DEPTH-TO-BASEMENT MAPPING USING FRACTAL TECHNIQUE: APPLICATION TO THE CHAD BASIN, NORTHEASTERN NIGERIA

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Abstract

Fractals are being recently used to describe many geophysical processes and rock properties. The estimation of crystalline basement depths from power spectra of magnetic data presumes some knowledge of the basement magnetization behaviour. Replacing the common assumption of non correlation of magnetization with a fractal description leads to a reduction in these estimates, producing more realistic results. The fractal method requires the knowledge of the scaling exponent at source level. It is shown in this work that the scaling exponent varies with depth in a linear form, and can thus be obtained at source level. Application to aeromagnetic data from the Chad basin north eastern Nigeria produced a basement relief which range from depths of 2.47 km to 5.40 km with an average of 3.92 ± 0.66 km.

Keywords: *Fractal, depth, basement, spectra, aeromagnetic*

1.0 Introduction

Mandelbrot (1975) used fractals to describe natural and irregular shapes. The essential feature of a fractal is the existence of a similar structure at all length of scales. Because the spatial variation of most geophysical parameters such as density and susceptibility often look similar at a wide range of scale, they can therefore be described using fractals (Sunmonu and Dimri, 2001; Bansal and Dimri ,2005). A fractal distribution is one that has a power-law dependence or is scale invariant, and Turcotte (1992) showed that for a time series to be fractal, the power spectral density S of the time series must have a power law dependence of frequency f (that is $S f^{\gamma}$), where γ is called the scaling exponent and f is the frequency of the time series from which the power spectrum is computed.

During the last three decades, spectral analysis of magnetic anomalies based on statistical models has been used in a variety of geologic applications, such as the estimation of the average depth to top of the magnetic basement and the crustal thickness. The statistical method developed by Spector and Grant (1970) has been used extensively in the interpretation of magnetic anomalies. The method is based on the expression of the power spectrum for the total-field magnetic anomaly produced by a uniformly magnetized rectangular prism of thickness t, length a, width b, and magnetization M. buried at a depth h (Bhattacharyya, 1966), Spector and Grant (1970) modelled the crustal magnetic layer, by an ensemble of vertical prisms, where the ensemble is characterized by a joint probability distribution of the parameters, h, t, a, b, M, and the local inclination I and declination D of the geomagnetic field. Based upon the assumption that the parameters for each prism are independent of the parameters of all other prisms, and that the probability distribution for each parameter is uniform, Spector and Grant (1970) stated that the shape of power-density spectrum of the magnetic anomaly is in large part controlled by the average depth of the ensemble. Specifically, the spectrum decays exponentially with wavenumber, at a rate of decay proportional to the average depth to the top of the ensemble. This is often used to estimate depth to magnetic sources from magnetic anomalies. Fedi *et al* (1997) discovered that the red character of the ensemble power spectrum is better approximated by a power law rather than an exponential form as suggested by Spector and Grant (1970). Fedi *et al* (1997) analysed the variation of the prisms horizontal dimensions with the frequency of the time series and found that they had the same decaying exponent of about B2.9, especially for the larger values of a, and therefore concluded that in most cases of practical use by the Spector and Grant method, that is for a , b and t not too small, estimates of h can be obtained by dividing the power spectrum by the correcting factor of $f^{B2.9}$. The depth obtained by applying this correction factor was lower than that obtained by the Spector and Grant method which leads to overestimation of depths. Fedi *et al.* (1997) also made some analysis with theoretical examples and observed that the corrected depths sometimes leads to underestimation, especially when the ensemble is deep. They therefore suggested that both the corrected and uncorrected spectrum should be plotted first and after noting the changes in slopes, the corrected spectrum is then accepted only if its change in slope is not significant.

Garcia B Abdeslem and Ness (1994) observed that factors other than depths also contribute to the overall shape of the spectrum and such factors should be used in its interpretation. In their study using theoretical models, they observed that the shape of the spectrum is determined by the depth, thickness and horizontal dimensions of the prisms in the ensemble. They also observed that the non-linear signature in the shape of the spectrum is mainly caused by the contribution of the horizontal dimension of the source. This in essence may cause incorrect depth estimates particularly when the low wavenumber part of the spectrum is used. They noted that the change in the slope of the spectrum, which quite often is interpreted as an intermediate horizon, arises because of the effect of the horizontal dimension of the prisms in the ensemble, and not because of the presence of a second or even a third independent ensemble. Garcia-Abdeslem and Ness (1994) therefore posed the interpretation of the power spectrum as an inverse problem. The solution of the forward problem was achieved in the Fourier domain where the spectrum is given by the product of the mathematical expectations of single-valued functions that describe depth, thickness and horizontal dimensions of prisms in the ensemble. The solution of the inverse problem was achieved iteratively, starting with an initial set of model parameters. Generally the method of modeling source distribution by simple bodies such as prisms is based on source geometries which may not always reflect the natural variations of a source parameter. Maus and Dimri (1995) used the fractal approach and modeled the sources of the potential field by a random function with scaling properties, defined on a half-space with its top at a specified depth beneath the observation plane. This model has however been shown by Maus and Dimri (1996) not to produce reliable depth results when applied to real data. Maus and Dimri (1996) observed that the theoretical power spectrum for a basin with source-free sediments falls off rapidly at high frequency, whereas the power spectra of real data tend to have flat tails.

When magnetic data are continued downwards, the depth at which the power spectrum flattens out, termed the white depth, can be taken as an estimate of the depth to the magnetic source distribution (Hahn *et al.*, 1976). This is equivalent to the depth estimation procedure for an ensemble of magnetic sources introduced by Spector and Grant (1970). It can be shown that by setting h = 0 the power spectrum becomes constant, under the assumption of uncorrelated magnetization values, and have a magnetization power spectra that is flat or white. Therefore if a relatively non magnetic sedimentary strata overlie a more magnetic crystalline basement, the depth to basement can be estimated from aeromagnetic data by using this procedure. The results obtained by Hahn *et al.* (1976) indicated that the spectrum becomes flat only when some sort of correlation in magnetization was assumed. It should however be noted that by using a fractal model, the source scaling exponent γ reflects the degree of correlation in magnetization (Pilkington and Todoeschuck, 1990) and density (Mumtaz. and Naci ,2002). When $\gamma = 0$, we have the commonly considered case of white noise, where values of magnetization are completely uncorrelated. When $\gamma > 0$ the values are anti-correlated in that successive values are likely to have opposite signs. When $\gamma < 0$, the values are correlated to a degree that increases with decreasing γ. Pilkington *et al.* (1994) have shown that a correlated fractal model ($\gamma < 0$), of the crustal magnetization distribution is preferred over the standard assumption of independent, spectrally white susceptibility/magnetization distribution ($\gamma = 0$). In this model, the spectra is corrected by simply dividing by the appropriate power spectral slope provided by a given fractal magnetization distribution. However Pilkington *et al.* (1994) in their work used previously determined value of scaling exponent by Gregotski *et al.* (1991) from borehole information for their spectral correction in order to obtain the basement depth over their area of study.In this study, a procedure is described, which enables the prediction of the value of the exponents at the source level without a priori knowledge of the source depth.

2.0 **Methods**

The investigation of the variation of scaling exponent with depth is done using a theoretical model. The model consist of an ensemble of 40 prisms and the magnetic field is calculated on a 64 x 64 regular grid at 1.0 km intervals using the method of Bhatacharyya (1964). The 40 prisms are uniformly magnetized in the direction of the geomagnetic field and assuming an ambient field of 33,000 gamma. Figure 1 shows the field generated by the ensemble.

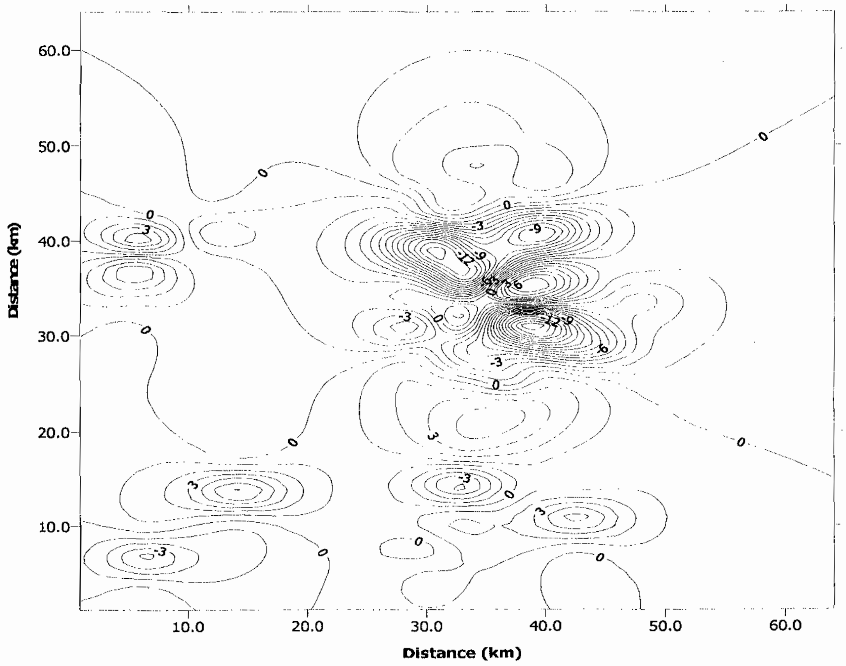

Fig. 1: Theoretical field generated by an ensemble of 40 prisms. Contour interval is 1nT and the geomagnetic field inclination is 2 degrees

The dimensions and parameters of the prisms in the ensemble is shown in Table 1 and the average depth of the prisms is 2.98 0.23 km.

The field in Figure 1 was continued downwards at various levels at 1.0 km intervals, and at each of these levels the magnetic field scaling exponent β is determined by plotting log Power against log frequency and then finding the slope (see Figure 2).

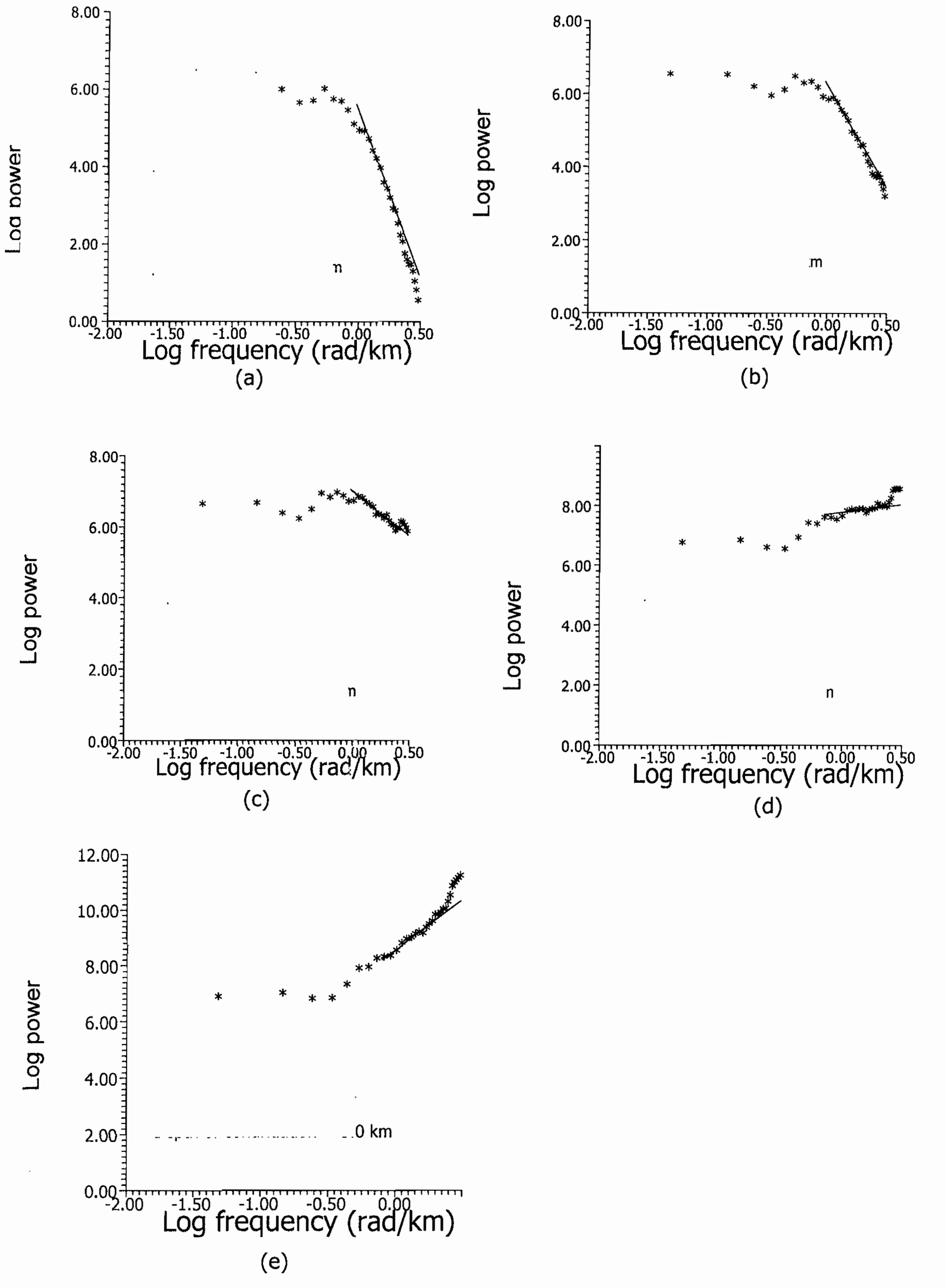

Figure 2: Plot of log of power against log of frequency for various levels of continuation of the field in figure1. (a) Depth of continuation = 1.0 km; slope = -7.6, (b) Depth of continuation = 2.0 km; slope = -4.8, (c) Depth of continuation = 3.0 km; slope = -2.0, (d) Depth of continuation = 4.0 km; slope = 0.9, (e) Depth of continuation = 5.0 km; slope = 3.7

Table 1: The parameters of 40 prisms in an ensemble model. The geomagnetic inclination and declination are both 2.0 degrees.

LENGTH a_1 (km)	LENGTH a_2 (km)	WIDTH b_1 (km)	WIDTH b_2 (km)	DEPTH TO TOP h_1(km)	DEPTH TO BOTTOM h_2(km)	SUSCEPTIBILITY (S.I. Unit)
7	8	29	30.5	3	6	0.001
28	29.3	29.3	28.6	3	6	0.001
29	30.5	27	28	3	6	0.001
28	29.3	28	28.6	3	6	0.001
35	36	31	35	2.8	6	0.001
33	35.5	4	4.3	2.8	6	0.001
31	35	35	36	2.8	6	0.001
13	15.5	34.5	34.3	2.8	6	0.001
41	43	40	41.5	3.2	6	0.001
40.5	42.3	41	41.4	3.2	6	0.001
40	41.5	11	13	3.2	6	0.001
40.5	42.3	41.8	41.4	3.2	6	0.001
38.5	41.5	28	29.5	3.1	6	0.001
13.3	15.5	31.4	34.4	3.1	6	0.001
28	29.5	38.5	41.5	3.1	6	0.001
33.3	35.5	37.4	34.4	3.1	6	0.001
28	29.5	43.5	45	2.9	6	0.001
5.8	7.3	5.4	7.5	2.9	6	0.001
43.5	45	28	29.5	2.9	6	0.001
35.8	37.3	32.6	36.5	2.9	6	0.001
26.5	32	33	34.5	3.3	6	0.001
29.8	33.3	32.1	31.5	3.3	6	0.001
13	14.5	16.5	12	3.3	6	0.001
29.8	33.3	30.9	31.5	3.3	6	0.001
38	41.5	39.5	40.5	2.7	6	0.001
28.8	31	39.6	39.9	2.7	6	0.001
39.5	40.5	8	4.5	2.7	6	0.001
38.8	41	40.1	39.9	2.7	6	0.001
37	41	30	32.5	2.6	6	0.001
33.5	36.8	33	35.1	2.6	6	0.001
30	32.5	37	41	2.6	6	0.001
33.5	36.8	37.3	35.1	2.6	6	0.001
24.5	26	36	38	2.9	6	0.001
30.3	32	34.1	31.1	2.9	6	0.001
36	38	4.5	6	2.9	6	0.001
30.3	32	28.1	31.1	2.9	6	0.001
47	48.5	33	34.5	3.3	6	0.001
40	41.5	37.3	40.8	3.3	6	0.001
33	34.5	47	48.5	3.3	6	0.001
10	11.5	44.3	40.8	3.3	6	0.001

Table 2 gives the summary of results for the β values obtained at each level and Figure 3 shows the plot of the β values against the depths of continuations and it can be seen that the variation is linear.

Table 2: The summary of results for the values obtained at each level of continuation in Figure 2.

Depth of Continuation (km)	Field Scaling exponent
1.0	7.6
2.0	4.0
3.0	2.0
4.0	-0.90
5.0	-3.7

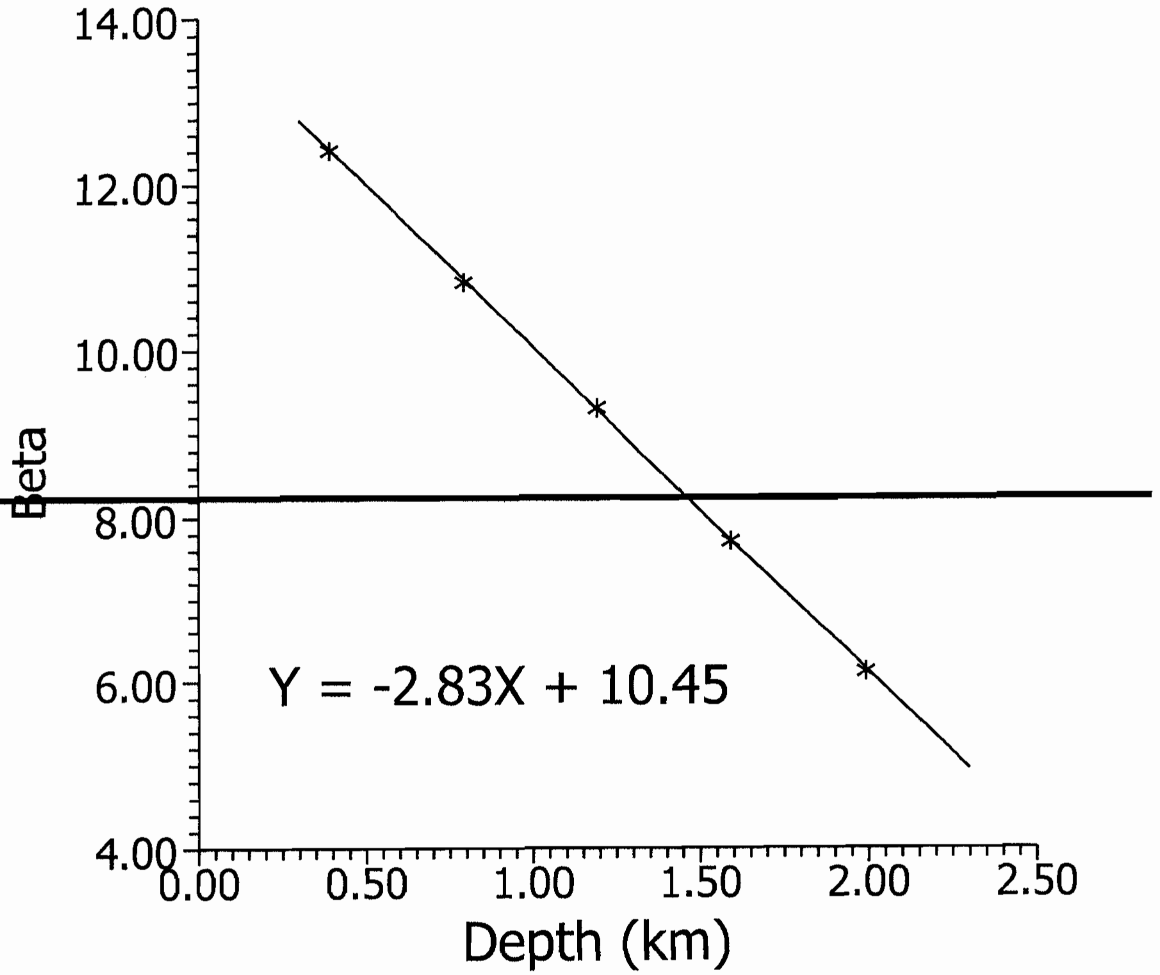

Fig. 3: The plot of the beta values against the depth of continuations

The decrease in value with depth of the scaling exponents is an indication that high values of scaling exponents reflects the contribution of longer wavelengths while a low value reflects the contributions from shorter wavelengths. This behavior is probably due to the fact that downward continuation amplifies short wavelengths. Finally the scaling exponent of the source is obtained using $\gamma = \beta + 1$ (Maus and Dimri, 1994) . The regression equation in Figure 3 is given by $\beta=-2.83d + 10.45$ (1)

Substituting $\gamma = \beta + 1$

We have $\gamma = -2.83d + 11.45$ (2)

where β = scaling exponent of field,
γ = scaling exponent of source
d = depth of continuation

Equation 2 can be easily incorporated into a computer program meant for the calculation of the white depths so that the scaling exponent at the source level and the depth to the source are being simultaneously determined.

3.0 Depth-to-basement mapping of the Chad basin

The Nigerian sector of the Chad basin is a plain which slopes gently towards Lake Chad. It is devoid of rock outcrops and is covered by superficial deposits of sand and clay. It is a large sedimentary basin which occupies the northeastern corner of Nigeria (Figure 4). The Chad basin exhibits platform sedimentation in a geotectonic setting related to rifting, and sedimentation is believed to have started during the Albian (Carter et al, 1963). Knowledge of the geology of Chad basin is deduced primarily from lithologic information obtained from boreholes (Avbovbo *et al.*, 1986). The area of study lies between geographic latitudes $12^0\ 00^1$ N and $13^0\ 30^1$ N and longitude $11^0\ 30^1$ E and $13^0\ 30^1$ E. It is covered by aeromagnetic map sheets 021 to 024, 042 to 045 and 064 to 067, and has an area of approximately 37,000 km^2.The aeromagnetic map sheets were digitized along flight lines and the data from the twelve maps were combined to form a composite map and the data in the composite map were then interpolated onto a regular grid at 1.0 km intervals. A regional-residual operation was performed on the data by fitting a first order polynomial, and the residual map of the area is shown in figure 5.

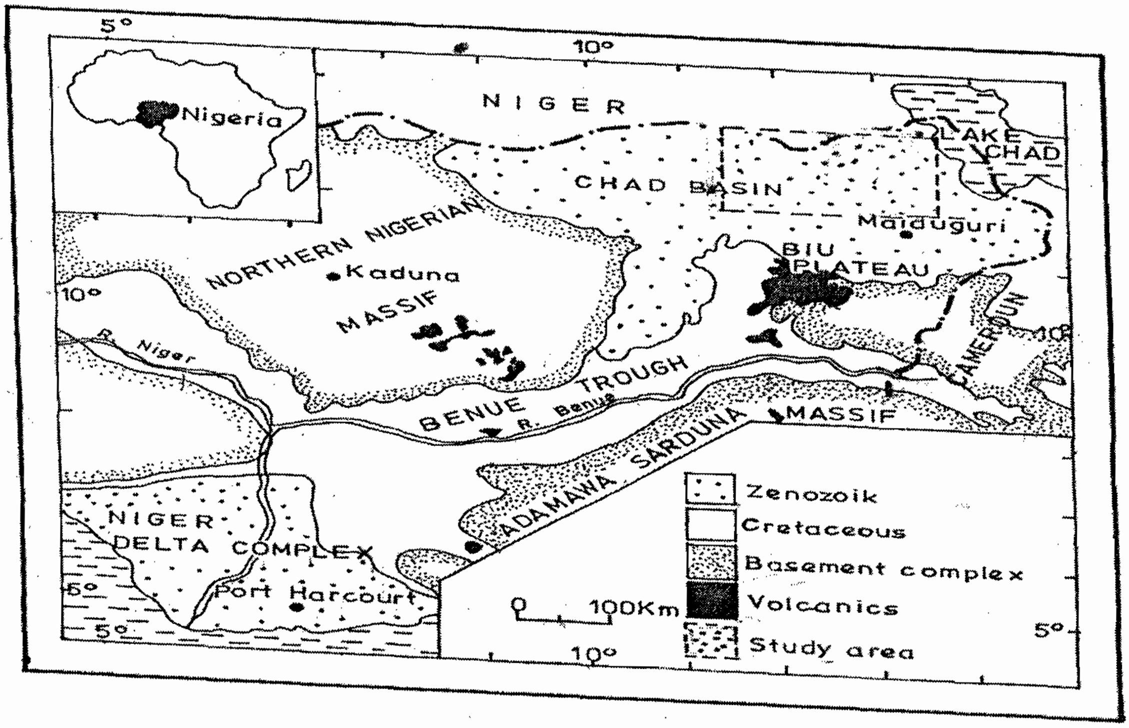

Fig. 4: Geological map of a part of Nigeria showing the Nigerian sector of the Chad basin and the location of the study area. (After Whiteman, 1982)

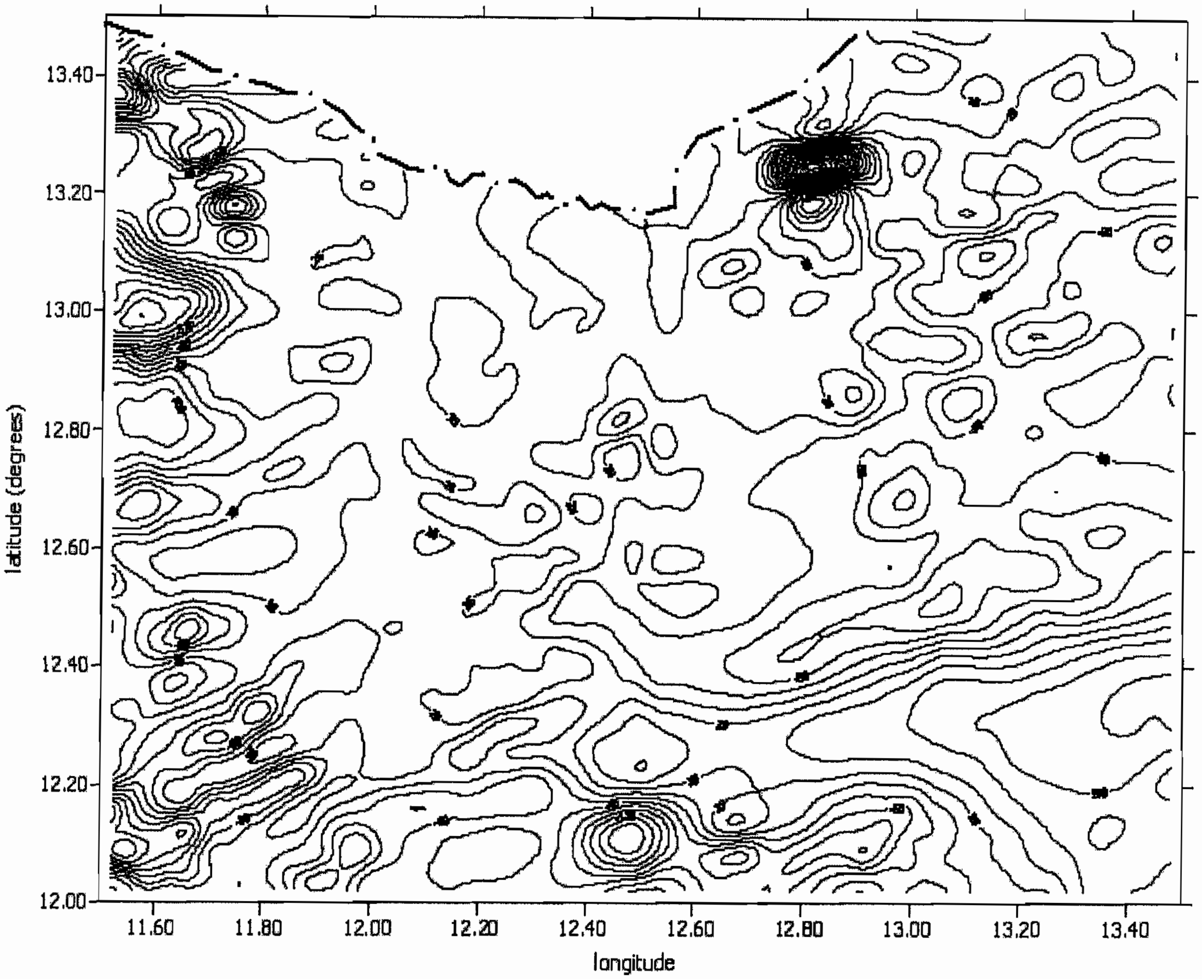

Fig. 5: Residual map of the area of study. Contour interval is 25 nT.

Traditionally the determination of depths from spectral analysis usually involves dividing the composite map uniformly into subsections, such that analysis performed produces the solutions (depths) at the center of each subsection. But this kind of division has the disadvantage that some relevant anomalies could be truncated at the boarders of adjacent subsections. Therefore, in this work a data window of 32 by 32 points was moved round the composite map, making sure that all relevant anomalies fall within the window without any being truncated. Figure 6 shows the distribution of points representing the centers of the windows where the depths were estimated. A total of 47 subsections were obtained and the numbers associated with each point in figure 6 are used in representing the results for each subsection. The procedure in section 2.0 above was then applied to the aeromagnetic data of the area. The data for each subsection were continued downwards five levels up to 6.0 km in .equal steps, and Table 3 shows the summary of the results which is the regression equation obtained for each subsection.

A FORTRAN 77 computer program was written to incorporate the regression equations in Table 3 and computation of the white spectra for each subsection was done. Figure 7 shows the regression plot for the variation of beta with depth and their corresponding white spectra for the three subsections. The summary of the depths producing the white spectra of all the subsections are shown in Table 4. Using the result in Table 4, the basement topography within the area of study was plotted and this is shown in Figure 8. This result indicates that the depth of basement within the area varies from about 2.47 km to about 5.40. km with an average value of 3.92. 0.66 km, and the basement relief generally varies from low values to high values and then to low values again as one moves from west to east of the surface plot of figure 8 and this supports the folding pattern suggested by Avbovbo *et al.*(1986). Avbovbo, *et al.*, (1986) described that the folds have their axis along north-east to south-west direction, and an observation of the surface plot of the basement depths (Figure 8) indicates the existence of a trough at the center of the map that is flanked by the two crests to the west and east of the map.

Table 3: The summary of the results indicating the regression equation obtained for each subsection.

SUBSECTION	REGRESSION EQUATION
SUB01	β = -2.22d + 5.48
SUB02	β = -2.52d + 6.88
SUB03	β = -1.69d + 4.40
SUB04	β = -2.53d + 8.26
SUB05	β = -1.79d + 4.69
SUB06	β = -1.42d + 4.07
SUB07	β = -2.64d + 5.68
SUB08	β = -1.75d + 4.62
SUB09	β = -2.16d + 5.57
SUB10	β = -1.55d + 4.38
SUB11	β = -2.08d + 5.76
SUB12	β = -1.61d + 4.02
SUB13	β = -2.04d + 5.56
SUB14	β = -2.23d + 5.09
SUB15	β = -1.94d + 5.75
SUB16	β = -2.32d + 5.93
SUB17	β = -1.88d + 4.80
SUB18	β = -2.32d + 6.32
SUB19	β = -2.42d + 5.76
SUB20	β = -1.97d + 6.26
SUB21	β = -2.17d + 5.58
SUB22	β = -2.42d + 5.39
SUB23	β = -2.72d + 5.69
SUB24	β = -2.72d + 7.13

Table 3 continued

SUB2[illegible]	[illegible]	β = -2.00d + 5.32
SUB2[illegible]	[illegible]	β = -1.99d + 5.57
SUB2[illegible]	[illegible]	β = -2.77d + 6.48
[illegible]	[illegible]	β = -1.77d + 4.22
[illegible]	[illegible]	β = -1.15d + 3.14
[illegible]	[illegible]	β = -1.23d + 3.52
[illegible]	[illegible]	β = -1.46d + 3.24
[illegible]	[illegible]	β = -1.56d + 4.25
[illegible]	[illegible]	β = -2.19d + 5.54
[illegible]	[illegible]	β = -2.53d + 8.39
[illegible]	[illegible]	β = -1.55d + 4.70
[illegible]	[illegible]	β = -1.79d + 5.21
[illegible]	[illegible]	β = -1.75d + 5.25
[illegible]	[illegible]	β = -1.78d + 4.71
[illegible]	[illegible]	β = -1.98d + 6.35
[illegible]	[illegible]	β = -2.19d + 4.34
[illegible]	[illegible]	β = -1.88d + 5.23
[illegible]	[illegible]	β = -1.88d + 4.99
[illegible]	[illegible]	β = -2.78d + 6.99
[illegible]	[illegible]	β = -1.93d + 6.02
[illegible]	[illegible]	β = -3.11d + 6.98
[illegible]	[illegible]	β = -2.38d + 5.53
[illegible]	[illegible]	β = -2.23d + 6.61

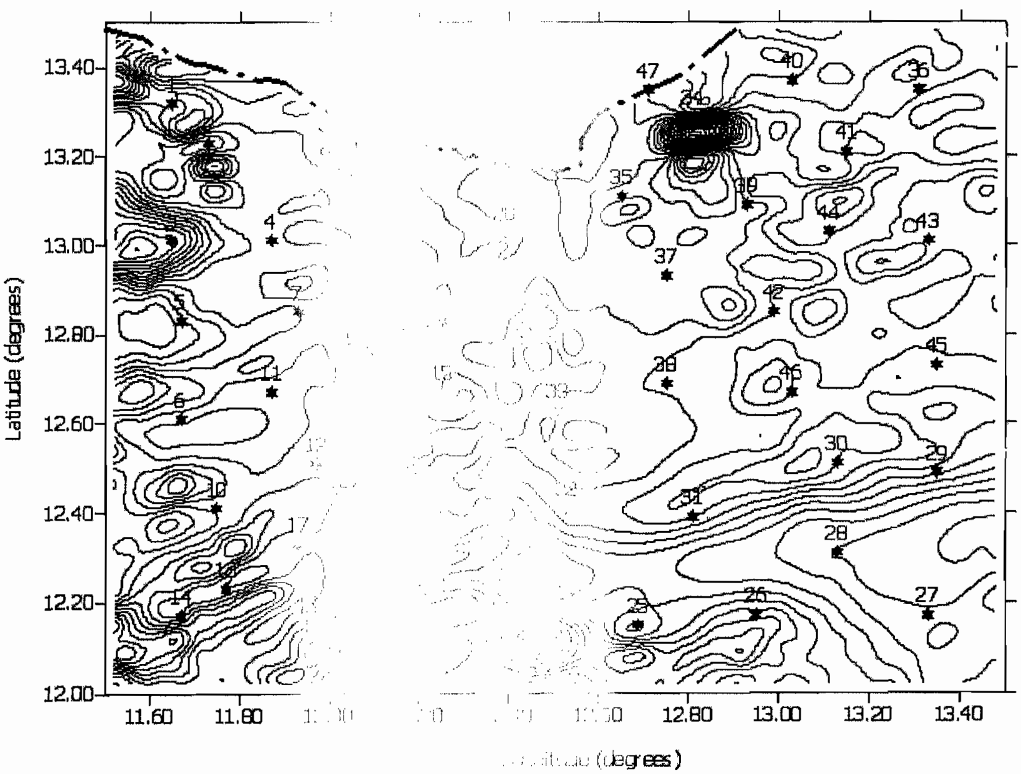

Fig. 6: The distribution of [illegible] centers of each subsection or window.

Table 4: The summary of the depths and scaling exponents obtained from the white spectra of each subsection.

Subsection	Depth to magnetic source (km)	Source scaling exponent .()
SUB01	2.87	1.58
SUB02	3.50	2.86
SUB03	4.64	3.73
SUB04	4.13	3.11
SUB05	3.99	2.81
SUB06	4.80	2.83
SUB07	3.24	3.88
SUB08	3.83	2.41
SUB09	3.99	3.69
SUB10	4.21	2.32
SUB11	3.83	2.79
SUB12	4.09	2.79
SUB13	3.78	2.72
SUB14	3.63	3.70
SUB15	4.37	3.20
SUB16	4.02	4.16
SUB17	3.79	2.75
SUB18	3.86	3.40
SUB19	3.50	3.55
SUB20	4.31	2.73
SUB21	3.93	3.60
SUB22	2.94	2.56
SUB23	2.94	3.37
SUB24	3.45	3.32
SUB25	3.68	2.56
SUB26	4.17	3.24
SUB27	2.71	2.13
SUB28	3.81	2.87
SUB29	4.55	1.97
SUB30	4.48	2.86
SUB31	3.66	2.21
SUB32	4.77	3.38
SUB33	3.73	3.29
SUB34	4.23	3.24
SUB35	5.27	3.65
SUB36	3.89	2.11
SUB37	5.13	4.06
SUB38	5.08	4.69
SUB39	5.40	4.85
SUB40	3.71	4.45
SUB41	3.42	1.63
SUB42	3.77	2.53
SUB43	2.96	2.35
SUB44	4.59	3.31
SUB45	2.47	2.07
SUB46	3.08	2.61
SUB47	4.33	3.74

Hydrocarbon accumulation is usually determined by the thickness of the sediments of the basin, and also by the kind of geological structures existing within the basement that forms traps for the oil and gas. If the folding pattern of the basement within the area of study is extended outwards and towards the east, one would expect to obtain another trough around the Lake Chad area. This is an indication that greater thickness of sediments around the Lake Chad area, was probably the reason for the hydrocarbon accumulation discovered in the area. The maximum depth to basement of 5.40 km occurring within the trough in the area of study might probably be large enough to produce pressure for hydrocarbon formation and can therefore be a possible sight of hydrocarbon accumulation.

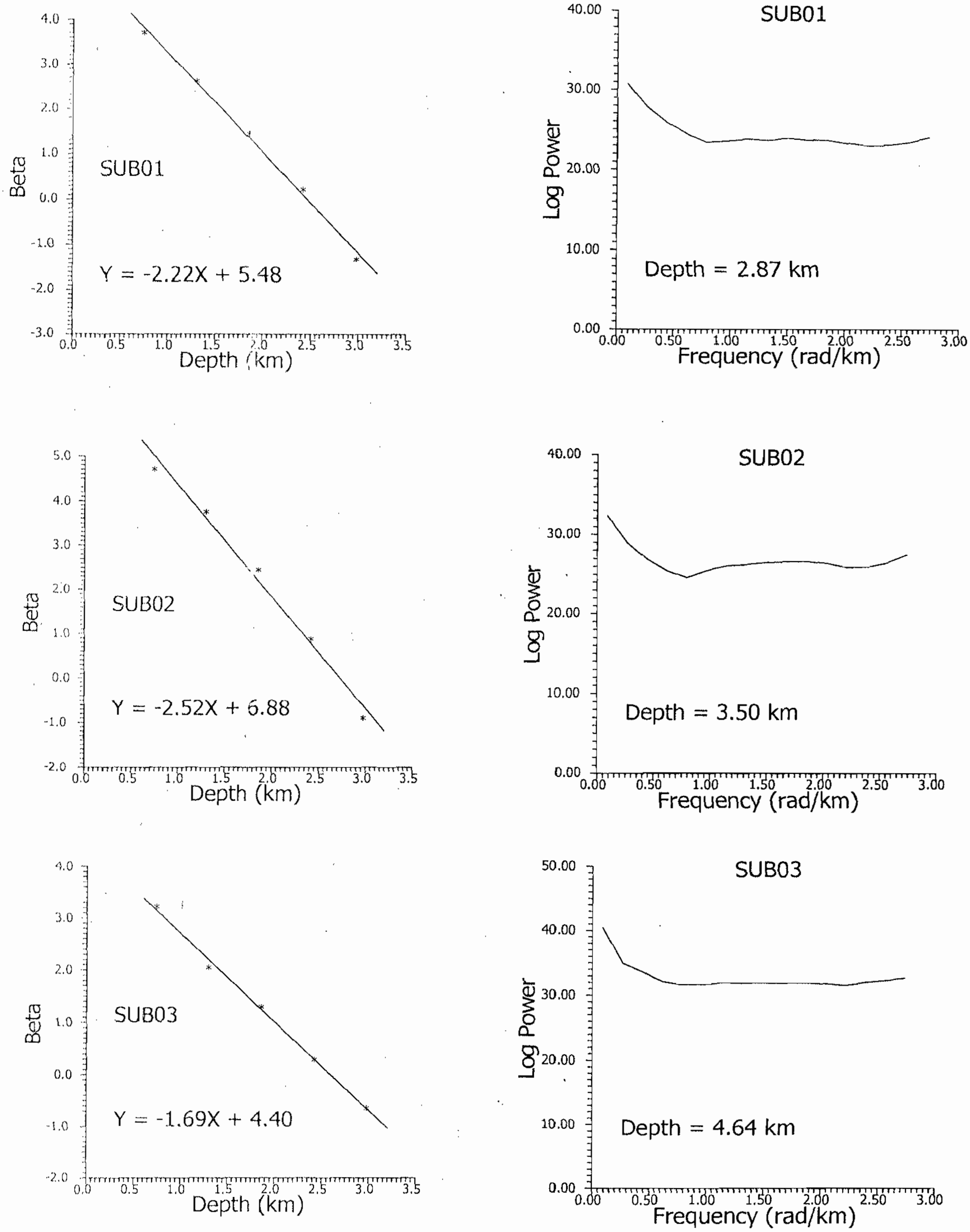

Fig. 7: Graphs of the variation of beta with the depths of continuation with their corresponding white spectra for SUB01 to SUB03 in the regression equation Y and X represents b and depth respectively.

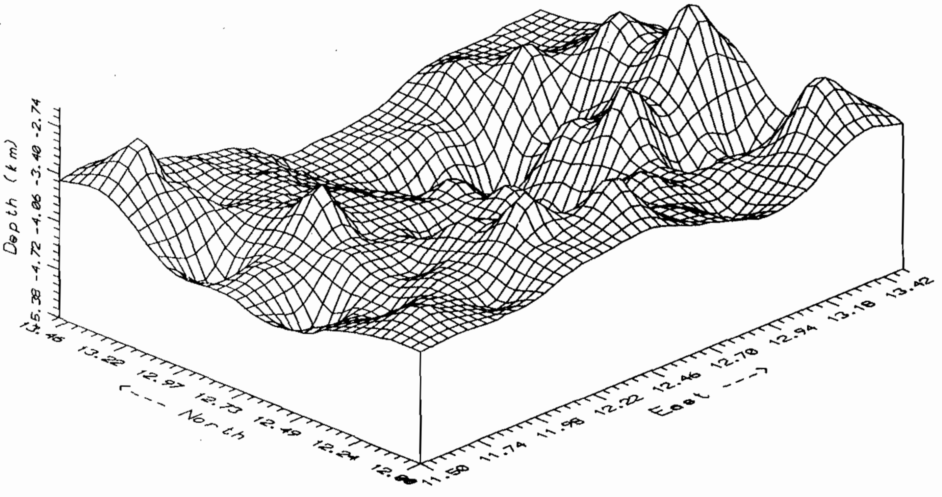

Fig. 8: Surface plot of the basement relief of the area of study.

4.0 Conclusion

The concept of fractals and three techniques of estimating depth from the power spectrum of magnetic field have been discussed and then applied to the aeromagnetic data over the Chad basin of north-eastern Nigeria. The techniques discussed are the Spector and Grant (1970) model, the Fedi *et al.,* (1997) model and the Maus and Dimri (1995) model. Therefore in this work the variation of scaling exponent with depth was investigated and it was discovered that the variation is linear. The result of depth to magnetic sources obtained from the area of study indicated that the basement depth varies from 2.47 km to 5.40 km with an average of 3.92 0.66 km, and it was also observed that the basement relief generally reveals a folding pattern.

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