DETERMINATION OF UPPER MANTLE CONDUCTIVITY USING QUIET DAY IONOSPHERIC CURRENT

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Abstract

The electrical conductivity of the upper mantle has been studied using data from L'Aquila (Latitude of 42°23'N; longitude of 13°19'E). The method of analysis employed was the Gauss spherical Harmonic analysis (SHA) technique. The results of the study indicate that the average electrical conductivity of the upper mantle in L'Aquila rose from very low sub-crustal value of 0.07 S/m to 0.17 S/m at about 290 km depth and dropped to a value of about 0.16 S/m at a mean depth of approximately 380 km and again rises sharply to a value of about 0.28 S/m at 705 km with no indication of leveling off. Results further show good correspondence between high conductivity and low seismic velocity zone. The cause of enhanced conductivity obtained from this study in the area under study could be attributed to the complex structure of the lower crust and upper mantle which consequently produces large lateral variation in electrical conductivity.

Keywords: Sq, spherical harmonic analysis, conductivity, ionospheric current
And upper mantle.

1. Introduction
The systematic flow of varying electric current in the part of the earth's upper atmosphere, the ionosphere, give rise to a magnetic field which in turn induces electric current in the earth's crust and mantle. The composite of both the external field and the internal field is measurable at the earth's surface in magnetic observatories. Such measurements, when made on quiet electrical conductivity of the upper mantle in the region concerned. This work is concerned with the use of fields due to ionospheric current variation at quiet conditions in L'Aquila, Italy to obtain the conductivity-depth profile of the area.

Schumucker (1970) initiated the method of obtaining the electrical conductivity-depth profile from the SHA external and internal coefficients of fields. Campbell et al. (1992, 1993) used this method to investigate the external source currents and deep - earth electrical conductivity for the India - Siberia region. Arora et al. (1995) also investigated the upper mantle electrical conductivity in the Himalayan region. They determined the conductivity profile of the earth at depths of about 50 to 500 km using the quiet
The years 1989 to 1999. Both horizontal (H) and vertical (z) components of the field from the solar quiet (Sq) day variations in the region were utilized.

3. Method of Analysis
Spherical harmonic analysis (SHA) technique devised by Gauss (1838) was used to separate the external and internal fields and we henceforth used the SHA coefficients generated to determine the deep-Earth conductivity.

The theory of upper mantle conductivity determination begins with the Maxwell's formulation of electromagnetic field equations. Maxwell (1873) had generated a concise list of differential equations which summarized the laws governing electric and magnetic processes. The magnetic fields in these equations are due to real currents flowing in the ionosphere or equivalent (substitute) currents.

For the case in which the fields are measured about the surface of a sphere across which current does not flow, Gauss (1838) provided a separable series solution of Maxwell's differential equations for his spherical harmonic analysis (SHA) of the Earth's main field. Schuster (1889, 1908) used the Gauss SHA technique for Sq and showed that ionospheric current flow in the upper atmosphere of the earth was the cause of the quiet daily field variations. With the separated external and internal fields, Chapman (1919) succeeded in achieving the first recognizable earth conductivity determinations and also drew greater attention to the method of considering the

2. Source of Data
The data set obtained from the geomagnetic observatory in L'Aquila consists of the hourly mean values of the five quietest days for each month and from

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magnetic observatories as moving below an external Sq current system fixed with respect to the sun in order that the 24 hour Sq variation at a station represent 360° of longitude.

Schmucker (1970) provided equations expressing a conductivity and depth where an induced current could flow for the fields. Campbell (1998) amended and henceforth summarized Schmucker's work with the following equations:

\[ d_n^m = z - p \]  \hspace{1cm} (1)

where \( d_n^m \) is depth; \( z \) and \( p \) are real and imaginary parts of a certain complex number transfer function and \( m \) and \( n \) are order and degree, respectively.

\[ \sigma_n^m = \frac{5.4 \times 10^4}{n(n \pi)^{2}} \text{ S/m} \]  \hspace{1cm} (2)

where \( \sigma_n^m \) is conductivity.

\[ Z = R \frac{\left[ n(a_n^m)^2 - (a_n^m)(b_n^m) + (a_n^m + b_n^m) \right]}{n(n-1)(a_n^m)^2 + (b_n^m)^2} \]  \hspace{1cm} (3)

\[ R = \frac{\left[ (a_n^m)^2 - (a_n^m)(b_n^m) + (a_n^m + b_n^m) \right]}{n(n-1)(a_n^m)^2 + (b_n^m)^2} \]  \hspace{1cm} (4)

where \((a_n^m)^2\) and \((b_n^m)^2\) are the external cosine and sine SHA coefficients \((a_n^m)\) and \((b_n^m)\) are the internal cosine and sine SHA coefficients. \(A_n^m\) and \(B_n^m\) are coefficient sums for the external and internal SHA coefficients, respectively.

This work however involves the use of only a single observatory data quiet unlike those of Campbell and Stiffmacher (1987,1988b), Arora et al (1995) and Campbell et al (1998) and instead of the one or two-year data used by most of the early researchers, a six-year data was employed in this present study in the L'Aquila region. The data processing routine employed was adapted from that of Campbell et al. (1998) and it is shown in Fig. 1.

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<tr>
<td>Obtain field records of H and Z for the mentioned years</td>
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<td>Evaluate monthly mean values of the five selected quiet days per month in each year for H- and for Z- fields</td>
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<td>Carry out a spectral (Fourier) analysis of each component and obtain spectral analysis (4 harmonics) of each Fourier (sine and cosine) coefficients for each of the years and hence generate the intermediate coefficients</td>
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<tr>
<td>Compute the spherical harmonic analysis coefficients obtaining the external and internal pairs of values of order, m and degree, n</td>
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<tr>
<td>Determine conductivity versus depth values for each month and for each pair of m and n</td>
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<td>Obtain locally weighted regression fits for depth versus conductivity profile</td>
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<tr>
<td>Calculate the external and internal current for any month at the given latitude and longitude.</td>
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**Fig. 1:** Data processing routine
4. Results and Discussion

From the flow diagram, Fig. 1, the monthly mean values of the five selected quiet days for each month of the six years utilized were evaluated for both the H- and the Z-components and a Fourier analysis was carried out to obtain the Fourier coefficients and the Fourier-analyzed values of H and Z. Associated Legendre polynomials (Schmidt functions) for the study area (colatitude 47°37') were computed. The intermediate coefficients were calculated using the Fourier-analyzed values of H- and Z-components. Spherical harmonic analysis coefficients were determined from the intermediate coefficients and the coefficient sums $A_n^m$ and $B_n^m$ were evaluated. The real and imaginary parts, $z$ and $p$ respectively of the special transfer function related to the depth, $d$ (km) and conductivity $\sigma$ (S/m) of the earth were determined from the separated external and internal SHA coefficients and the values of $d_n^m$ and for various pairs of $m$ and $n$ were evaluated with $n = 1$ to 12, $m = 1$ to 4 and $m = n$. The conductivity-depth values were analyzed graphically and the statistical method employed was the locally weighted regression fitting described by Cleveland (1979). This gave the most likely profile for each year's data. By using the lowest regression fitting with two fitting iterations and appropriate smoothing factors, the profiles obtained are as shown in Figs. 2 to 7.

Fig. 2: Conductivity depth-profile of L'Aquila for 1989 Sq data

Fig. 3: Conductivity depth profile of L'Aquila for 1990 Sq data
Fig. 4: Conductivity-depth profile of L'Aquila for 1995 Sq data

Fig. 5: Conductivity-depth profile of L'Aquila for 1996 Sq data

Fig. 6: Conductivity-depth profile of L'Aquila for 1997 Sq data

Fig. 7: Conductivity-depth profile of L'Aquila for 1999 Sq data
The open squares in Figures 2 to 7 illustrate the conductivity-depth computation results. The solid lines are the regression fitted values. From Fig. 2, it is clearly seen that, the conductivity rises from a low sub-crustal value of about $0.06 \text{ S/m}^2$ to an almost relative maximum of about $0.13 \text{ S/m}$ near 295 km depth. A relative minimum conductivity of about $0.12 \text{ S/m}$ occur near 450 km depth. Thereafter, the conductivity rises sharply toward a value of $0.17 \text{ S/m}$ at 780 km depth.

In Fig. 3, the conductivity is observed to rise moderately from a very low sub-crustal value to a value of about $0.09\text{S/m}$ near 300km depth. Thereafter it decreases gently to about $0.08 \text{ S/m}$ at 380 km depth and rises rapidly from there to about $0.25 \text{ S/m}$ at 800 km depth with no sign of leveling off. In fact, it is clearly evident from these profiles, including those of Figs. 4, 5, 6, and 7, that:

(i) The profiles exhibit about the same trend of variation of conductivity with depth. This is suggestive in the similarity of the shape of the regression fittings.

(ii) The conductivity in all the profiles increases generally with increase in depth except at few depth ranges. This is in excellent agreement with global models.

(iii) Almost all the profiles show two turning points -relative maxima and minima with the maxima always before the minima.

Correlating the conductivity-depth profiles with seismic wave velocity models, the shapes of Figs. 2, 3, 4 and 6 suggest some mantle zonation. On this ground, it becomes important to match our models with seismological and petrological boundaries. According to Arora et al. (1995), current seismic wave velocity models for the mantle such as PREM (Dziewonski and Anderson, 1981) and IASP 91 (Kenneth and Engdahl, 1991) show abrupt rise in velocity at 410 km and 670 km depth and provide strong evidence for the existence of a zone of low velocity gradient between about 100 and 220 km depth. The conductivity profiles in this present study do not resolve a discontinuity at the base of the lithosphere (before the low velocity layer) of 80 km depth, but rather they rise progressively to a mean depth of about 300km (Figs. 2, 3, 4, and 5). This feature of these profiles is in agreement with those of other regional profiles such as those of Campbell and Stiffmacher (1988), Arora et al. (1995) and Cambell et al. (1998).

The low velocity layer between 80 km (i.e base of the lithosphere) and 220 km (i.e base of the asthenosphere) show a very good correspondence with the high conductivity zones indicated by the present study in L'Aquila. The general correspondence observed between high
conductivity zones and low velocity zones here is in agreement with the global results of Tarrits (1992), even though processes giving rise to this corelation is yet to be discovered. It can be stated that the rapid rise in conductivity in all the profiles from a mean depth of 380 km to 800 km is a cumulative effect of the distinct sharp increases in conductivity at the worldwide discontinuities of 410 km, 670 km and 770 km. In terms of the rapid increase in conductivity in the depth interval of 380-800 km, the present conductivity models are similar to the global models such as parker (1971), Bott (1982), Schmucker (1985) and Stacey (1992) which show a rapid rise below 350-500 km depth (Schutz and Larsen, 1990).

5. Conclusion
Considering all the profiles obtained in the study, we conclude that the average conductivity of the upper mantle in L'Aquila rises from a low sub-crustal value of 0.07 S/m to a value of about 0.17 S/m at a mean depth of about 290 km; fall to a value of about 0.16 S/m at a mean depth of approximately 380 km and rises sharply to a value of about 0.28 S/m at 750 km depth with no indication of leveling off. Comparing the results of this study with those of similar works carried out at other regions such as the work of Arora et al (1995) in the Himalayan region and that of Campbell et al. (1998) in the Australian region, it is obvious that the results are reasonable. Nevertheless, the conductivity values of this presnt study are noted to be slightly higher than those of previous research works at comparable depths.

The cause of the enhanced conductivity results obtained in L'Aquila may be generally due to the complicated structure of the crust and upper mantle which results in large lateral variations in electrical conductivity (Lowrie, 1997). It is therefore recommended that further research work be carried out in the study area perhaps with multiple observatory stations to confirm these findings.

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