APPLICATION OF LAPLACE INTERPOLATION IN THE ANALYSIS OF GEOPOTENTIAL DATA

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Abstract

Geophysical data is often collected at irregular intervals along a profile or over a surface area. But most methods for the treatment of geophysical data often require that any data collected at irregular intervals have to be interpolated to obtain values at regular grid. Unlike the common 2-dimensional interpolation procedures, the Laplace (finite-difference) method can be applied to regions of high data gradients without distortions and smoothing. However, by itself, this method is not convenient for the interpolation of geophysical data, which often consists of regions of widely variable data densities. In this paper a procedure is developed which allows that by combining it with the method of quadratic weighting, the Laplace method can be successfully applied to interpolate two-dimensional geophysical data. These methods were applied to some geopotential data. The results show that there is no significant difference between aeromagnetic maps derived from data as observed and maps obtained when the data is interpolated in a region of thick sedimentary formation. This is attributed to the fact that the magnetic body in such region are deeply buried. However, interpolated aeromagnetic map over a region of outcropping granitic bodies exposes more shallow features which are otherwise not seen on the map derived from the observed data that are not interpolated. Similar observation was recorded for a gravity anomaly map produced from data collected in basement terrain. It was concluded that since it is impossible to observe every point in a given surface area in order to produce an accurate map that will reflect the distribution of the various shallow subsurface anomalies, it is better to interpolate the observed data prior to the production of the desired geopotential map.

Keywords: Geophysical, geopotential, Lapalce interpretation, finite difference, quadratic weighting.

Introduction

We sometimes know the value of a function at a set of points, but we do not have an analytic expression that tells us how to calculate its value at an arbitrary point. In geophysics, the function values result from the measurements of physical quantities *e.g.* potential fields, electric currents, ground movements, radiation intensities etc. The task of interpolation is to estimate the function at various points within the range of known values. Most of the accurate and reliable methods available for the quantitative analysis of geophysical data (*e.g.* mathematical filtration techniques, equivalent source schemes, modelling procedures, *etc.*) are suitable only for equally spaced data. Interpolation is sometimes employed to fill gaps in a set of otherwise evenly spaced seismograms (Cabrera and Levy, 1984; Ronen, 1987). Recently, a method has been developed for the computation of Fourier transforms of

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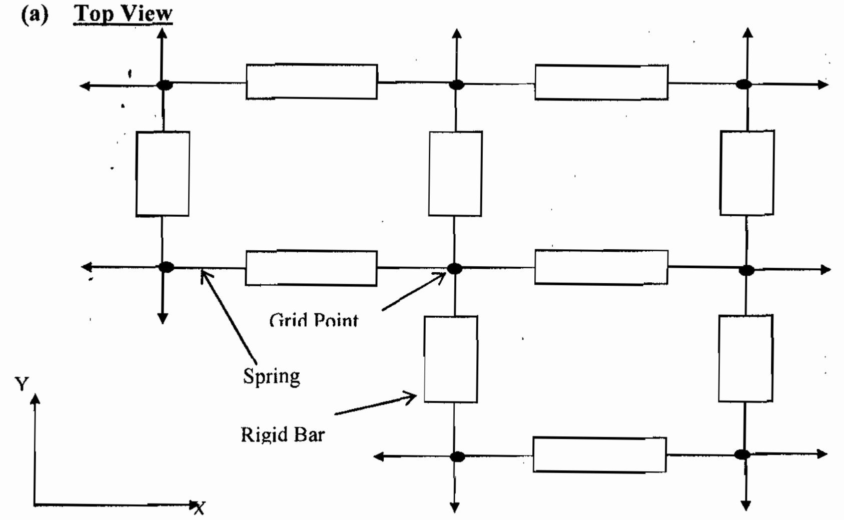
unequally spaced data (Press *et al.*, 1992). But this method is not only computationally cumbersome but, most importantly, it sidesteps the use of the Fast Fourier Transform algorithm - the main attraction for the use of Fourier methods in the analysis of geophysical data. Unfortunately, for reasons of technical feasibility, geophysical data is often collected at irregular intervals. Thus reliable interpolation schemes are necessary prior to the interpretation of such data. The earliest (and still widely used) form of interpolation is by 'hand'. This consists of plotting measured values on a graph or sheet and establishing a regular grid at which points, function values are visually deduced. The major setbacks in this method are the lack of objectivity and slowness. With the advent of high speed calculating machines, the interpolation of geophysical data is more reliably done by using mathematical functions and numerical procedures.

The Laplace interpolation technique is a two dimensional Finite Difference method (Taylor, 1976), which falls under the category that is usually described as 'numerical surfaces' (Crain, 1970). In our treatment of gravity and magnetic data we found that the use of common interpolation schemes, *e.g.* bicubic spline interpolation (Bhattacharyya, 1969; Press *et al.*, 1992), frequency domain methods (Ronen, 1987), leads to the distortion of the data. A good method like Quadratic Weighting leads to smoothing of data in regions of high field gradients (Crain and Bhattacharyya, 1967; Davis, 1973). We have found that Laplace Interpolation is fast and reliable and that in regions of high data density it preserves certain important features of the original data such as: resolution and shapes of individual anomalies, magnitudes and location of peaks and troughs, and steepness of field gradients. Details of the technique and its application are given below.

Laplace Interpolation

Like other Finite Difference methods, Laplace Interpolation is based on the assumption that the desired surface satisfies the differential equation of static equilibrium (minimization of the potential energy). This equation is then approximated by finite differences and then solved iteratively. The iterative nature of the solutions makes the method highly suitable to computer application. In his formulation of the method, Taylor (1976) likened the two-dimensional function surface to a network of rigid bars and elastic springs intersecting at the grid points as shown in Figure 1. Between each pair of bars there are two connecting springs that meet end-to-end at a grid point (Figure 1) and apparently holding the bars in a straight line. The grid points that are also data points are constrained, while the others which are not data points can be pulled up or down. The equilibrium position of the net, under the influence of the elasticity (of springs) and tension (in bars), and constrained at the data points, gives the z(i,j) values generated for the non-data points by this method.



(b) <u>Side View</u> **(Along x - direction)**

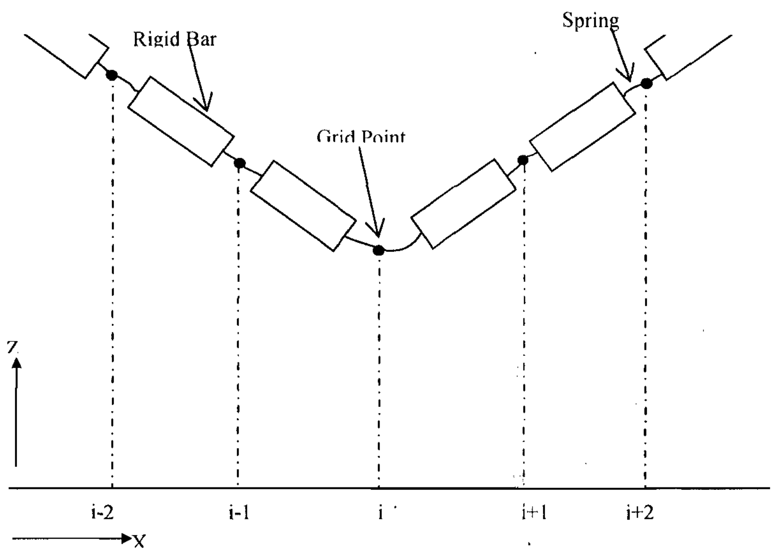


Fig. 1: Network of rigid bars and elastic springs

A parameter *k* is defined for the force due to the springs such that by varying this parameter, a trade-off between smoothness of the surface and avoidance of overshoots in region of sparse data can be obtained. But, because of the requirements of smoothness at data points, the avoidance of over or under-shoots in regions of sparse data is not usually successful. To minimize this effect, appropriate regions of the grid should be blanked out (left undefined). This is a major weakness of the Laplace method for its application to geophysical data.

Finite difference equations

The equilibrium condition at a grid point under forces due to spring elasticity and tension can be expressed as (Taylor, 1976):

$$\delta_x^2(z)+\delta_y^2(z)-k\,(\delta_x^4+\delta_y^4)(z)=0 \qquad (1)$$

This is applied only to grid points, which are not data points.

For such points the finite difference form of equation (1) applied to a point and its neighbors is of form:

$$-kz(i-2,j)-kz(i,j-2)-kz(i+2,j)-kz(i,j+2)++(1+4k)z(i-1,j)+(1+4k)z(i,j-1)$$
$$+(1+4k)z(i+1,j)++(1+4k)z(i,j+1)-(4+12k)z(i,j)=0 \qquad (2)$$

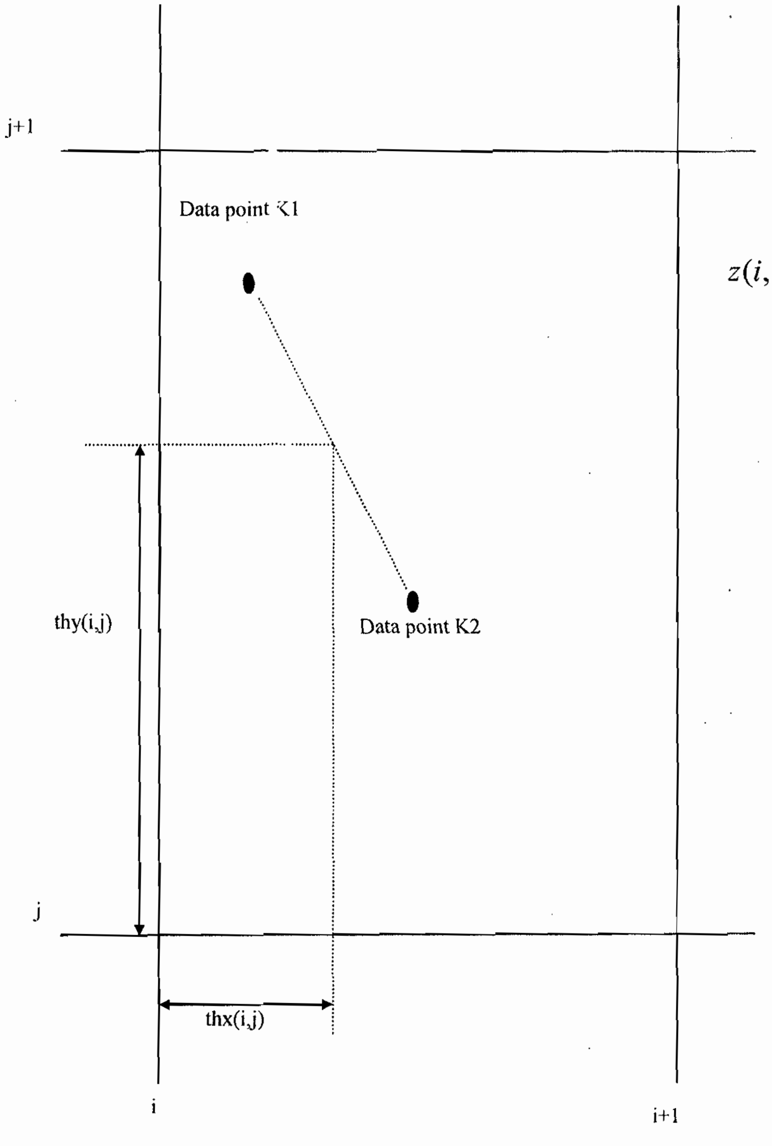
From this we get:

$$z(i,j)=\frac{-k}{4+12k}\left[z(i-2,j)+z(i,j-2)+z(i+2,j)+z(i,j+2)\right]+$$

$$+\frac{1+4k}{4+12k}\left[z(i-1,j)+z(i,j-1)+z(i+1,j)+z(i,j+1)\right] \qquad (3)$$

Steps in the calculation

(i) Initially, each data point is shifted to the nearest grid intersection. This shift will be enhanced if the sample interval (or the grid spacing) is appreciably small. If a given intersection is nearest to two or more data points, then z(i,j) is set as the average of the two or more values (z_p). Also, the true position of the data point (or average of more than one) relative to the nearest intersection is saved in arrays thx(i,j) and thy(i,j). The array element zth(i,j) is initially set equal to z(i,j), and the array element kz(i,j) records the number of data points falling closest to the mesh point (i,j) (Figure 2).



$$z(i,j) = zth(i,j)$$
$$= \frac{z_p(k1) + z_p(k2)}{2}$$
$$kz(i,j) = 2$$

Fig. 2: Location of the approximated non-grid point with respect to the grid intersection

(ii) Equation (3) is solved iteratively over the net of grid points. At each grid point, which contains no data, the new z-value is found by solving equation (3) for it in terms of the surrounding points. Call this valve z′(m+1) on the $(m+1)^{th}$ iteration. Then the new estimate is obtained as:

$$z(m+1) = z(m) + w[\, z'(m+1) - z(m)\,]$$

Where w is the relaxation factor, and should be between 1 and 2. By selecting the value of w properly, the rate of convergence can be greatly improved. The optimum relaxation factor w_c can be estimated as (Taylor, 1976):

$$w_c = \frac{2}{1 + (1 - \lambda^2)^{\frac{1}{2}}} \quad (4)$$

where λ is a constrained sine function given by Froberg (1974) as:

$$\lambda = 1 - \sin^2\left(\frac{\pi}{N}\right) \quad (5)$$

In equation (5), N is the total number of spacing in a square grid, which has *n* grid points on each side. Convergence was conveniently obtained by taking N = (NX +NY), where NX and NY are the total number of grid spacing in the x and y directions respectively. For a square grid, NX = NY = $n(n-1)$. The iteration process starts with w = 1.0. Every ten iterations w_c is estimated using equation (4), then the new w is calculated as w_c - (2.0 - w_c) x 0.25, just to keep w from increasing too rapidly. The value of k is set at zero for the first 10 iterations and then changed to 10.0 for subsequent iterations.

(iii) On every 5th iteration, and at every mesh point containing one or more data points, a paraboloid of form

$$f(x,y) = a + bx + cy + dx^2 + ey^2$$

is fitted through the z values at the point (i,j) and its four neighbors to the North, South, East and West. We then calculate

$$fth = f[\,thx(i,j)\,,thy(i,j)]$$

and then adjust $z(i,j)$ to

$$z(i,j) = zth\,(i,j) - fth$$

(iv) The iteration is carried on until the estimated error is less than $10{,}000^{-1}$ of the range of data ($z_{pmax} - z_{pmin}$).

Application

As stated above, the Laplace method necessitates the blanking out of regions of sparse data. Thus the method can only be conveniently applied to data that has uniform density. This is not usually the case with geophysical data; for example aeromagnetic maps often exhibit low relief over thick sedimentary formations, whereas in regions of outcropping igneous rocks the maps exhibit very high relief. In other words, digitized data obtained from such maps are sparse over thick sedimentary formation (due to unavoidably large sampling intervals), but dense over regions of outcropping igneous rocks (because of the unavoidably small sampling intervals).

In our treatment of gravity and magnetic data we found that a faithful interpolation can be done if we restrict the Laplace Technique to regions of high data density. In regions of low data density we apply the method of quadratic weighting (Crain and Bhattacharyya, 1967; Davis, 1973), which is reliable when field gradients or reliefs are low. In quadratic weighting the function value at a point is obtained as a weighted average, *f(x,y)*, of (say *n'*) neighbouring points. This is given by:

$$f(x,y) = \frac{\sum_{i=1}^{n'} w_i(x,y)\, G_i}{\sum_{i=1}^{n'} w_i(x,y)} \quad (6)$$

where G_i is the observed value at the location (x_i, y_i) and $w_i(x,y)$ is the weighting function given by

$$w_i = \frac{1}{x_i^2 + y_i^2} \qquad (7)$$

In the algorithm that we eventually developed for the interpolation of geophysical data, the grid points are divided into three categories:

(i)grid points which contain data points;

(ii)grid points which are not data points but, which have neighbouring data points;

(iii) grid points which are not data points and which have no neighbouring data points.

Points in category (iii) occur in regions of low function gradients and can be satisfactorily interpolated using the weighting method. A minimum of five and a maximum of ten surrounding points are used for calculating the interpolated values. Points in category (i) need no interpolation. Points in category (ii) are in regions of dense data and can be rapidly and satisfactorily interpolated using the Laplace method.

Results

A FORTRAN program (DINTL3) written by the authors was used to carry out the interpolation of random surface data in the manner outlined above. The program was applied to sparse aeromagnetic data, dense aeromagnetic and gravity data. The sparse aeromagnetic data was obtained by digitizing (along flight lines) the aeromagnetic sheet No. NB-32-X-1 of the Cameroon Department of National Resources (Kangkolo, 1996).

The map (called here Map No. 1) extends from Latitude 5.75°N to 6.00°N and from Longitude 9.00°E to 9.25°E. Figure 3 shows a contour map of the random data obtained from Map No. 1. The data is then interpolated on a 26 x 26 grid to get 676 equally spaced data points. Figure 4 shows a contour map of the interpolated data. The dense aeromagnetic data was obtained by digitizing (along flight lines) the aeromagnetic sheet from the Geological Survey of Nigeria. The map extends from Latitude 11.00°N to 12.00°N and from Longitude 8.00°E to 9.00°E. Figure 5 shows a contour map of the random data. The data is then interpolated on a 51 x 51 grid to get 2601 equally spaced data points. Figure 6 shows a contour map of the interpolated data. The gravity data is obtained from a gravity survey carried out in the Kwello area of Kaduna State, Nigeria Akaolisa (1997), which was a follow-up of a gravity survey carried out in the same area earlier compiled by Osazuwa *et al.* (1994). Figure 7 shows a contour map of the random data (taken along motorable roads in the area). The data is then interpolated on a 41 x 31 grid to get 1271 equally spaced points. Figure 8 shows a contour map of the interpolated data.

Discussion

There is no significant difference between the non-interpolated map (Figure 3) and the interpolated map (Figure 4) derived from the grided data of aeromagnetic data over regions of thick sedimentary formations. In such areas, the magnetic bodies occur at appreciable depth. Therefore, the method of production of the required map may not alter significantly the surface geologic boundaries. In the case o data obtained from areas of granitic outcrops, there is significant difference between the aeromagnetic maps produced from the non-interpolated data (Figure 5) and the aeromagnetic map produced from interpolated data (Figure 6). The implication of this is that maps produced from interpolated data expose the shallow (or near surface), but high frequency geologic features. Similar enhancement of near surface features through interpolation is noticed in the gravity anomaly map in Figure 8 as compared with the non-interpolated gravity anomaly map in Figure 7.

While the interpolated map of the gravity data (Figure 8) is very smooth, the interpolated map of the aeromagnetic data (Figure 6) is relatively noisy.

This is due to the bipolar nature of magnetic anomalies, which is absent in gravity anomalies. It is essential that a smoothing filter should still be applied to interpolated magnetic maps in order to remove any noise that might be present.

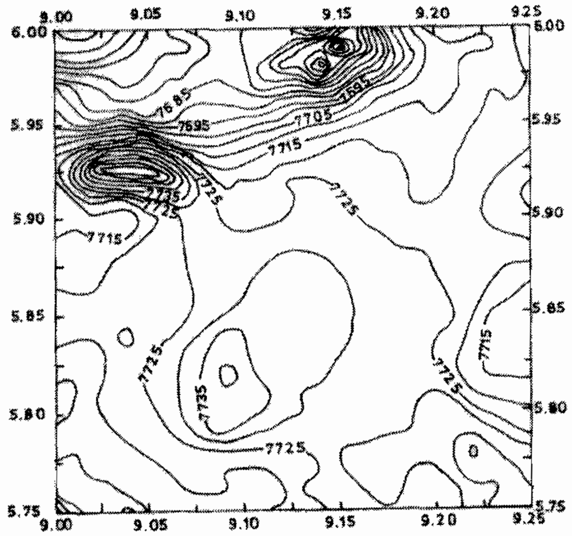


Fig. 3: Map No. 1 before interpolation

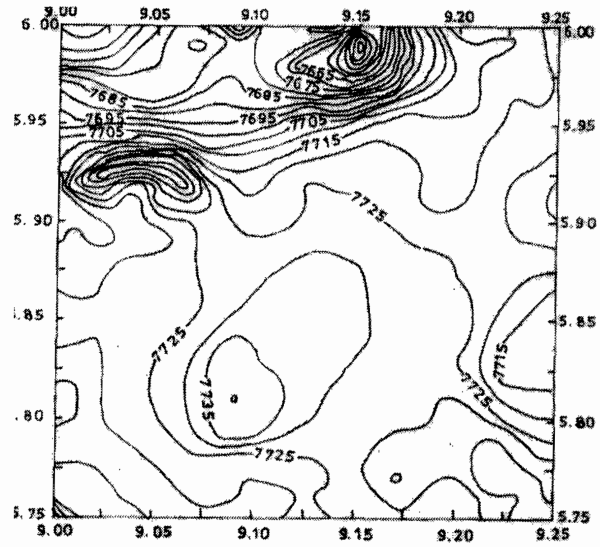


Fig. 4: Map No. 1 after interpolation

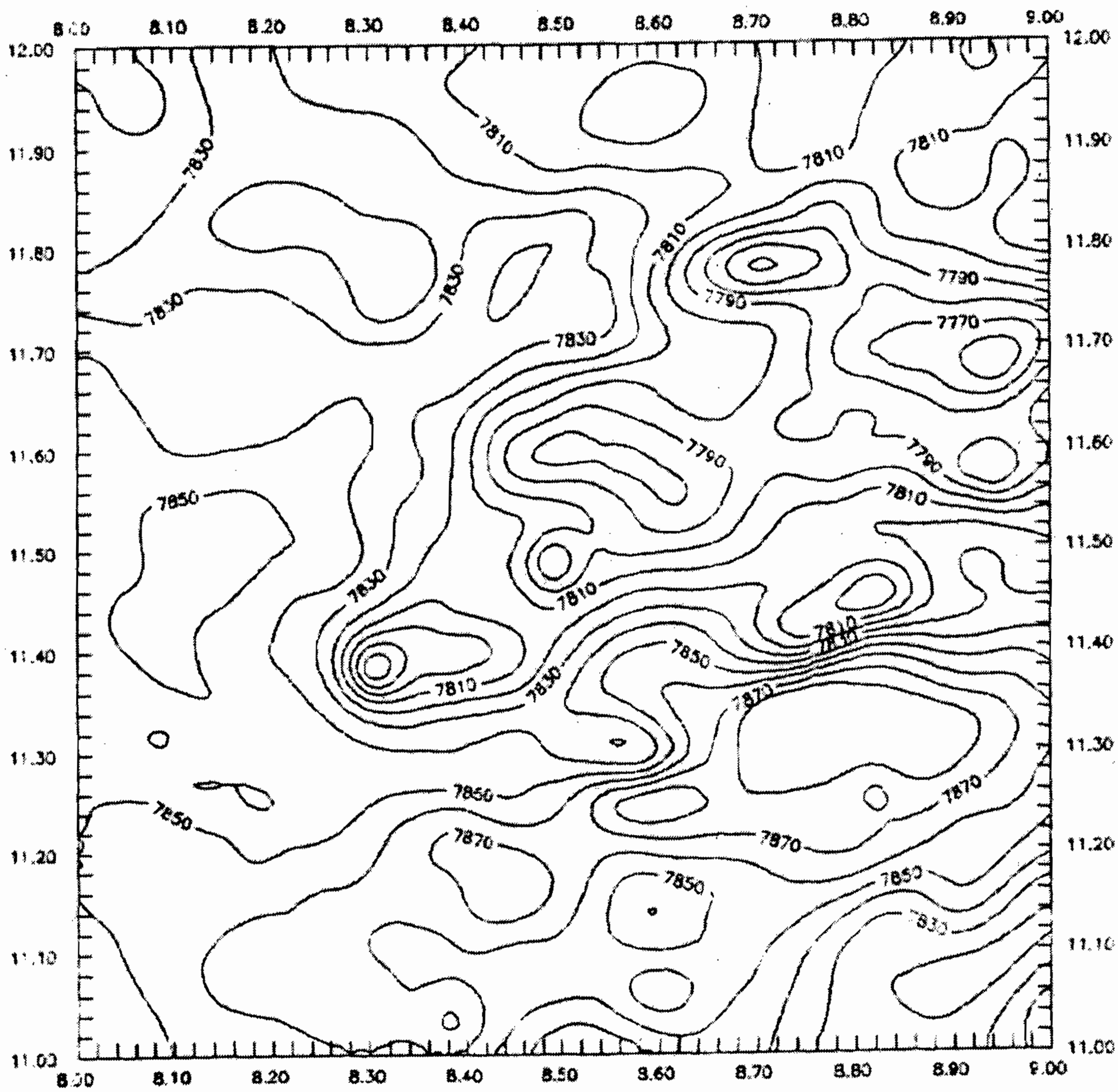


Fig. 5: Aeromagnetic map before interpolation

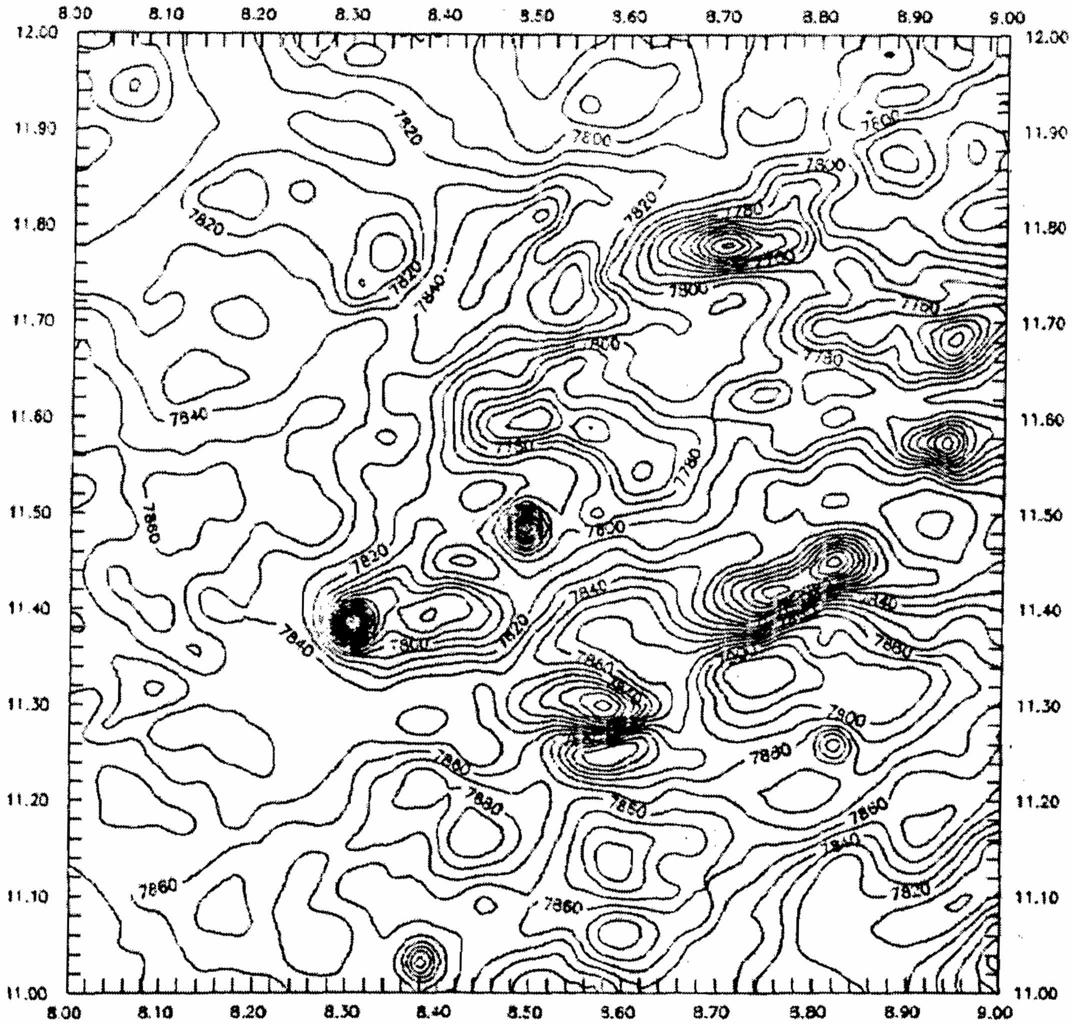


Fig. 6: Aeromagnetic map after interpolation

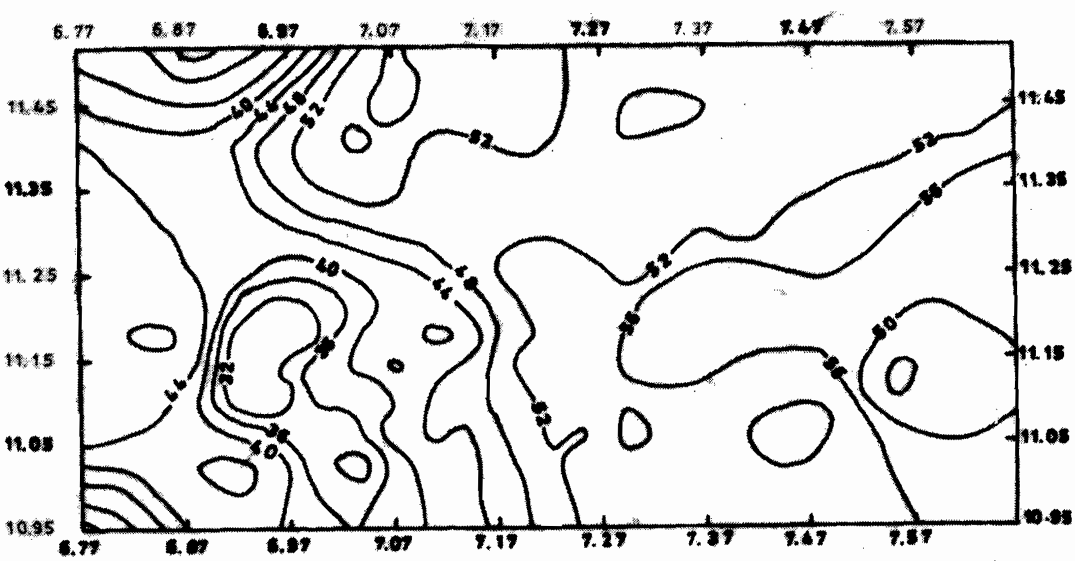


Fig. 7: Random Gravity Field Data

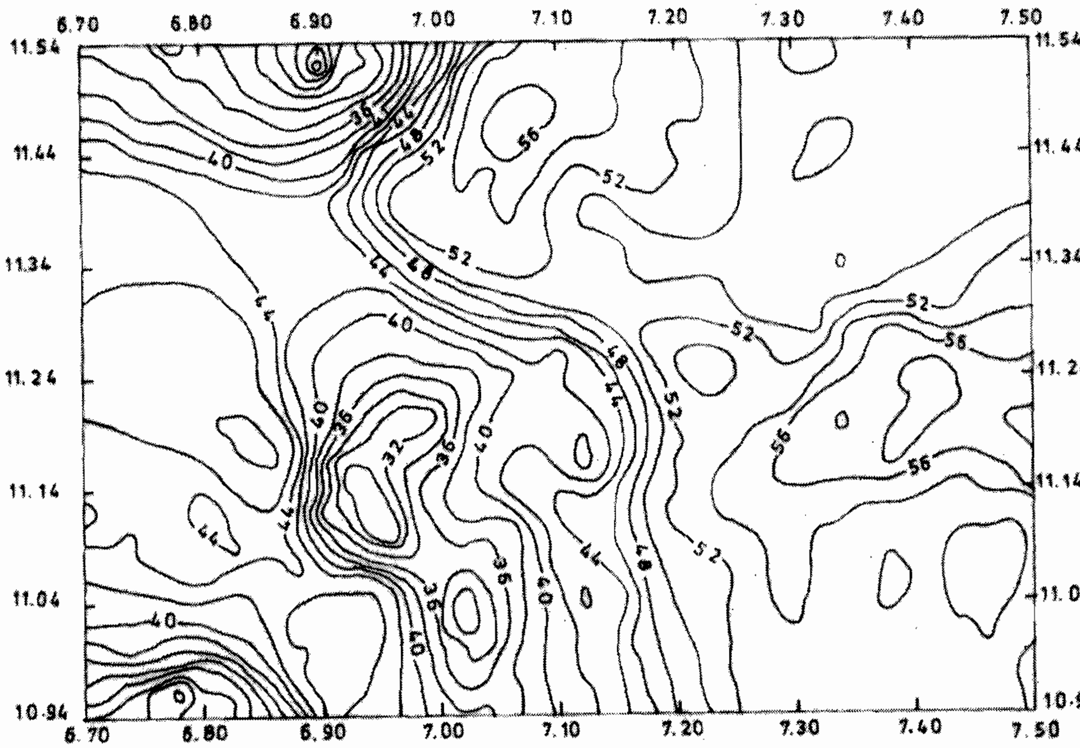


Fig. 8: Interpolated Gravity Field Data

Conclusion

An interpolation procedure is developed which allows the densification of geopotential fields in areas where the observed data are sparse. This involves the use of quadratic weighting in regions of low data density and the restriction of the Laplace (finite-difference) method to regions of relatively high data density where the use of traditional interpolation techniques would lead to distortion of data and smoothing. The interpolation methods are fast in operation and can efficiently applied to aeromagnetic (or ground magnetic) data obtained over regions, which in some areas are characterised by thick sedimentary cover and in other areas where igneous rocks outcrop or occur at shallow depths. The interpolation techniques also apply to gravity data, which are often collected along road networks that are not evenly distributed in the survey area. Comparison of the set of random maps with the set of interpolated maps shows them to be in reasonably good agreement. However, the interpolated maps show better enhancement of the near surface geologic features, which consequently leads to higher gradient of linear features and higher relief generally. If interpolation is not done with care unnecessary enhancement, in the form of noise, can be introduced into the map. This, however, can be removed by applying appropriate smoothing filter.

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