SOME ASPECTS OF THE SEDIMENT TRANSPORT CAPACITIES OF OGBESE AND OWENA RIVERS

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Abstract. In this paper, the effect of sediment transport capacities of Ogbese and Owena Rivers were examined. The data were obtained from field investigations carried out on the two rivers respectively. The result obtained is in the form of power law relationship. Calibrating yield the values for $k_1$ and $k_2$ which are the erosivity coefficients. When tested, the predicted models for the two rivers performed well when compared with the established models ($R^2 = 0.998, 0.929$). It is concluded that models which will be useful for scientific and engineering applications on the two rivers have been developed.

1. INTRODUCTION

The movement of sediments such as sand, silt or gravel by flowing water is of interest to a wide range of engineering disciplines. However, very few of the problems involved can be considered to be even approximately solved and consequently there are few universal quantitative laws. Indeed, there is a great diversity of opinion generated by the apparently contradictory findings of different investigators.

It is also evident from the numerous sediment transport models that have been evolved, that is ultimately necessary to resort to the use of empirical relationships, which may be applicable only to a given location or set of experimental results. Many of the more commonly used of these models have been tested recently over an extensive range of flume and field data [1]. It would appear that, in the absence of more detailed studies of the various governing phenomena, the most relationships empirically from field data. It is generally accepted that in a unidirectional flow the sediment can be considered to be transported in two regions: abed load region where sediment is transported in a layer adjacent to the stationary part of the bed supported by inter-particle collisions, and a suspended load region in which gravitational forces are overcome by fluid turbulence. Owing to observational difficulties the transition between the suspended load region and the bed load region has for a long time remained undefined.

Two of the relevant topics that have gained the most general acceptance are the form of the turbulent velocity distribution and the suspended concentration distribution. However, in order to apply the respective theories in a predictive sense it is necessary to predetermine the magnitude of fluid velocity and sediment concentration at a given reference level above an assumed datum. These distributions cannot be considered to be wholly applicable in the region very close to the sediment because of the neglect of the governing forces in that area. It is the understanding and formulation of bed load transport and hence of total sediment transport that is at present particularly unsatisfactory.

Some of these difficulties may be overcome by adopting the following hypotheses. First it is suggested that there is continuity of both fluid and sediment velocity and also of sediment concentration at a transitional level between bed load and suspended load. Further, the exact definition of this level should not be of crucial importance as the order of magnitude of sediment flux above and below the transitional level will be similar under the previous assumption. Based on these ideas a predictive mathematical model for sediment transport in uniform flow is presented. Although not attempting a complete description of the physical processes involved, the model relies on the simplest possible model of bed load and suspended load behaviour capable of predictive use.

2. SUSPENDED SEDIMENT AND TURBULENT FLOWS

Several reasonably similar formulations for the turbulent velocity distribution and sediment concentration distribution exist [2,3,4,5]. The most consistent of these is due to Hunt and Fleming [5] and may be summarized as follows: -

The shear stress is assumed to vary linearly with depth so that

$$\tau_d = \tau_o (1 - y/h) \quad (1)$$

where $\tau_o = \rho gh \sigma$ (2)
and $y$ is the distance above the bed, $g$ is acceleration due to gravity, $S$ is the energy slope, $h$ is the total depth and $p$ is the density of the fluid.

In the momentum transfer theory the mixing length $l$ of an eddy for momentum exchange is given by Prandtl's relationship

$$
\tau = \rho l \frac{d(u)}{dy} \frac{dU}{dy}
$$

(3)

where $U$ is the fluid velocity.

From von Karman's similarity principle

$$
l = \kappa \frac{dU}{d^2U} \text{dy}^2
$$

(4)

where $\kappa$ is von Karman's constant.

and

$$
U = \frac{U_*}{k} \left( \frac{1 - y}{h} \right)^{1/2} + B \ln \left[ \frac{h}{B (1 - y/h)^{1/2}} \right] + \text{const}
$$

(7)

The second constant of integration may be eliminated by defining a velocity at a given value of depth, most conveniently the maximum velocity at $y = h$. When $B = 1$, the result is given by Von Karman in which the velocity gradient is infinite at the end. This is not a serious objection as eqn (7) cannot be applied in regions very close to the bed because of the neglect of viscous terms in eqn (3). Consequently eqn (7) is valid only in the fully turbulent region where Reynolds stress predominates.

The continuity equation for suspended sediment may be written as

$$
\varepsilon_s \frac{\partial C}{\partial y} + \kappa \frac{\partial C}{\partial y} (\varepsilon_s - \varepsilon_w) + (1 - \varepsilon)CW = 0
$$

(8)

where $\varepsilon_s$ and $\varepsilon_w$ are diffusion coefficients for sediment and water respectively, $C$ is the volumetric concentration and $W$ is the representative fall velocity for the sediment. Assuming $\varepsilon_s = \varepsilon_w$.

$$
\varepsilon_s \frac{\partial C}{\partial y} + (1 - \varepsilon)W = 0
$$

(9)

the solution to eqn (9) is

$$
\ln \frac{C}{1-C} = -W \int \frac{dy}{\varepsilon_s} + \text{const}
$$

(10)

by analogy with Boussinesq.

thus

$$
gS (h-y) = \kappa \frac{(dU)^2}{dy} \frac{d^2U}{dy^2}
$$

(5)

and integrating

$$
\frac{dU}{dy} = \frac{\mu_* \tau}{2khB - (1 - y/h)^{1/2}}
$$

(6)

where $\mu_* \varepsilon_s (ghS)$, the shear velocity, and $B$ is a constant of integration.

Integrating again, the velocity distribution is given by

$$
U = \frac{U_*}{k} \left( \frac{1 - y}{h} \right)^{1/2} + B \ln \left[ \frac{h}{B (1 - y/h)^{1/2}} \right] + \text{const}
$$

(7)

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$$

(10)

by analogy with Boussinesq.

$$
\varepsilon_s = \frac{\tau_0 (1 - \frac{y}{h})}{\rho \frac{dU}{dy}}
$$

(11)

which from eqn (6) leads to

$$
\varepsilon_s = 2khU_* \left( \frac{1 - y}{h} \right) \left( B - (1 - \frac{y}{h})^{1/2} \right)
$$

(12)

substituting into eqn (10) and integrating

$$
(\frac{C}{1-C} \kappa \frac{1-C}{C}) = \left( \frac{1 - \frac{y}{h}}{1 - \frac{\varepsilon_s}{\kappa}} \right)^{1/2} \frac{B - (1 - \frac{y}{h})^{1/2}}{B - (1 - \frac{y}{h})^{1/2}}
$$

(13)

where $C_s$ is the concentration at a reference level $y = \varepsilon$ and $q = \frac{W}{kB_U}$.

The total rate of suspended sediment transport in the region $\varepsilon \leq y \leq h$ is

$$
T_s = \rho S \int \frac{cU}{dy} dy
$$

(14)

Where $S$ is the density of solids.

By assigning a value to the velocity $U_e$ at $y = \varepsilon$, eqns (7) and (13) are combined to give

$$
T_s = \rho S \int \frac{cU}{dy} dy
$$

(14)
3. BED LOAD

The attractive force theory is used in which it is assumed that when the shear force acting on the bed exceeds a critical value there is movement of sediment. At flow stages just above the threshold motion, the type of movement may be rolling or hopping, which is known generally as saltation. This is due to the combination of lift and drag forces in the region of high velocity gradient near the bed. At higher stages of flow the velocity gradient is substantially reduced because of the presence of sediment particles, thus reducing the lift forces, and a collision layer of particles is formed contributing the bed load layer. In the steady state, the shear force in excess of the critical shear force must be dispersed within the layer otherwise successive layers of sediment would be removed from the bed resulting in infinite scour. This type of argument was originally proposed by Bagnold who suggested that the excess shear stress is dispersed by resisting forces in the fully developed bed load layer. He proposed that the presence of sediment in high concentrations has the effect of increasing the apparent viscosity of the equivalent fluid/sediment mixture. The shear force dispersed is expressed as [ ]

\[
\tau_d = \mu_s \frac{dU}{dy} \tag{16}
\]

where \( \mu_s \) is the effective viscosity and \( \frac{dU}{dy} \) represents the viscosity: defining the inverse of the slope \( \rho \) as the normal pressure due to the submerged weight of the particle eqn (16) becomes

\[
\frac{\tau_d}{\rho} = \tan \alpha \tag{17}
\]

where \( \alpha \) is the friction angle approximately equal to the static angle of repose.

It is necessary to make some assumptions concerning the distribution of concentration within the bed load layer. Some authors [2,6,7] have assumed the concentration to be sensibly constant. However, [8] has observed that there is an attenuation of concentration with height above the bed, which is a more plausible assumption if the lower layers within the bed load are to support the particles in the upper layers.

Across an element of thickness \( dy \) the change in dispersed shear is

\[
d\tau_d = (\rho_s - \rho) g \tan \alpha \, dy \tag{18}
\]

Assuming that the layer is supported entirely by grain interaction and that fluid pressure is unchanged, and also assumes that the local shear is proportional to the local concentration then

\[
\frac{d\tau_d}{c} = kdc \tag{19}
\]

where \( k \) is an unknown parameter. From eqns (18) and (19)

\[
\frac{k}{c} \frac{dc}{dy} = (\rho_s - \rho) \tan \alpha = \text{const} \tag{20}
\]

Integrating

\[
\ln \left( \frac{c}{C_m} \right) = (\rho_s - \rho) g \tan \alpha \tag{21}
\]

Now assuming that \( k \) is independent of \( y \), when \( y = 0 \) the concentration should be a maximum \( C_m \), so that

\[
\ln \left( \frac{c}{C_m} \right) = (\rho_s - \rho) g K y \tan \alpha \tag{22}
\]

If \( C_s \) is the concentration at the top of the load (reference level) at \( y = e \), then

\[
C = C_m \left( \frac{c}{C_s} \right)^{y/e} \tag{23}
\]

This analysis is considerably oversimplified, but suggests that the distribution should take a form of the exponential form. Writing \( C = \phi (y) = \phi (Y') \) where \( Y' = y/e \) and \( \tau_d = \tau_e \tau_s \)

where \( \tau_s \) is the critical shear corresponding to the threshold of movement, the bed load thickness, assumed to be the transition level, therefore can be expressed as

\[
e = \frac{\tau_e - \tau_s}{(\rho_s - \rho) g \tan \alpha \int \phi (Y') \, dY'} \tag{24}
\]

If the channel has a bed slope \( \theta \) eqn (24) becomes

\[
e = \frac{\tau_e - \tau_s}{g(\rho_s - \rho) (\tan \alpha - \tan \theta) \int \phi (Y') \, dY'} \tag{25}
\]

Now from eqn (18)

\[
-d\tau_d/\,dy = \phi(y) g (\rho_s - \rho) \tan \alpha \tag{26}
\]

which combined with eqn (16) gives

\[
\phi(y) (\rho_s - \rho) \tan \alpha = \frac{d(\mu_s / \rho)}{dy} \tag{27}
\]

As is assumed that \( \mu_s \) is dependent on the local concentration, it must also be a function of \( y \) for a given \( \phi \) (y), then eqn (27) may be solved by numerical method applying the boundary conditions, at \( y = 0 \), \( U = 0 \) and at \( y = e \)

\[
\frac{du}{dy} = 0 \text{ (because } \tau_d = 0) \tag{28}
\]

The velocity distribution in the bed load can thus be evaluated and the total rate of sediment transport in the bed region \( 0 \leq y \leq a \) is given by
\[ T_b = \int \phi(y) \, U \, dy \]  
(29)

One of the many results that can be used in the proposed model comes from (1), in which a particle mobility number \( F_{sp} \) was defined as

\[
F_{sp} = \frac{u_n}{\sqrt{gD(s-1)}} = \frac{u_n^2}{\sqrt{gD(s-1)}} \left( \frac{g}{(32)\log_{10}(ah/D)} \right) \frac{D_{sp}}{D} \]  
(30)

where \( U \) is the mean velocity, \( s \) is the specific gravity of the sediment, \( D \) is the particle diameter and \( n \) is an empirical co-efficient of about 10, and where the best fit for \( n \) was found to be

\[ n = 1 - 0.56 \log_{10} D_{sp} \]  
(31)

in the range \( 60 \geq D_{sp} \geq 1 \). The dimensionless grain size is then defined as

\[ D_{sp} = D \left[ \frac{g(s-1)}{v} \right]^{1/3} \]  
(32)

where \( v \) is the kinematic viscosity.

The exponent \( n \) in eqn (30) reflects the proportion of applied shear that is available to transport the sediment load and is analogous to a slope separation procedure. Eqn (30) is basically derived from the von Karman-Prandtl [9] logarithmic velocity distribution which assumes shear stress to be constant throughout the depth and that the velocity tends to zero at some finite distance above the bed determined by a linear measure of grain roughness. It also assumes that the von Karman coefficient retains its clear fluid constant value of approximately 0.4 for all stages of flow and sediment concentration.

It was found that in the case of the proposed models a better fit to the data was given when the depth ‘h’ in eqn (30) was replaced by the hydraulic radius. It is also possible that an alternative value for ‘n’ might be considered. However, this would require a revaluation of the relationship for ‘n’ using eqns (30) – (32). With the aforementioned modifications, an estimate of the grain shear velocity and thus the shear force can be made and this corrected value used in the relevant equations by replacing \( u^* \) by \( u^{*n} \).

4. Von Karman’s constant

The constant introduced in eqn (4) was originally proposed by von Karman (1930). However, it has been found that in sediment-laden flow the value tends to decrease depending on the velocity and sediment distributions adopted. It has been suggested that the presence of sediment tends to dampen turbulence and reduce momentum transfer or alternatively that apparent changes in \( k \) are due to variations in bed roughness. However, variations in the value of \( k \) alone do not account for the velocity distribution in the region close to the bed. The proposed model can deal satisfactorily with this aspect by deriving the velocity distribution in the bed load from conditions at the bed where the turbulence originates.

5. RESEARCH METHODOLOGY

The research procedure consisted of two main approaches: Field Data Collection and model Development. Field investigations were conducted on the two rivers, Ogbese and Owena to obtain the relevant data, water discharges and sediment loads.

5.1 Field Investigation Approach

Field measurements of sediments transport consisted of the following procedures (a) selection of the sampling stations (b) survey of the cross-section profile of the river (c) suspended load sampling and (d) Bed load sampling. The choice of a site for field stations were made such that the results obtained were representative of conditions in the stream as well as providing the necessary information. The following sampling stations were identified for this research work on Ogbese and Owena rivers respectively. These are shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1: Sampling stations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ogbese River</strong></td>
</tr>
<tr>
<td>Stations</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td><strong>Owena River</strong></td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>E</td>
</tr>
<tr>
<td>F</td>
</tr>
</tbody>
</table>

However, two stations were selected, one on each river, at reaches where each river discharge is perennial, and, facility for river sampling are available. These were Ogbese Bridge crossing for Ogbese River on the Akure – Owo highway and Owena Bridge crossing for Owena River on the Akure – Ondo highway. These sampling had to be done at stations where there are man – made
structures, such as bridges within the river for the purpose of convenience.

Already existing at each of the stations was a water level recorder for measurement of river level, which was converted to discharge through calibration with current meter. Stream velocities were obtained at these stations by relating the cross-sectional profiles obtained by survey on the river to the corresponding discharges. Depths were determined by sounding using each sounder and poles to dip into the stream vertical down to the riverbed. Sediment discharge data were obtained using the slough bottle DHH – 59 samples for suspended load and Bed Load Transport meter “Artemis” (BTMA) instrument for bed load.

BTMA is a modern equipment designed and develop by the Netherlands Rijkswaterstaat. The instrument consists of a frame with a steering fin, which keeps the apparatus in the right direction relative to the current. In that frame, a catch – basket is fixed with an entrance opening of 8½ x 5m. The BTMA was lowered into the river bottom and it remained there for few minutes (mostly 2 minutes). The amount of sediment caught was measured after emptying into a glass cylinder container. This procedure was repeated five times in order to arrive at an average figure for the streambed load discharged.

The volumes of both the suspended load samples and bed load samples were recorded in the standard recommended form. Ten beds – load observations were taken each time by lowering BTMA instrument into river bottom and leaving it there for two minutes.

Using the appropriate conversion table the amount of bed load transported in m³/day/km was computed from the amount of sediment caught by the BTMA instrument. The mean of these ten-computed bed loads was taken as the average quantity of sand – transport per unit width over a height up to 5cm above the bottom. An addition of 15% to this amount, which is considered as a salutation load was assumed to be transported over the height from 5cm to 10cm which still belonged to the bed loads but which was not directly measured by BTMA instrument.

5.2 Computation of Sediment Loads

a. Computation of Bed Load

The catches of the samples were averaged and the volume of the average catch or the complete catch converted to daily transport m³ / day / km / by means of the conversion formula for the International Organization for Standardization [10]

\[ S = \frac{2GB}{0.0857} \]  

(32)

where S is the transport in kg / day through cross section, G is average dry weight of all samples taken in the cross section, B is the width of the river (m) and T is the Sampling time of one measurement.

b. Computation of Suspended Sediment Load

According to many researchers such as [11], suspended load is carried by water across a stream section above the bed layer. It consists of particles that stay suspended from an appreciable length of time and usually consists of sand and finer fractions.

From the collected sediment transport sample, the concentration C₃ (mg/l) was determined in the laboratory, from which the daily sediment yield (Qₛ, ton / m² / day) was obtained, using the equation:

\[ Qₛ = QC₃ K \]  

(33)

where Qₛ is the Sediment discharge, Q is the water discharge, C₃ is the concentration of suspended sediment and K is a constant with value of 86.4

The value of K incorporated has already the specific gravity of 2.65 typically used for soil volume conversion.

Dry weight of Sediment × 1000 = \[ \text{in parts per million} \]

Weight of Water + Sediment mixture

Sediment concentration in parts per million C₃ was converted to milligram per litre by a factor Cₛ based on the ISO 4363 [10], guidelines for measurement of suspended sediments in rivers. The following relation was used

Cₛ (milligram per litre) = C₃ Cₛ (Part per million)

C was given as 0.0864 by report No 14 US Federal International – Agency on sedimentation projects (1963)

c. Analysis of Data

The data were either subjected to one – way analysis of variance (ANOVA), or F – statistics test, in accordance with the procedure of [12] with the coefficient of variation of 1 to 5%, and to correlation and multiple regression analysis [13]. Means were separated by multiple range test [14]. The results obtained were tested with some established river models such as [1,7,15,16,17]. These models were chosen because they are deterministic in nature, and can be applied to rivers in both temperate and tropical regions of the world.

In this study, four different statistical methods of comparisons were used to compare the results obtained with these established relations. The procedure is as follows

(1) A discrepancy ratio

\[ R = \frac{X_c}{X_m} \]
In which $X_m$ and $X_m$ = computed and measured bed material concentration in Kilogram/cubic meter, respectively.

$\delta = \sqrt{\frac{\sum (X_e - X_m)^2}{N - 1}}$

In which $N =$ data set number, and $N$ runs from 1, 2, .........

(3) The mean normalized error (MNE)

$$MNE = 100 \frac{\sum (X_e - X_m)}{N}$$

(4) Mean Prediction Factor (MPF)

$$MPF = \frac{1}{N} \sum \text{large of } \left( \frac{X_m}{X_m} - \frac{X_e}{X_e} \right)$$

d. Effective viscosity

The concept of effective viscosity has been introduced in eqn (16). The setting velocity of a single particle differs from the general settling motion of dispersed particles at high concentrations. A few closely spaced particles may settle faster than the same individual particle, whereas the converse may be true when the particles are dispersed throughout the fluid.

Elata and Ipplen [18] found that a uniform suspension of neutrally buoyant spheres behaved as a Newtonian fluid but appeared to exhibit an effectively higher viscosity dependent on and increasing with volumetric concentration. Some investigators have interpreted changes in fall velocity as an effective change in viscosity. The viscosity of dilute suspensions was given by [2] as

$$\mu_e/\mu = 1 + k_1$$

where $k_1 = 2.5$ for low concentrations < 2% [19].

McNown [20] proposed a relationship using linear concentration $\lambda$ such that

$$\mu_e/\mu = 1 + k_2 \lambda$$

where

$$\lambda = \frac{1}{(c/c_*)^{1/3} - 1}$$

where $c_*$ is the maximum value of $c$ when particles are in contact, 0.74 for uniform spheres at maximum packing and approximately 0.65 for natural sands) and $k_2 = 0.7$.

The results have been obtained by measuring relative fall velocities in conditions, which differ from those that prevail within the bed load layer. A better assessment of the behaviour of a layered or anisotropic fluid/sediment mixture is required, rather than the relative motion between the fluid and the sediment in idealised uniform dispersions. An important series of experiments that most closely represent the desired conditions was carried out by Bugnold (1954) who found the relationships for grain shear dispersion dependent on the stage of flow as

$$t_f (viscous) = 2.2 \lambda^{1.5} \mu Ud/y$$

$$t_f (inertial) = 0.017 \rho (\lambda D)^2 (dU/dy)^2$$

The results were found to lead to two dimensionless numbers, which can be used to define the flow. It was assumed in the present model that within the bed load the viscous or non-inertial state prevails. Combining the fluid and particle components from eqn (33) and (36) and assuming the velocity gradient of the fluid and sediment to be sensibly equal then

$$\mu_e/\mu = 1 + k_1 \epsilon + 2.2 \lambda^{1.5}$$

Eqs (33), (34) and (38) can be combined mathematically or graphically into a single relationship.

6 DISCUSSION OF RESULTS

The representative particle size was taken to be $D_m$ and the corresponding fall velocity used. The critical or threshold stress can be practically determined only by visual observation. Despite many differing experimental results the most generally accepted relationship is that of Shields (1936) and this was adopted for the analysis. However, this may not be analogous to the quasithreshold conditions in the presence of a bed load and bed forms. Taking a guide from eqn (23) some tests were carried out to determine the relative merits of differing forms of the concentration distribution in the bed load. Using as a basis

$$\phi (y) = C_{m0}(c/c_m)^{\alpha + \beta}$$

A suitable value for $\beta$ was found to be 3 and $C_{m0}$ was taken as 0.52, this being the maximum concentration consistent with grain movement. Using eqns (26), (27) and (39) and the effective viscosity relationship, the velocity distribution in the bed load can be determined. These equations are solved numerically and a typical volumetric concentration and viscosity ratio $\mu_e/\mu$ is shown in Fig. 1.

As insufficient data were available, it was assumed that the value of $B$ is unity. Given the mean or maximum velocity of flow the value of $k$ can be calculated from eqns (7) and (25). Then using eqns (15), (29), (30) and (39) and assuming continuity of velocity and particle concentration at the transition level, the only unknown parameter is
Fig. 1: Effective viscosity relationships for Ogbese River.

Fig. 2: Reference concentration derived from field data for Ogbese and Osun Rivers.
the reference concentration \( C_v \). Therefore using experimental data it was possible to evaluate the reference concentration at the transition level.

Two sources of data were used to evaluate \( C_v \). Those were those obtained at Ogbese River with mean diameter \( D_{50} \) range 0.19 – 0.98 mm and Owena River with mean diameter \( D_{50} \) range 0.30 – 1.2 mm.

The basic quantities used for the analysis were slope, depth, width, discharge, median particle diameter, temperature and total sediment transport rate \( T = T_b + T_s \). The optimized values of reference concentration are shown in Fig. 2 plotted against \( D_{50} \) (\( T_s \cdot T_{th} \)) where \( T_s \) is the dimensionless grain shear and suffix \( c \) refers to threshold condition. Two cubic curves were fitted on Hunt and Fleming [5] experimental data and also on Ogbese river field data.

Various sediment transport models were tested against more than 1000 sets of data [11]. Those with the highest percentage of predicted results have discrepancy ratios \( \frac{T_{calculated}}{T_{observed}} \) in the range 0.5 – 2 and compared well with those due to [1,15]. The former model was restricted to Froude numbers of less than 0.8 by the originators, but the latter has no restrictions imposed. The proposed models were tested together with these two models first against the data used to optimize the relationship for \( C_v \). The results are shown in Table 2 for different ranges of application. The overall performance of the proposed models lie between the two models tested. The model of [1] gave the better results.

Table 2: Comparison of test models

<table>
<thead>
<tr>
<th>Theory</th>
<th>Range of applicability</th>
<th>Percentage of data ( 0.5 &lt; T_{th} &lt; 2 )</th>
<th>Mean discrepancy ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ackers and White [1]</td>
<td>No restriction</td>
<td>80</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>( Fr \leq 0.8 )</td>
<td>75</td>
<td>1.31</td>
</tr>
<tr>
<td></td>
<td>No ripple</td>
<td>88</td>
<td>1.23</td>
</tr>
<tr>
<td>Engelund and Hansen [15]</td>
<td>No restriction</td>
<td>66</td>
<td>4.80</td>
</tr>
<tr>
<td></td>
<td>( Fr \leq 0.8 )</td>
<td>61</td>
<td>6.55</td>
</tr>
<tr>
<td></td>
<td>No ripple</td>
<td>78</td>
<td>2.50</td>
</tr>
<tr>
<td>Hunt and Fleming [5]</td>
<td>No restriction</td>
<td>72</td>
<td>1.51</td>
</tr>
<tr>
<td></td>
<td>( Fr \leq 0.8 )</td>
<td>63</td>
<td>1.69</td>
</tr>
<tr>
<td></td>
<td>No ripple</td>
<td>81</td>
<td>1.20</td>
</tr>
<tr>
<td>Proposed model</td>
<td>No restriction</td>
<td>74</td>
<td>1.52</td>
</tr>
<tr>
<td></td>
<td>( Fr \leq 0.8 )</td>
<td>66</td>
<td>1.70</td>
</tr>
<tr>
<td></td>
<td>No ripple</td>
<td>82</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Owena River 0.30 – 1.2 mm
Ogbose River 0.19 – 0.98 mm

The models were also tested by replacing the relationship for particle shear velocity (eqns (30) – (32) by a relationship proposed by [22]. The results were of the same order and gave marginally poorer predictive results.

From Fig. 2 it can be seen that it may have been more appropriate to consider a family of curves related to \( C_v \). It was found that by fitting a curve to an individual particle sizes a significantly improved result is given. For example a fit to the 1.35 mm sand would give 76% of discrepancy ratios in the more restricted range 0.75 – 1.5 with a mean discrepancy ratio of 1.002.

7 CONCLUSION

A mathematical model for sediment transport in unidirectional flow has been developed. Conditions of continuity of both velocity and concentration are assumed between the bed load and suspended load regions. The concept of an effective viscosity and a non-uniform concentration distribution in the bed load appears to be satisfactory. However, further research work is in progress.

REFERENCES


Notations:
- $\alpha$: empirical coefficient
- $B$: constant of integration
- $C$: volumetric concentration
- $C_a$: volumetric reference concentration at $y = a$
- $D$: grain diameter
- $D_{gr}$: dimensionless grain size thickness of bed load
- $F_{gr}$: dimensionless mobility number
- $g$: mean depth of flow
- $D$: mean depth of flow $k_1$, $k_2$, $k$ empirical coefficients
- $l$: mixing length
- $n$: empirical variable
- $p$: normal pressure
- $S$: energy slope
- $U$: mean flow velocity
- $U$: flow velocity at $y = a$
- $\mu$: shear velocity
- $\mu$: effective shear velocity
- $V$: flow velocity
- $\alpha$: angle of friction
- $\beta$: empirical coefficient
- $\sum F$: diffusion coefficient for sediment
- $\sum F$: diffusion coefficient for fluid
- $\theta$: angle of inclination of channel
- $k$: von karman's constant
- $\lambda$: linear concentration
- $\mu_c$: effective viscosity of fluid/sediment mixture
- $\rho$: density of fluid
- $\rho_s$: density of sediment
- $\tau$: shear stress
- $\tau_c$: critical shear stress
- $\tau_y$: shear stress at $y = 0$
- $\tau_d$: dispersed shear stress
- $\tau_{gr}^{*}$: dimensionless shear stress
- $\omega$: fall velocity of particle