tag{5}$$

$$Q_1 = \begin{bmatrix} -(\beta_1 + \beta_2 + \beta_3) & \beta_1 & \beta_2 & \beta_3 \\ \alpha_1 & -(\alpha_1 + \beta_2 + \beta_3) & 0 & 0 \\ \alpha_2 & 0 & -(\alpha_2 + \beta_1 + \beta_3) & 0 \\ \alpha_3 & 0 & 0 & -(\alpha_3 + \beta_1 + \beta_2) \end{bmatrix}$$

$$MTSF = \frac{N_1}{D_1}$$

Where

$$N_1 = (\alpha_1 + \beta_2 + \beta_3)(\alpha_2 + \beta_1 + \beta_3)(\alpha_3 + \beta_1 + \beta_2) + \beta_1(\alpha_2 + \beta_1 + \beta_3)(\alpha_3 + \beta_1 + \beta_2) + \beta_3(\alpha_1 + \beta_2 + \beta_3)$$
$$(\alpha_2 + \beta_1 + \beta_3) + \beta_2(\alpha_1 + \beta_2 + \beta_3)(\alpha_3 + \beta_1 + \beta_2) \qquad +$$

Sample efficiency (%) $= \dfrac{W_{B+S} - W_B}{0.077} \times 100$

$$D_1 = \alpha_2\beta_1\beta_3^2 + \alpha_3\beta_2\beta_3^2 + \alpha_3\beta_1\beta_2^2 + 2\alpha_1\beta_1\beta_2\beta_3 + \alpha_2\alpha_3\beta_1\beta_2 + 2\alpha_3\beta_1\beta_2\beta_3 + \alpha_2\alpha_3\beta_1\beta_3 + 2\alpha_2\beta_1\beta_2\beta_3$$
$$+ \alpha_1\alpha_3\beta_1\beta_2 + \alpha_1\alpha_3\beta_2\beta_3 + \alpha_1\alpha_2\beta_1\beta_3 + \alpha_1\alpha_2\beta_2\beta_3 + 4\beta_1\beta_2\beta_3^2 + \alpha_1\beta_1^2\beta_3 + \alpha_2\beta_1^2\beta_2 + \alpha_2\beta_1\beta_2^2 + 4\beta_1^2\beta_2\beta_3$$
$$+ 4\beta_1\beta_2^2\beta_3 + \alpha_2\beta_1^2\beta_3 \, \alpha_1\beta_1\beta_2^2 + \alpha_1\beta_2^2\beta_2 + \alpha_2\beta_2^2\beta_3 + \alpha_1\beta_1^2\beta_2 + \alpha_3\beta_1^2\beta_2 + \alpha_3\beta_1^2\beta_3 + \alpha_3\beta_1\beta_3^2 + \alpha_3\beta_2^2\beta_3$$
$$+ \alpha_1\beta_1\beta_3^2 \, \alpha_1\beta_2\beta_3^2 + \alpha_2\beta_2\beta_3^2 + \beta_1^3\beta_2 + 2\beta_1^2\beta_2^2 + \beta_1^3\beta_3 + 2\beta_1^2\beta_3^2 + \beta_1\beta_2^3 + \beta_2^3\beta_3 + 2\beta_2^2\beta_3^2 + \beta_1\beta_3^3 + \beta_2\beta_3^3$$

Mean time to system failure analysis for a linear consecutive 2-out-of-3 warm standby system:
From Figure 1, the up states of the system are: $S_0(O_1, O_2, O_S)$, $S_1(F_{R1}, O_2, O_3)$, $S_4(O_1, O_2, F_{R3})$ and down states are $S_2(O_1, F_{R2}, O_3)$, $S_3(F_{R1}, \overline{O}_{2S}, F_{W3})$, $S_5(F_{R1}, F_{W2}, \overline{O}_{3S})$, $S_6(\overline{O}_{1S}, F_{R2}, F_{W3})$, $S_7(F_{R1}, F_{W2}, F_{R3})$. Define $P_i(t)$ to be the probability that the system at time $t, (t \geq 0)$ is in state S_i. Let $P(t)$ be the probability row vector at time t, the initial condition for this paper are $P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0)] = [1,0,0,0,0,0,0,0]$

It is difficult to evaluate the transient solutions hence we delete the rows and columns of absorbing state of matrix A and take the transpose to produce a new matrix, say Q_2 (El said, 2008 and Haggag, 2009).

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTSF_2 = P(0)(-Q_2^{-1})\begin{bmatrix}1\\1\\1\end{bmatrix} = \frac{N_2}{D_2} \tag{6}$$

Where $Q_2 = \begin{bmatrix} -(\beta_1 + \beta_2 + \beta_3) & \beta_1 & \beta_3 \\ \alpha_1 & -(\alpha_1 + \beta_2 + \beta_3) & 0 \\ \alpha_3 & 0 & 0 \quad -(\alpha_3 + \beta_1 + \beta_2) \end{bmatrix}$

$$N_2 = (\alpha_1 + \beta_2 + \beta_3)(\alpha_3 + \beta_1 + \beta_2) + \beta_1(\alpha_3 + \beta_1 + \beta_2) + \beta_3(\alpha_1 + \beta_2 + \beta_3)$$

$$D_2 = 3\beta_1\beta_2\beta_3 + \alpha_3\beta_1\beta_3 + \alpha_1\beta_1\beta_2 + \alpha_3\beta_1\beta_2 + \alpha_1\alpha_3\beta_2 + \alpha_3\beta_2\beta_3 + \alpha_1\beta_1\beta_3 + \alpha_1\beta_2\beta_3 + \beta_1^2\beta_2 + 2\beta_1\beta_2^2$$
$$+ \beta_1^2\beta_3 + \alpha_1\beta_2^2 + \alpha_3\beta_2^2 + 2\beta_2^2\beta_3 + \beta_1\beta_3^2 + \beta_2\beta_3^2 + \beta_2^3$$

Availability analysis for a random 2-out-of-3 warm standby system:
For the analysis of availability case of Figure 1 using the same initial conditions for this problem as
$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0)] = [1,0,0,0,0,0,0,0]$$

The differential equations can be expressed as

$$\begin{bmatrix}\dot{P_0}\\\dot{P_1}\\\dot{P_2}\\\dot{P_3}\\\dot{P_4}\\\dot{P_5}\\\dot{P_6}\\\dot{P_7}\end{bmatrix} = \begin{bmatrix} -a_{11} & \alpha_1 & \alpha_2 & 0 & \alpha_3 & 0 & 0 & 0 \\ \beta_1 & -a_{22} & 0 & \alpha_3 & 0 & \alpha_2 & 0 & 0 \\ \beta_2 & 0 & -a_{33} & 0 & 0 & \alpha_1 & \beta_3 & 0 \\ 0 & \beta_3 & 0 & -a_{44} & \beta_1 & 0 & 0 & \alpha_2 \\ \beta_3 & 0 & 0 & \alpha_1 & -a_{55} & 0 & \alpha_2 & 0 \\ 0 & \beta_2 & \beta_1 & 0 & 0 & -a_{66} & 0 & \alpha_3 \\ 0 & 0 & \beta_3 & 0 & \beta_2 & 0 & -a_{77} & \alpha_1 \\ 0 & 0 & 0 & \beta_2 & 0 & \beta_3 & \beta_1 & -a_{88} \end{bmatrix}\begin{bmatrix}P_0\\P_1\\P_2\\P_3\\P_4\\P_5\\P_6\\P_7\end{bmatrix}$$

The states S_0, S_1, S_2, S_3 and S_4 in Figure 1 are the only working states of the system. The steady-state availability is sum of the probability of operational states. Thus, the steady-state availability is given by
$$A(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_4(\infty) \tag{7}$$
In the steady state, the derivatives of the state probabilities become zero and therefore equation (2) become
$$AP(\infty) = 0 \tag{8}$$
which is in matrix form

$$
\begin{bmatrix}
-a_{11} & \alpha_1 & \alpha_2 & 0 & \alpha_3 & 0 & 0 & 0 \\
\beta_1 & -a_{22} & 0 & \alpha_3 & 0 & \alpha_2 & 0 & 0 \\
\beta_2 & 0 & -a_{33} & 0 & 0 & \alpha_1 & \beta_3 & 0 \\
0 & \beta_3 & 0 & -a_{44} & \beta_1 & 0 & 0 & \alpha_2 \\
\beta_3 & 0 & 0 & \alpha_1 & -a_{55} & 0 & \alpha_2 & 0 \\
0 & \beta_2 & \beta_1 & 0 & 0 & -a_{66} & 0 & \alpha_3 \\
0 & 0 & \beta_3 & 0 & \beta_2 & 0 & -a_{77} & \alpha_1 \\
0 & 0 & 0 & \beta_2 & 0 & \beta_3 & \beta_1 & -a_{88}
\end{bmatrix}
\begin{bmatrix}
\overline{P_0} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
$$

Using the following normalizing condition

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) + P_6(\infty) + P_7(\infty) = 1 \qquad (9)$$

We substitute (9) in the **last** row **of** (8) to give the following system of linear equations in matrix form:

$$
\begin{bmatrix}
-a_{11} & \alpha_1 & \alpha_2 & 0 & \alpha_3 & 0 & 0 & 0 \\
\beta_1 & -a_{22} & 0 & \alpha_3 & 0 & \alpha_2 & 0 & 0 \\
\beta_2 & 0 & -a_{33} & 0 & 0 & \alpha_1 & \beta_3 & 0 \\
0 & \beta_3 & 0 & -a_{44} & \beta_1 & 0 & 0 & \alpha_2 \\
\beta_3 & 0 & 0 & \alpha_1 & -a_{55} & 0 & \alpha_2 & 0 \\
0 & \beta_2 & \beta_1 & 0 & 0 & -a_{66} & 0 & \alpha_3 \\
0 & 0 & \beta_3 & 0 & \beta_2 & 0 & -a_{77} & \alpha_1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
\overline{P_0(\infty)} \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty)
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1
\end{bmatrix}
$$

and solve for $P_0(\infty), P_1(\infty), P_2(\infty), P_3(\infty), P_4(\infty)$

RESULTS AND DISCUSSIONS

In this section, we numerically compared the results for MTSF and availability for the developed models. For each model the following set of parameter values are fixed throughout the simulations in Figures 2-8 for consistency:

If we put $\alpha_1 = 0.7$, $\alpha_2 = 0.7$, $\alpha_3 = 0.9$, $0.1 \le \beta_1 \le 1$, $\beta_2 = 0.5$, $\beta_3 = 0.4$ **and vary** β_1 we **obtain** the following:

Fig.2: **Show**s relation between β_1 and MTSF, **for** $0 \le \beta_1 \le 1$

Fig.3: **Show**s relation between β_1 and Availability, **for** $0 \le \beta_1 \le 1$

Fig. 4: **Show**s relation between β_1 and MTSF, **for** $1 \le \beta_1 \le 1.2$

Fig. 5: **Show**s relation between β_1 and MTSF, **for** $1.2 \le \beta_1 \le 1.4$

Fig. 6: **Show**s relation between β_1 and MTSF for **the two cases**, **for** $0 \le \beta_1 \le 1$

Fig. 7: **Show**s relation between β_1 and MTSF for **the two cases**, **for** $1 \le \beta_1 \le 1.2$

Fig. 8: **Show**s relation between β_1 and MTSF for **the two cases**, **for** $1.2 \le \beta_1 \le 1.4$
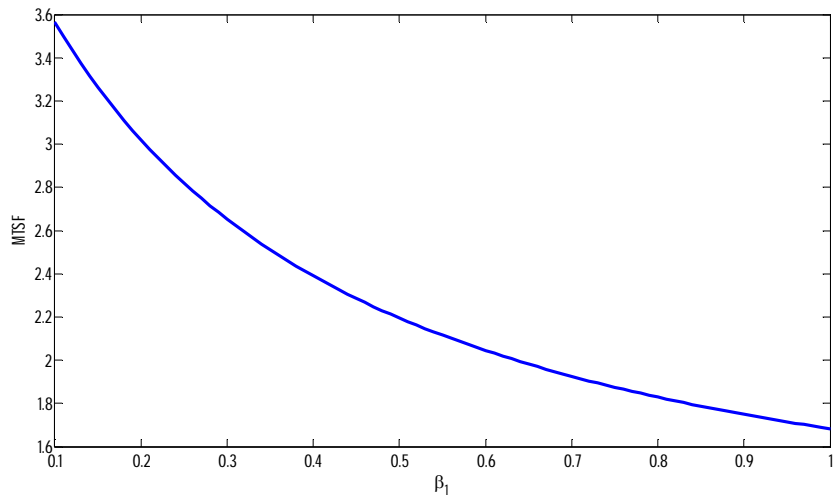
Figure 2: Plot of MTSF against β_1
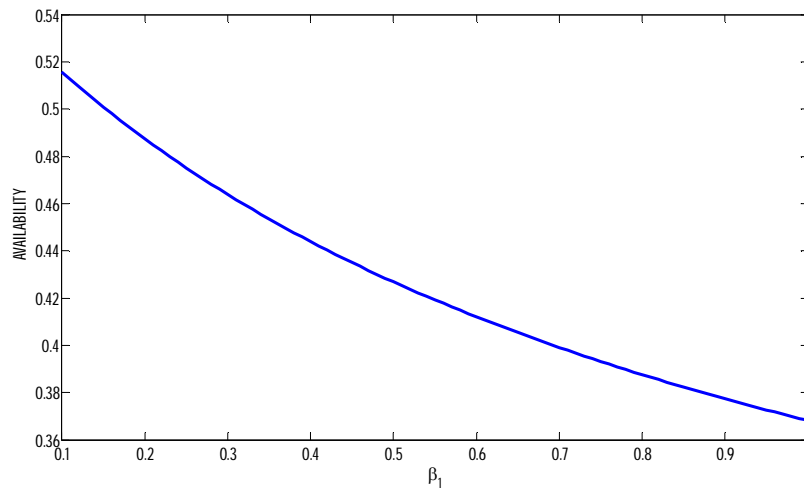
Figure 3: Plot of Availability against β_1
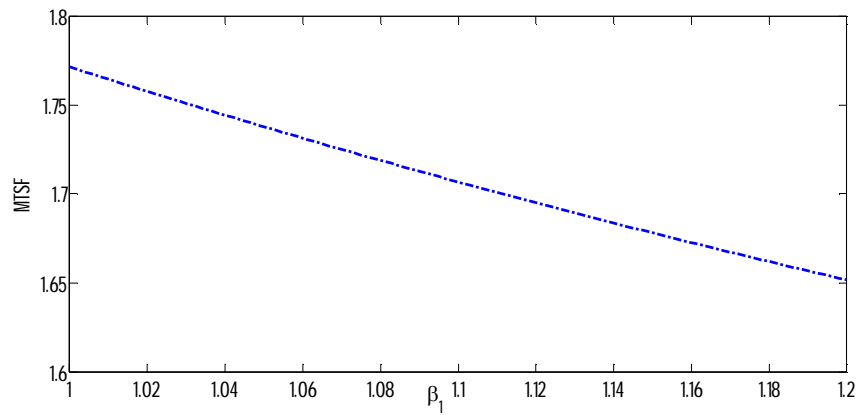
Figure 4: Plot of MTSF against β_1
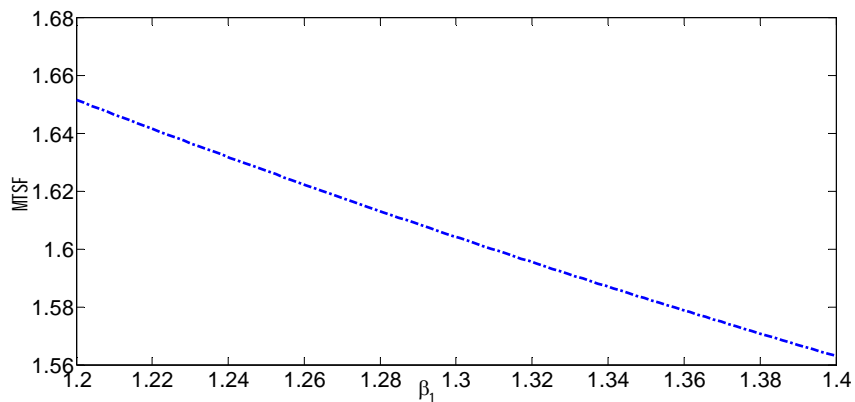
Figure 5: Plot of MTSF against β_1
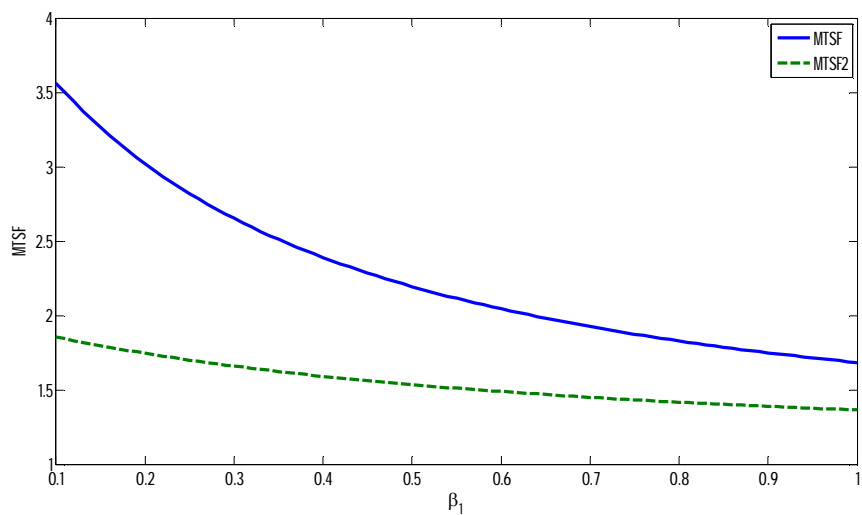
Figure 6: Plot of MTSF against β_1
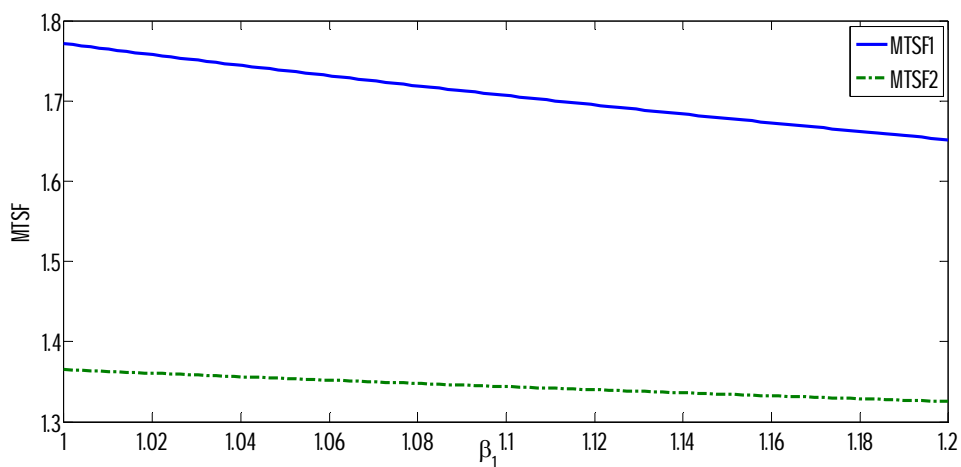
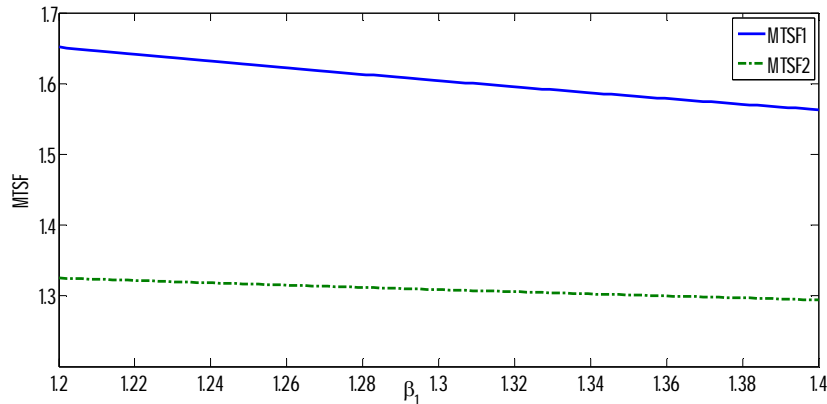

Figure 7: Plot of MTSF against β_1

Figure 8: Plot of MTSF against β_1

Numerical results of MTSF and availability are depicted in Figures 2 – 8. Figures 2 and 3 show that the MTSF and availability decreased as type I failure rate β_1 increases. We varied the value of β_1 in Figures 4 and 5, MTSF in this case decreased with increase in the value of β_1. However, the value of MTSF is better in Figure 4 than Figures 3 and 5. Thus, the optimal value of MTSF is in Figures 4. For different values of β_1, we plotted the graphs in Figures 6 – 8 to compare the MTSF when the system is viewed as random 2-out-of-3 warm standby with linear consecutive 2-out-of-3 warm standby system. It is clear that MTSF in each case decreased with increase in the value of β_1 which reflected the effect of failure rate on life span of the system. However, MTSF for linear consecutive 2-out-of-3 warm standby $MTSF_2$ decreased more than the MTSF for a random 2-out-of-3 warm standby system $MTSF_1$. Thus, $MTSF_2 < MTSF_1$.

CONCLUSION
In this study we developed explicit expressions for MTSF and availability of three redundant warm standby system. The MTSF and availability decreases with failure rate, thus, as the failure rates increased, both MTSF and availability decreased as can be seen in Figures 2 and 3. A random 2-out-of-3 warm standby system is more effective than a linear consecutive 2-out-of-3 warm standby system based on MTSF.

ACKNOWLEDGMENTS

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