RELATIONSHIP BETWEEN THE EXPOSURE OUTPUTS OF SINGLE-PHASE AND THREE-PHASE DIAGNOSTIC MACHINES: IMPLICATIONS FOR PATIENT EXPOSURE DETERMINATION IN DEVELOPING COUNTRIES

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Abstract

The indirect determination of the patient dose sometimes becomes necessary. This requires the measurement of the radiation output in free air for the determination of the entrance surface dose (ESD) used in the determination of the effective and population doses. Methods developed for these include direct and indirect measurements of exposure and calculation employing the empirical, semi-empirical and Monte Carlo methods. The latter is computer intensive and cumbersome and, in many cases are not quite representative of the true situation in the diagnostic centers. Measurement methods require large input in instrumentation than are available in many developing nations. Finally, most of the existing methods are based on three-phase x-ray machines with the implication that they are not directly applicable to the situations affecting thousands of diagnostic centers in many countries. This report discusses results of the models developed for estimating output from single-phase and three-phase machines with moderately high accuracies and discusses how to use them. The advantage is that with only electronic calculators, the models can be used to estimate the output with greater ease.

Keywords: Exposure output, entrance surface dose, voltage ripple, effective peak voltage, target angle

Introduction

Knowledge of radiation exposure (X), expressed in units of milliGray (abbreviated mGy), of patients undergoing diagnostic examinations is needed for the administration of radiation health. Such exposure may be retrospective occurring after the patient has been exposed or it may be necessary before irradiation in order to provide a pre-irradiation knowledge of the dose to expect from a particular diagnostic procedure. This is particularly important in cases where the examination is complex requiring many cycles of irradiation as in the case of the scoliosis examination where the patient who is usually relatively young and requires about 22 radiological examinations over a period of three years (Chamberlain et al, 2000). The effective dose expected to be delivered to the patient from selected radiological parameters may require the measurement of exposure or exposure rate as a function of the tube voltage V in units of kilovolt peak (abbreviated kVp), filtration (F) in mm Al and waveform together with tables of percent depth doses or tissue-air ratios which may be difficult. Alternatively, the entrance surface dose (ESD) could be estimated from an analytical formula that relates output, V, backscatter factors and the focus-to-detector distance (fdd) that is next used for calculating the organ and the effective dose. The use of analytical formulae (Chaney and Batchelor, 1981) minimizes additional work by the radiology department, facilitate extended dose studies to a much larger number of examinations that would be either less cost effective with thermoluminescence dosimetry (TLD) measurements or incapable of
providing information for the full appraisal of the nature of all the output. It also enables the estimate of doses lower than the measurement sensitivity of the TLD and eliminates errors associated with the interpolation of data from tables and graphs.

But most published empirical methods, tables, graphs and x-ray spectra codes for estimation of the output were derived from measurements made from three-phase, full wave machines with constant potentials or very low voltage ripple due either to the fact that single-phase machines are no longer used in clinical practice in most developed countries where the empirical methods originated or the electrical characteristics of the single-phase x-ray machines with unrectified power are less stable producing greater voltage ripples and generally lower exposure than the three-phase machines. However single-phase machines are still in use for radiography in many countries around the world with several unascertained hundreds in Nigeria and many other developing countries. Good radiation protection programs are required for these x-ray machines irrespective of phase and operational difficulties encountered by their users.

Few empirical analytical formulae with corrections for the target angle, graphs and tables for estimating X from V, F, heel effect, wave form and target angle have been reported (Chzney and Batchelor 1981, George et al. 2004, McCullough and Cameron 1970, Boone and Siebert 1997). Ay et al (2004a) provided a comprehensive summary on computational x-ray spectra codes based on x-ray beams from tubes with tungsten targets. These used empirical, semi-empirical models and Monte Carlo calculation (Cranley et al 1997, Novotny: http://www.bmtpl.akh- lern.ac.at/people/nworo1/nworo1.html). But these codes, aside of being based on 3-phase constant potential machines or machines with low voltage ripple, also require varying levels of computing facilities than are affordable in many diagnostic centres in developing countries. Many of the facilities also lack the capability for radiation measurement due to the unavailability of appropriate instrumentations and staff training. So, a code for calculating the output from single and three-phase machines with less dependence on computer and expertise in radiological physics is desirable. A suitable empirical method that can be modified to do our work is the one that expressed the output as power function of V (Institute of Physical Sciences, 1994). Robson (2001) used this to calculate x-ray output generated by mammography machines.

The objective of this work is to report the development of an empirical analytical formula that accurately calculates the radiation output for both single and three-phase x-ray machines within given limits. The work involved the establishment of an empirical code for calculating the output from measured X, exposure times (t), V and F from a single-phase x-ray machine, with tungsten target and target angle of 16°. The broad-based data of Birch et al (1979) on exposures from three-phase x-ray generators operated on constant potential power and machines with low voltage ripples were used to derive a filtration and voltage dependent factor to convert single-phase output values to three-phase equivalents. The results are in good agreement with the data by Chaney and Batchelor (1981).

**Measurements**
The x-ray machine, a GEC Medical Apollo, Roentgen 501 with an x-ray tube made by Machlett at the out-patient department of UCH, Ibadan was used for the measurements. It is a single-phase machine fitted with tungsten anode and 16° anode target angle. The inherent filtration (indicated on tube housing) of the x-ray tube is 0.7 mm Al and the machine is being operated without additional filtration. UCH is one of the busiest hospitals in the country in terms of volume of patients being treated.

An initial quality control (QC) check was performed on the machine using a Vicoreen's (1995) x-ray test device (4000 M+). The device which was also used for the measurements of x-ray output, V, and exposure time (t seconds(s)) is fitted with a 36-cm³ flat-plate ionization chamber and digital readout meter that has been factory-calibrated at 3.0 mm Al was used to measure x-ray exposure to an accuracy of ±10% at various set values of V, t, added filtrations and a set current of 100 mA at a 0.10
cm. V and t were set to the values commonly used in the radiology department. In each measurement, the values of V, t and the phase of the power indicated by the device were recorded. The aluminium filters added in order to increase the total filtration for the present measurements are of thicknesses 0, 1, 2 and 3 mm and are of high quality (99% in purity) obtained from the Secondary Standard Dosimetry Laboratory (SSDL) established through the IAEA at the Department of Physics in the University of Ibadan. The detector system was a microprocessor instrument pre-programmed to determine the phase of two-peak (single-phase) and six-peak or more (three-phase) machines and also calculate the mean, effective and the maximum peak voltage of the waveform. According to Victoreen (1995), the effective peak voltage is physically similar to the quantity measured by the voltage test cassette method in which two stored detector waveforms are integrated independently and their ratio is used to calculate the effective V that has low voltage ripple. At all measurements, the difference between the effective V and either the mean or the maximum V is within ±1% so that the tube voltages are equal for all practical purposes. The presence of voltage ripples can make modeling of output of single-phase machines difficult in comparison with that of a three-phase machine but this effect can be considerably reduced by the use of the effective peak voltage (Victoreen 1995). Hence in this work, we made use of effective voltage as displayed by the Victoreen device placing the detector on the central axis of the beam so that its front-to-rear axis is oriented along the anode-to-cathode axis of the tube to eliminate the 'heel effect' on output measurement (Victoreen 1995).

The inaccuracy in these parameters are possibly due to a repair work performed in 2003 on the machine, aimed at improving image quality since an earlier quality control measurement on this same machine (Ogundare et al. 2004) showed that the measured and set values of these radiological parameters were accurate. There is no record of any quality assurance measurements on the machine after the repair. The fact that after-repair QC was not done, may suggest that the engineers have limited knowledge of radiation protection, which is the case in many developing countries. Hence there was no concern about patient dose, but only image quality.

Output model

1. Variation of output with tube current-exposure time product, beam size and total filtration (F)

The measured exposures (X) were plotted against the product of the tube current in milliAmperes abbreviated mA and t in seconds(s), equal to t (mAs), at varying beam sizes and F. Therefore the outputs, simply defined as the exposure per unit t, were determined as the gradients of the graphs with the unit of milliGray (mGy) per mAs. Figure 1(a) and (b) show the variation of the exposure with t and F at 40x30 and 40x35 (cm x cm) beam sizes. It can be observed that the exposure increased linearly with t and in Figure 2, that there was little variation between X and beam size at the beam sizes employed as had been observed by Victoreen (1995). These trends were true for all data at 0.7, 1.7, 2.7 and 3.7 mm Al and all beam sizes employed and it can be argued that the scatter due to increased volume of air that was irradiated and recorded by the detector was insignificant.

Results and Discussion

Initial quality control measurements

The results of an initial quality control measurements on the x-ray machine used in this work showed that the percentage difference between the measured and set V values is more than 100% in all cases, this is by far greater than the acceptable limit of ±10% in diagnostic radiology. The percentage differences between the measured and set t were mostly greater than 25% of the set values, hence there was also a bad accuracy for t and had to be corrected.
Figure 1(b)

The measured outputs are shown in Table 1 against measured V, with F of 0.7 mm Al (used in UCH for their examinations) and those measured at other total filtrations of 1.7, 2.7 and 3.7 mm Al. The table shows that the exposure output increased with V and decreased as F increased as expected. Therefore the exposure outputs were fitted with V as the independent variable for each of the four values of F (0.7, 1.7, 2.7 and 3.7 mm Al) using EXCEL 2000 on a Pentium IV PC. The fitted curve at each F satisfied a power function with real and positive coefficient and exponent as presented in equation (1) below in agreement with the Institute of Physical Sciences (1994) with the exclusion of the inverse square term that is satisfied by x-ray exposures along the central axis used in this work:

\[
Output = a(V)^b \left( \frac{100}{\text{fdd}} \right)^2
\]  

where a and b are fitting coefficient and exponent respectively, V and fdd are as defined above. The inverse square factor in the parenthesis on the right side of equation (1) is for correcting the output for distances other than 100 cm.

Table 1: Comparison between measured output and calculated output using Equation (1).

<table>
<thead>
<tr>
<th>Total Filtration (mm Al)</th>
<th>Tube Voltage (kVp)</th>
<th>Measured Output (mGy/mAs)</th>
<th>Calculated Output (mGy/mAs)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>115.7</td>
<td>1.18</td>
<td>1.19</td>
<td>-1.24</td>
</tr>
<tr>
<td></td>
<td>117.6</td>
<td>1.25</td>
<td>1.24</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>121.5</td>
<td>1.33</td>
<td>1.32</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>129.3</td>
<td>1.50</td>
<td>1.51</td>
<td>-0.50</td>
</tr>
<tr>
<td></td>
<td>136.9</td>
<td>1.70</td>
<td>1.70</td>
<td>0.06</td>
</tr>
<tr>
<td>0.7</td>
<td>114.5</td>
<td>1.02</td>
<td>1.01</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>119.9</td>
<td>1.99</td>
<td>1.10</td>
<td>-1.05</td>
</tr>
<tr>
<td></td>
<td>121.7</td>
<td>1.14</td>
<td>1.13</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>128.9</td>
<td>1.24</td>
<td>1.26</td>
<td>-1.62</td>
</tr>
<tr>
<td></td>
<td>138.8</td>
<td>1.46</td>
<td>1.45</td>
<td>0.97</td>
</tr>
</tbody>
</table>

III Variation of coefficient $a$ and exponent $b$ in equation (1) with filtration

The exponents ($b$) obtained from the fitted curves from the output values in column 3 of Table 1, were observed to depend on $F$, as illustrated in Figure 3. Again, using the EXCEL 2000 computer software and Pentium IV PC, the values of $b$ were fitted as a cubic function of $F$ with real coefficients as in equation (2):

$$b = -0.1656F^3 + 1.2373F^2 - 2.4478F + 3.2576; \quad R^2 = 1.0000 \quad (2)$$

where $F$ is the total filtration in mm Al and $R$ is the correlation coefficient of the relationship between $b$ and $F$. Values of $b$ with $F$ at 0.7 to 4 mm Al are presented in Table 2 column 3.

![Fig. 3: Variation of exponent $b$ (Eq. (1)) with total filtration](image)

![Fig. 4: Variation of coefficient $a$ (Eq. (1)) with total filtration.](image)
The values of $a$ obtained from the power fits of output against $V$ were $6 \times 10^7$, $2 \times 10^6$, $9 \times 10^5$, and $1 \times 10^5$ at $F$ of 0.7, 1.7, 2.7 and 3.7 mm Al respectively with coefficients of correlation, $R^2$, that ranged between 0.9304 and 0.9962. More accurate values of $a$ were determined by substituting the values of the output, $b$ and $V$ back in equation (1) using equation (3) below at each $F$ and taking the averages so that $a$ becomes independent of $V$ with negligible differences in the results stated above as presented in Table 2.

$$a = \exp(\ln(output) - b \ln(V))$$

(3)

Using equation (1), without the inverse square term, for the determination of output in a mammography unit operated with $V$ in the range of 25-32 kVp, Robson (2001) obtained an approximate value of $b$ as 3 for x-ray tubes with Molybdenum target and filter (Mo/Mo) and used equation (3) to determine coefficient $a$. With tungsten target tubes and similar voltage range, the Institute of Physical Sciences (1994), obtained typical values of $b$ that ranged from 2 to 3. These results and our own are in agreement and confirm that Equation (1) is also valid for filtrations commonly employed in diagnostic radiology using single-phase machines. Besides, the relationship between $b$ and $F$ established in the present work will result in more accurate calculations of the output than when generic values of $b$ are used as well as provide accurate output for beams filtered with amount of aluminum not included in the work of Robson (2001) and the Institute of Physical Sciences (1994).

<table>
<thead>
<tr>
<th>Total Filtration (mm Al)</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>5.72x10^7</td>
<td>2.0936</td>
</tr>
<tr>
<td>1.0</td>
<td>1.45x10^6</td>
<td>1.8815</td>
</tr>
<tr>
<td>1.5</td>
<td>1.90x10^6</td>
<td>1.8109</td>
</tr>
<tr>
<td>1.7</td>
<td>1.51x10^6</td>
<td>1.6855</td>
</tr>
<tr>
<td>2.0</td>
<td>7.80x10^7</td>
<td>1.9864</td>
</tr>
<tr>
<td>2.5</td>
<td>1.63x10^7</td>
<td>2.2837</td>
</tr>
<tr>
<td>2.7</td>
<td>9.35x10^7</td>
<td>2.4090</td>
</tr>
<tr>
<td>3.0</td>
<td>4.30x10^7</td>
<td>2.5787</td>
</tr>
<tr>
<td>3.5</td>
<td>1.91x10^7</td>
<td>2.7471</td>
</tr>
<tr>
<td>3.7</td>
<td>1.40x10^7</td>
<td>2.7512</td>
</tr>
<tr>
<td>4.0</td>
<td>2.21x10^6</td>
<td>2.6648</td>
</tr>
</tbody>
</table>

It was assumed that the distribution of $a$ with the F should be Gaussian so that by careful interpolation, its values at other filtrations can be mapped. This we did by dividing the ordinate and the abscissa of a graph of $a$ against $F$ into a large number of grids, manually drawing a curve to join the points and reading off the points at other filtration values. This resulted in Figure 4 from which a can be read. The work of Birch et al (1979) who used a constant potential x-ray machine was used to adjust the values of $a$ to obtain greater accuracies at some of the filtrations not employed in our measurements.

There are many diagnostic rooms using machines with a wide spectrum of filtrations and the present form should be useful to them. The outputs calculated using the model in equation (1) and the values of $a$ and $b$ chosen from Table 2 (or figure 3) are presented in Table 1 (column 4) and were compared to the measured values (column 3) at $F$ of 0.7 mm Al to 3.7 mm Al.

The mostly low percentage differences (Table 1 column 5) between measured and calculated outputs showed that there is good agreement between the two. Therefore if $\tau$ (mAs), fdd, V(kVp), and $F$(mm Al), of a single-phase tungsten anode x-ray machine with low voltage ripple are accurately known, the x-ray output at any point along the central axis can be accurately calculated at 16° target angle using equation (1).

But the work by Ay et al (2004 b) showed that the variation of the exposure output with the distance along the central axis is not significant for x-ray tubes with tungsten anodes having target angles in the range from 14° to 18°. This implies that Equation (1) is valid for x-ray beams with tungsten anode and anode target angle in this range.

Application to Three-Phase Machines

Chaney and Batchelor (1981) gave a conversion factor of 1.7⁶ that transforms the output from a three-phase, full wave machine operating in V range of 70 kVp – 120 kVp and F of 2.5 and 3.5 mm Al to the output of a single-phase machine that runs at peak potentials of similar quality. This factor produced single-phase output from the work of Birch et al (1979) and V of 150 kVp outside

the V and filtration range given above that were not in agreement with the results from equation (1). Therefore a more general conversion factor is required for the model in equation (1) to be applicable to three-phase x-ray machines. For this the output data of Birch et al (1979) tabulated for three-phase machines operating at constant voltage, tungsten anode with target angle of 17° and that have been widely used in developing x-ray codes (Ay et al 2004 a) were obtained at various operating voltages and F ranging from 1.5-4 mm Al. Corresponding output values were calculated for a single-phase machine using equation (1) corrected to 75 cm fdd used by Birch et al (1979). The ratios of corresponding outputs from the two sets of data were obtained and fitted as a function of the product of F and V again using the EXCEL 2000 software and Pentium IV PC. The equation that best fits these ratios is quadratic in the product of F and V as given below in equation (4).

\[ C_{iv} = 8F - 0.5(FV)^2 - 0.0801(FV) + 37.514 \]

\[ R^2 = 0.9997 \] (4)

where F is the total terminal filtration in mm Al and V is the tube voltage in kVp.

The expression in equation (4) is then multiplied into equation (1) to obtain equation (5) below. Hence for a three-phase machine the model becomes

\[ Output = C_{iv} at(V)^n \left( \frac{100}{fsd} \right)^2 \] (5)

The output data calculated using equation (5) and the diagnostic parameters employed by the authors are compared with those from the authors in Table 3 as function of the F and V. The generally low percent difference (Column 6) between the data of Birch et al (1979) and those calculated using equation (5) shows good agreement between the output values by the model and the data by the authors.

### Table 3: Comparison between the output data of Birch et al (1979), the data for single-phase and three-phase models (Eq. 1) and the data for three-phase model (Eq. 5) are at fdd of 75 cm

<table>
<thead>
<tr>
<th>Total Filtration (mm Al)</th>
<th>Tube Voltage (kVp)</th>
<th>Exposure Output</th>
<th>Present Models</th>
<th>%difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Birch et al 1979</td>
<td>1-phase (Eq. 1)</td>
<td>3-phase (Eq 5)</td>
</tr>
<tr>
<td>1.5</td>
<td>50</td>
<td>0.13</td>
<td>0.004</td>
<td>0.13</td>
</tr>
<tr>
<td>2.0</td>
<td>80</td>
<td>0.23</td>
<td>0.008</td>
<td>0.22</td>
</tr>
<tr>
<td>2.5</td>
<td>100</td>
<td>0.27</td>
<td>0.012</td>
<td>0.27</td>
</tr>
<tr>
<td>3.0</td>
<td>120</td>
<td>0.28</td>
<td>0.014</td>
<td>0.25</td>
</tr>
<tr>
<td>4.0</td>
<td>140</td>
<td>0.36</td>
<td>0.021</td>
<td>0.29</td>
</tr>
</tbody>
</table>

The output calculated using equation (5) at F of 2.5 mm Al and 3.5 mm Al with V in the range 70-150 kVp were compared with the results obtained by the method of Chaney and Batchelor (1981) and are plotted in figure 5 (a) and (b). It can be seen that there is good agreement among the data at both 2.5 mm Al and 3.5 mm Al from 70 to 120 kVp. Here the percentage difference ranged from a magnitude of 2 to 9 % but 15 % and 21% respectively at 150 kVp. The large discrepancy at 150 kVp (and some V in reference 9) is probably due to the fact that the method of the authors were recommended for 70 - 120 kVp and the two filtrations only. The conversion factor of 24.14 recommended by the authors is good only for V and F ranges that they considered in their work. The factor, as illustrated by equation (4), varies with total filtration and tube voltage. The model also tested valid for machines run at voltages having low ripples not greater than 10%.
filtration (addition of inherent and added filtration), machine phase (single or three phase) and ensure that the voltage ripple level is no more than 10%, beam size >760 cm².

Step 2. Suppose the following are true from step 1 above.
Tube voltage 80 kVp, inherent filtration is 0.6 mm Al, added (external) filtration is 1.9 mm Al, beam size 35 cm x 20 cm, fdd=75 cm; voltage ripple 10% (this condition is satisfied by three-phase machines on constant potential and single-phase machines with rectified voltage or peak voltage; if not sure then view the waveform on an MCA using a hyper pure germanium detector).

Step 3. Enter Table 2 at total filtration of 2.5 mm Al (1.9 mm Al +0.6 mm Al).
Get value of \( a = 0.0000183 \), \( b = 2.2837 \).
(a) For a single phase machine \( C_{31} \) (equation (4)) is unity \( C_{31} = 1 \)

Step 4. Substitute for \( V=80 \) kVp, \( a, b, fdd=75 \) and \( C_{31} \) in equation (5) using a calculator:
Output \( = 1 \times 0.0000183 \times 80^{2.2837} \times (100/75)^2 = 0.0072 \ mGy/mAs±6\% \).
(b) For a three-phase machine with similar exposure parameters, \( C_{31} \) is not unity.

Calculate \( C_{31} \) from equation (4) by substituting for \( V=80 \), \( F=2.5 \) mm Al:
\[ C_{31} = 8E-05 \times (2.5 \times 80)^2 - 0.0801 \times (2.5 \times 80) + 37.514 = 24.694 \]
The values of \( a, b, fdd \) and \( V \) are the same as for in this example.
Output \( = 24.694 \times 0.0000183 \times 80^{2.2837} \times (100/75)^2 = 0.178 \ mGy/mAs±2\% \).

*The error estimates are based on the maximum that can be reasonably committed as observed from the calculated data.
+To convert mR/mAs to mGy/mAs, the former was multiplied by 0.01 (~0.00873)
Conclusion
An analytic formula that can be used to estimate the output of x-ray machines with tungsten targets having single and three phase waveforms and low voltage ripples have been presented. The model is good for x-ray tubes with the target angles in the range 14°-18° and, generally, will produce moderately accurate output to better than ±5% for x-ray machines with voltage ripple no more than 10%. For use with machines having high voltage ripples, the effect of the high voltage ripple needs to be built into the model in order to obtain an accurate result. In contrast to the model of Chaney and Batchelor (1981), the present model will eliminate the error inherent in the use of a single factor for converting a three-phase output to a single-phase output and vice versa at different combinations of the tube voltage and total filtration for which the former model was not intended.

The present model is more applicable in radiation transport problems, than tabulated and graphical data and, also, x-ray spectra codes.

The fact that the present model allows the estimation of output from both single-phase and three-phase machines makes it more general than the others that are based on three-phase machines alone and are more important to many facilities located in countries still using single-phase x-ray machines for radiological purposes. The present model therefore becomes a good primary tool and complement to existing models in many centers.

The measurement of V, exposure time and output from the x-ray machine at the OPD of UCH revealed gross inaccuracies in V and exposure time indications as against what were measured in the earlier work by Ogundare et al (2004). This could have been caused by a repair that was carried out on the machine and the fact that there is no program for after-repair quality control measurements in place. The absence of after-repair quality control check is likely to be a common occurrence in many radiological facilities, especially in developing countries. We therefore recommend that national and international regulatory authorities pay attention to this in order to avoid unnecessary exposure to patients from such facilities.

References


Ogundare, F. O, C. Z. Uche, and F. A.