THE DEPENDENCE OF THE PERIOD ON ANGULAR AMPLITUDE OF A SIMPLE PENDULUM

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Abstract

The interesting properties of a pendulum are that the pendulum executes simple harmonic motion, and that the period of each swing is constant, and depends only on the pendulum length. While it is independent of the weight. The major aim of this paper is to ascertain the minimum angular amplitude at which the error in the period will be significant. this work was carried out by setting up a simple pendulum experiment. The pendulum was set swinging with a small. The timing of the oscillation was done as the bob passed through its rest position. The time for 50 oscillations was recorded for different lengths and angular amplitudes. It was observed that the period depends on length and angular amplitude of the pendulum. The variation of the period with the angular amplitude is not a linear relation, but a parabolic curve. At the minimum values of the curves obtained, the angular amplitude can be between 5° to 15° for any choice of length of pendulum

Keywords: *Pendulum, period, oscillation, dependence, angular amplitude.*

Introduction

A pendulum is a rigid body mounted on a fixed horizontal axis, which is free to swing to and fro under the influence of gravity. The interesting properties of a pendulum (if the angle of swing is small) are that the pendulum executes simple harmonic motion, and that the period of each swing is constant, independent of the weight and the displacement. But depends only on the pendulum length (Gron,1983).

The properties of the pendulum were first known to Ibn vunis (C.1200) but were introduced to European science by Galileo.

He was quick to realize the importance of the constant time of swing of the pendulum, and later on it occurred to him that a pendulum might be used to govern a clock more satisfactorily than the horizontal oscillating cross bar which were then in use. Before the death of Galileo in 1642, he left plans for the construction of a pendulum clock, but it was not until 1655, that the first successful pendulum clock was constructed by the Dutch scientist Christian Huygens.

There are several types of pendulum. The Katar's reversible pendulum, the Ballistic pendulum, the Tensional pendulum. The simple pendulum is the type of pendulum used for this investigation. It is the most common of all the pendulums that exist. Fig 1 shows the variation of T/T_0 with amplitude of a simple pendulum, where T_0 is the period related to a given length of pendulum related to a small amplitude.

It is only for very large amplitude that the period differ appreciably from T_0. For small amplitude, it is enough to take only the first corrective terms and even then substitute for with the result that

$$T = 2\pi\sqrt{\frac{L}{g}}(1 + \frac{1}{6}\Theta_1^2) \qquad (1)$$

where θ must be expressed in radians.

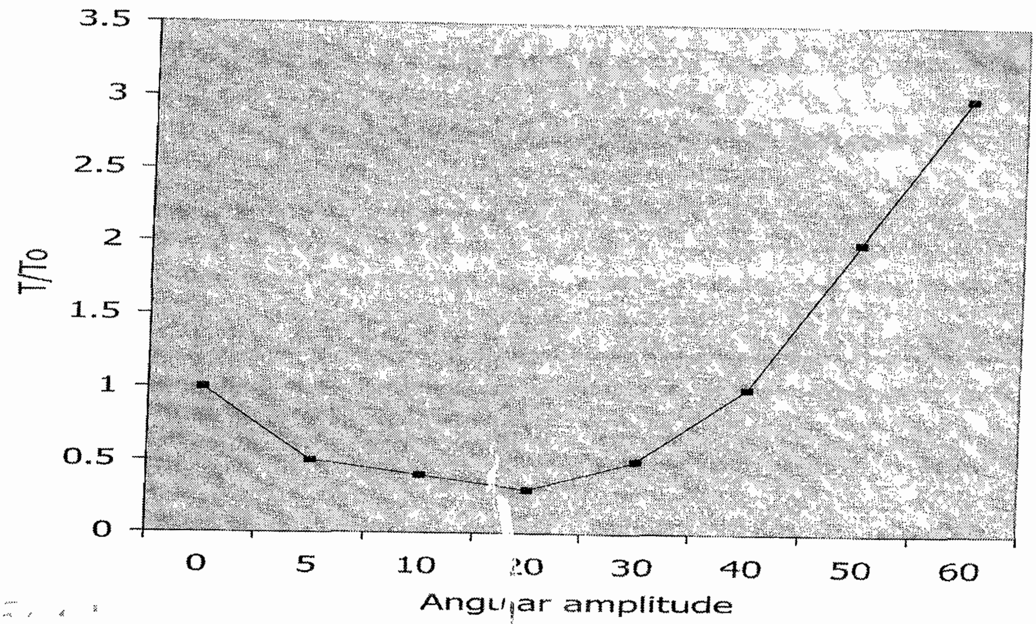

Fig. 1: Graph of T/T_o versus θ

Scope and Rational for the Study

For this investigation, one simple pendulum bob was used. The range of angular amplitude covered was 2.5° to 45°. At a measured angular amplitude, 50 complete oscillations were counted for different lengths of the pendulum ranging from 20cm to 100cm and for different angular displacements the periods (T) was calculated and recorded.

The major objective of this study is to ascertain the minimum angular amplitude at which the error in the period will be significant. A two percent level of significance was chosen. We noted that most authors insist that the angle of swing be less than 10° (Alanso and Finn, 1973 and Mcgraw-Hill, 1987).

Materials and Method

The materials used in performing this experiment are:

A spherical copper metal bob of diameter 0.85cm whose weight is one gramme with a hook attached to it. This served as the weight, which is to be displaced.

A retort stand, clamp, a split cork and a string of about two metres long. These were used in holding and suspending the pendulum bob.

A stopwatch, which was used in timing the number of oscillations, a metre rule and a large protractor, which were used in measuring the length of the pendulum and the angular displacement, respectively.

The simple pendulum was set up by tying one end of a string to the hook of the pendulum bob, while the other end of the string was suspended with the aid of the split cork to the stand. (Fig. 2 below).

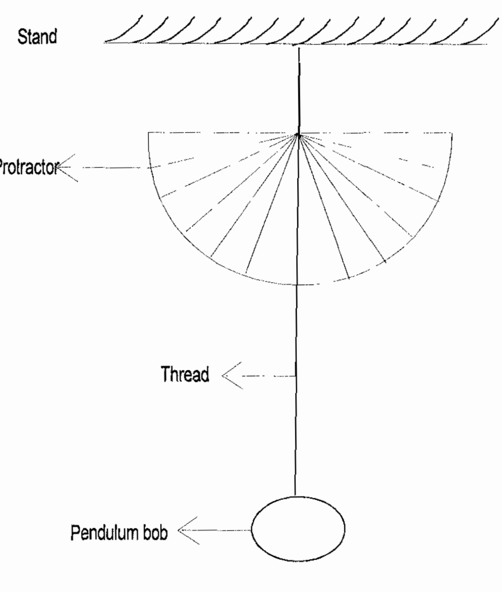

Fig. 2: The experimental set up of simple pendulum

The exact length of the pendulum was measured, which is the distance between the point of suspension and the middle of the bob. This initial length of pendulum, which was 20cm, was recorded. The equilibrium position of string, which was used as the reference point for timing the number of oscillations, was marked. The pendulum bob was displaced through an angle of 2.5°. With the aid of the protractor and the time for 50 oscillations was noted and the period (T) calculated. The timing for this value of L was repeated several times until a constant value was obtained. The experiment was repeated several times keeping L constant while varying the angular displacement, say 5°, 10°, 15°, 30° and 45°. The length L was increased to 30cm, 40cm, 50cm, 60cm, 80cm and 100cm, in each case, the corresponding values of T at the different angles were calculated and noted.

Result and discussion

The angular displacements for each length i.e. 20cm to 100cm, the periods (T) for each length at different angles and the percentage errors in T were calculated. Graphs of the percentage errors against the angular displacements è are shown in figures 3. From the graphs the angle èc for which the percentage error is less than or equal to two percent is comparable to the error in the approximation sin è = 0 in radians. This is greater than 36°.This is the maximum angle, which should be allowed in a simple pendulum experiment. The value of acceleration due to gravity g was deduced. The error in g was also calculated as

$$T = 2\pi g^{-1/2} L^{1/2} \qquad (2)$$

$$g^{1/2} = 2T^{-1} L^{1/2}$$

$$\frac{dg}{g} = -2\frac{dT}{T} + \frac{dL}{l} \qquad (3)$$

The reading error in the length, dL is 0.5mm = 0.05. While that of the period dT is 0.5 seconds. This is the error in reading 50 oscillations therefore; error in reading 1 oscillation is 0.01 seconds. These values with the corresponding values of L and T were substituted in equation (2) to obtain the fractional errors shown in table 1. The value of acceleration due to gravity corresponding to the least of the angular displacement, was taken at various lengths. The deviation of other values from $_m$ was determined, and called the error due to large amplitude. The angular displacement for each length ie 20cm to 100cm, the period (T) for each length at different angles and the percentage error in T were also calculated. The graphs of the percentage errors against the angular displacement are shown in figure 3.

From the curves the angle for which the percentage error is less than or equal to two percent is comparable to the error in the approximation Sin= in radians see Table 2. This is greater than or equal to 36°. This is the maximum angle, which should be allowed in a simple pendulum experiment, when two percent error is considered. Then considering the minimum values of the curves in figure 3, the minimum values lies between 5° and 15°, which should be the range of the angle to be allowed in a simple pendulum experiment.

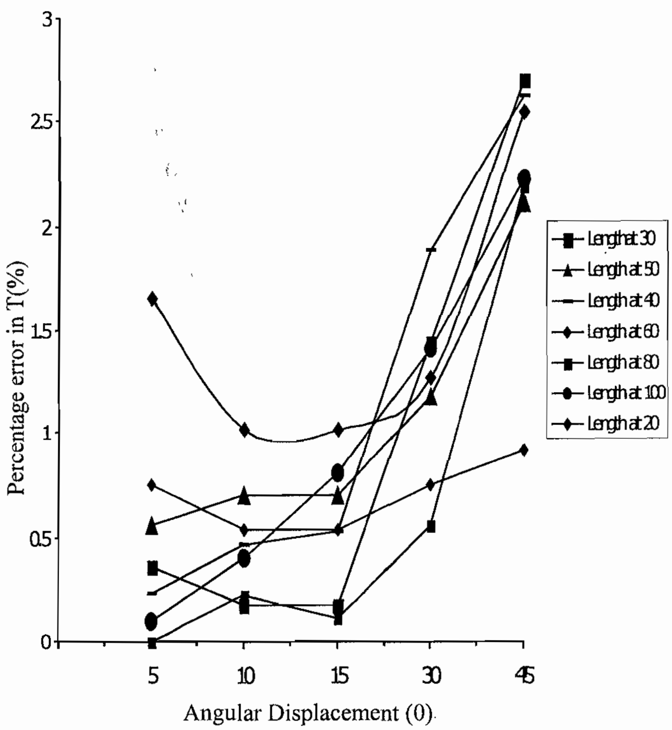

Fig. 3: Variation of percentage error in T with angular displacement

Table 1: Angular amplitude showing fractional errors in g

Length L(cm)	25°	5°	10°	15°	30°	45°
	g ± error(m/s²)					
20.00	9.33±0.02	9.41±0.02	9.44±0.02	9.33±0.02	9.08±0.02	8.71±0.02
30.00	9.61±0.02	9.68±0.02	9.44±0.02	9.57±0.02	9.34±0.02	9.04±0.02
40.00	9.79±0.01	9.75±0.01	9.69±0.01	9.66±0.01	9.47±0.01	9.12±0.01
50.00	9.79±0.01	9.90±0.01	9.79±0.01	9.79±0.01	9.57±0.01	9.39±0.01
60.00	9.60±0.01	9.93±0.01	9.83±0.01	9.83±0.01	9.58±0.01	9.49±0.01
80.00	9.96±0.01	9.97±0.01	10.01±0.01	9.44±0.01	9.85±0.01	9.61±0.01
100.00	10.06±0.01	10.04±0.01	9.98±0.01	9.90±0.01	9.86±0.01	9.63±0.01

Table 2: Approximation sin θ = θ

Θ=0°=0.0000 radians	SinΘ=0.0000	0.00
Θ=2 ° =0.0347 radians	SinΘ=0.0347	0.00
Θ=5 °=0.0875 radians	SinΘ=0.0872	0.11
Θ=10 °=0.1745 radians	SinΘ=0.1736	0.51
Θ=15 °=0.2618 radians	SinΘ=0.2588	1.14
Θ=30 °=0.5238 radians	SinΘ=0.5000	4.50
Θ=45 °=0.7854 radians	SinΘ=0.7071	9.96
Θ=60 °=1.0472 radians	SinΘ=0.8660	17.30

Conclusion

From the experimental results, it was observed that at different values of angular displacement, apart from the dependence of the period, on the length of the pendulum, the Angular amplitude also affects it. The variation of the period with The angular amplitude is a form of a parabolic curve. However, taking minimum value of the curves, the minimum angle which should be allowed in a simple pendulum experiment, is 5°-15°.

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