



AN ONLINE MODEL FOR ASSESSING STUDENTS' UNDERSTANDING WITH STEPWISE SOLVING OF CALCULUS QUESTIONS

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Abstract

Information Technology (IT) is a pivot of new phase lift to educational system. It has reformed the mode of learning on the part of learners from passiveness to activeness that promote mental reform and skills. In all learning, one of the ways to ascertain the level of students understanding in a given concepts network is to provide either formative or summative assessment. Hence, in this paper, an online-based model is developed for assessing students' understanding with stepwise solving of calculus questions. The system presents the students' understanding as it relates to retention and recall on those inter-concepts that they need to know.

Keywords: Information Technology, educational system, students' understanding, calculus questions

1. Introduction

With the rapid development of the Internet and the World Wide Web (WWW), Information Technology (IT) has gone beyond communication. It is now used as a means of research and findings, which has given rise to online learning. Education is one of the sectors that most benefited from the current technological advancement [1].

E-learning is referred to as the intentional use of networked information and communications technologies in learning and teaching [9]. The use of new multimedia technologies and the Internet in learning is seen as a means to improve accessibility, efficiency and quality of learning by facilitating access to resources and services as well as remote exchanges and collaboration [5] [4].

Mathematics involves observing, representing and investigating patterns in social and physical phenomena and within mathematical relationships. Mathematics offers ways of thinking, structuring, organizing and making sense of the world [3].

According to [2], Mathematics is credited with the potential to solve everyday problems which gives one the ability: to learn and apply rules, to understand that a problem can be solved in more than one way; to learn to see relationships among different entities; and to learn to know that specific operations are needed in solving a

problem. Michael and Smith [7] argued in favour of horizontal integration of Mathematics and Science with a view to enhancing learners' potentials. They recommended stepwise problem solving approach with the view to helping learners realize the connections or relationships within what they learn and perceive in their field of study in a broader perspective. The authors argued further that the approach improves learners' comprehension, synthesis and translation that lead to an integrated knowledge base, promotion of understanding, enhancement of problem solving abilities and self-directed learning.

Calculus as a topic in mathematics serves as a basis for mathematical modelling and representations. In the secondary school syllabus, it is a topic studied under the further mathematics, which makes it officially more "dreaded by students". With this attitude most students often do not learn in details the basis of this course before moving on to higher institutions where they all are assumed to be prepared to an extent. This system would help to bridge this distance by helping them learn early in a familiar environment which they can always remember as the years go by and as the need for complexity arises.

Assessment based on computer mediated is a form used to obtain the level of students comprehension on the instructional content

delivered based on a particular domain or discourse. From the literature, it is either assessment is designed separate from the learning tasks or they are integrated together in realistic situations. What is paramount is that the learning tasks and instructional content must be structured to obtain user mediated activities such as behavioral perspective or internal thought process as it relates to domain knowledge acquisition. Assessment provides an effective tool for communicating educational expectations and progress toward accomplishing those expectations. Assessment and learning are two sides of the same coin. As students engage in an assessment exercise, they should learn from it as well. This would improve student learning through effective feedback and enable teachers adjust their teaching strategy. Assessment can be described in form of diagnostic, formative and summative.

The emphasis of this paper is to develop an online-based model for assessing students' understanding and skills as they gainfully engaged in stepwise solving of calculus questions.

2. Information Technology in Nigerian Education and E-Assessment

The development of telecommunications in Nigeria began in 1886 when a cable connection was established between Lagos and the colonial office in London. Telephones especially mobile cells, were seen as a symbol of status and achievement [10]. IT applied to education enhances the delivery and access to knowledge, and improves the curriculum. It produces richer learning outcomes compared to education without IT. It encourages critical thinking and offers unlimited means of achieving educational goals [4]. The goals of IT in education should embrace these four approaches; emerging, applying, infusing and transforming approaches. It is estimated that 90% of Nigerian educational institutions are in the emerging phase, 7% in the applying phase, and 3% in the infusing and transforming phases, therefore, IT is still in its infancy in Nigeria [4].

Learning in an electronic environment is a great challenge in Nigeria because of so many factors [5]. Iloanusi[14] discussed extensively some of the major factors such as lack of technical and building infrastructure; and unavailability of steady and continuous power supply. The e-learning techniques mostly adopted by most of the Nigerian institution are in form of prepared

lectures on a CD-ROM that can be played as at when the need arises. This has limited advantage because of the number of students per computer system in which most of this facilities are not interactive enough as compared with when the lecture is been received in real time over the internet [10]. Despite all these hindrances and threats faced, government and private bodies are evolving to fund the implementation of IT in the Nigerian educational institutions at all levels (basic to tertiary institutions). Their statistics is however very low as most of the schools and institutions have still not seen any reason for the implementation of IT in their learning curriculum [4]. Studies have shown that computer attitudes are a strong predictor of performance and evaluation of computer literacy courses, while some other studies have shown that the use of computer in education has the potential of changing students' attitudes positively towards mathematics and computers [1].

Computers and electronic technology offer a wide spectrum to enrich educational assessment both in the classroom and in large-scale testing situations. Computer-based assessment vastly expands testing possibilities beyond the limitations of traditional paper-and-pencil tests. Through these and other technological innovations, the computer-based platform offers the potential for high quality *formative* assessment that can closely match instructional activities and goals, make meaningful contributions to the classroom, and perhaps offer instructive comparisons with large scale or summative tests [12].

The acceptance of computer-based assessment in learning by teachers and students in Nigerian schools is becoming widespread. The employers are conducting aptitude test for their job seekers through electronic means; the universities and other tertiary institutions are registering and conducting electronic examination for their students through the internet and other networking gadgets [11]. Various examination bodies in the country like the West Africa Examination Council (WAEC), National Examination Council (NECO), National Board for Technical Education (NABTEB), National Teacher Institute (NTI) register their students through electronic means, recently electronic examination had been widely adopted by nearly all the Nigeria University for post Unified Tertiary and Matriculation Examination (Post-UTME) otherwise called pre-admission screening [11].

Most of the e-assessment conducted in all these is in form of multiple-options questions but in this paper, students are presented with questions that must be answered in stepwise solving of calculus problems.

3. Mathematics and E-Learning

Technology-mediated learning is the umbrella term used in this section to describe E-learning as the learning-mediated environment through the use of technology where learners interact with learning materials, peers, and/or instructors to enhance knowledge and performance [15]. Early technology-mediated learning made use of primitive devices such as paper, pen, chalkboard and textbook to process and deliver mathematics education and other forms of learning. The advances in technology revolution opened up the use of mechanical devices, radio and television technologies such as audio-visual instructional movement, radio and television broadcast educational programme. The emergence of Information and communication technologies brought about E-learning and other universally acceptable terms that describe this concept. The resultant effect of these technologies has enabled E-learning to be facilitated through weblog, webcasts, screencasts, podcasts etc. The introduction of mobile and communication technologies enabled mobile E-learning solutions that characterized with seamless and pervasive communication.

Mathematics is way of describing relationships between numbers and other measurable quantities [6]. It serves as a basis for scientific and engineering inventions and discoveries. Most students are however math-phobic due to its complex look at first glance, and it is often times caused by poor pedagogy techniques. Incorporating e-learning into mathematics is a means of enhancing student-teacher interaction and to ensure that the right knowledge alone is passed down to the students. Teaching mathematics this way has however had some challenges. When instructors begin teaching online, they do not understand the differences between the online environment and the classroom [13]. But with adequate reinforcements and encouragements students are likely to get more involved in learning mathematics.

Most mathematical-learning applications do not require a step-by-step contribution from the student; rather only the final answers are

required to be chosen from different options of answers. This could not promote the reasoning skills of students which is part of mathematics phenomenon. An example is mathXL's teaching mathematics online as illustrated by [13] as shown in Figure 1.

This would however not be efficient for students in secondary schools as the steps to the final answer of the solutions are graded along with its final answer. Emphasis has been placed on reinforcements through multimedia, but it would not be complete without assessing the student activities as the course syllabus requires. Students have to be adequately supported during learning process and graded within the scope of the syllabus, simulating all sorts of students' activities that the instructors in traditional classroom settings obtain in visual cues from students through body language and voice expressions [13].

4. System Design

This section presents the mathematical modelling, the system architecture of the model and the system modules.

4.1 System Architecture

The conceptual view of the system consists of three major components known as model-view-controller architecture as depicted in Figure 2.

A. Model Component of the System

The model consists of application data, business rules, logic, and functions. It is responsible for the storage and modification of data into the database in an Entity- Relationship format which consists of the following:

User: The attributes of the user model are - email, username, password, surname, firstname and tutor. The user type is of Boolean data type which helps to differentiate a student-user from a tutor user.

Topic: The topic model has attributes - topic id and topic name.

Question: The question model contains the question text which holds a single question at a time. The relationship between the topic and question model is one-to-many, that is a topic "has many" questions while a question "belongs to" a topic

Answer: The answer model contains the question-id, with which it is related to the question model in a one-to--one scheme, and a student-id which relates to the student accessing

the questions to hold the input answers from the student.

Step: The step model contains the step-name, which holds a solution step to a question, step-score, which holds the score to each step-answer, and hint, which holds the hint associated with each step's answer.

B. View Component of the System

The view is responsible for generating the display of each model in the user interface, normally based on data in the model. Each user (whether student, tutor or administrator), communicates with the system via the view. Questions displayed to the students and the report viewed by the tutor is all rendered by the view.

C. Controller Component of the System

The controller mediates input, output and converts it to commands for the model or view. It

provides the user with input by arranging for relevant views to present themselves in appropriate places on the screen. It provides means for user output by presenting the user with menus or other means of giving commands and data. The controller receives such output translates it into the appropriate messages and pass these messages on to one or more of the views.

4.2 Mathematical Modelling of the System

The mathematical modelling technique was adopted from [16] and adapted into this paper. This presents the problems and students response to solving the problems in step-by step solution process. The mathematical notations of the model are described as shown in equations (1) – (3).

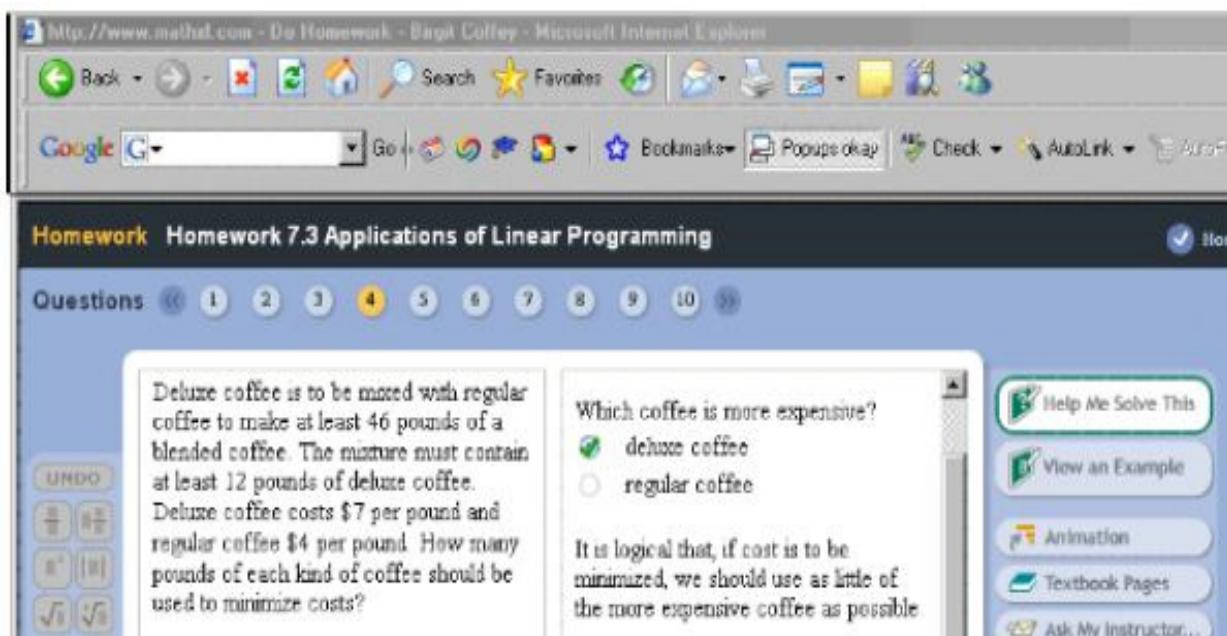


Figure 1: A sample page from MathXL's teaching mathematics online [13]

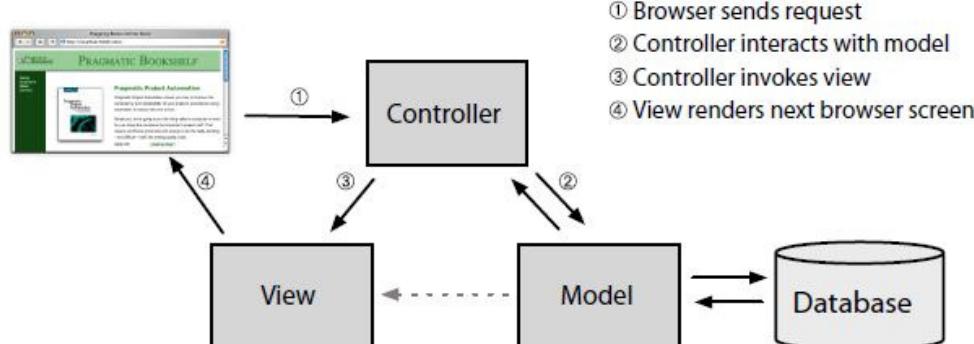


Figure 2: Model-View-Controller of the System [8]

There exist a problem, call it Prob (well-formulated in a mathematical sense such as differential equations and other properties) and defined a set, call it Σ to find all solution process to the problem, and defined condition, call it C , that the only solution elements satisfy in Σ . A solution is an element in Σ that satisfies the condition:

$$C(s) = \text{Prob}(\Sigma, C) \quad (1)$$

where equation (1) represents an implicit description of the solution set.

$$\text{Soln}(\Sigma, C) \quad (2)$$

where equation (2) represents an explicit description of the solution set obtained by the solution process; therefore, there exists:

$$\text{Prob}(\Sigma, C) = \text{Soln}(\Sigma, C) \quad (3)$$

as sets where Σ and C define the problem Prob in equation (3);

then let

$$\text{Prob}(\Sigma, C) = \{s \in \Sigma : C(s)\} \subseteq \Sigma \quad (4)$$

where equation (4) is the solution set during the solution process where the process is to obtain an explicit description of $\text{Soln}(\Sigma, C)$. Check the condition $C(s)$ for the steps of solution elements $s \in \Sigma$ to determine if it is indeed a yes or no

solution to the problem.

Assign 1 mark to each yes solution steps/process to the problem that spans up to maximum 5 marks solution step.

4.3 System Modules

The system modules comprise of the home page module, the registration module, the administrative module, the tutor module and the student module.

A. Home Page module

The system is accessed through the home page as shown in Figure 3 with a log-in interface for authentication and verification from where user is redirected to the main components of the system.

B. Registration Module

Registration module is the process through which users are allowed to register their personal information such as email, username, password, surname and firstname as depicted in Figure 4. Before any user can access the system, s/he must be registered which further provides log-in interface for authentication and verification.

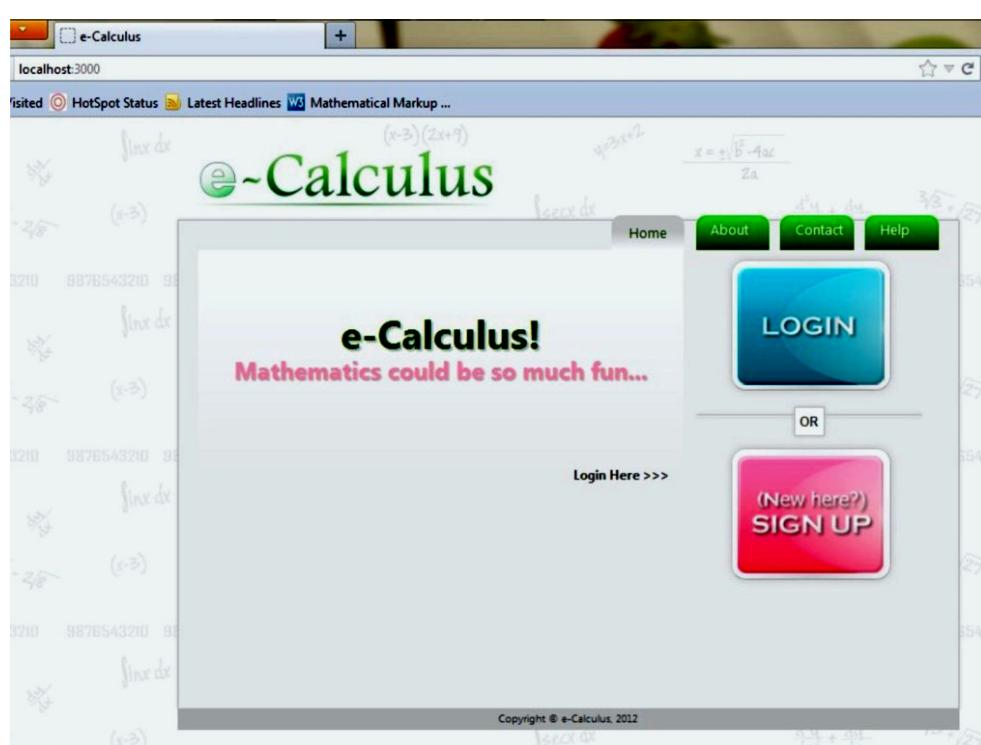


Figure 3: Home page [8]

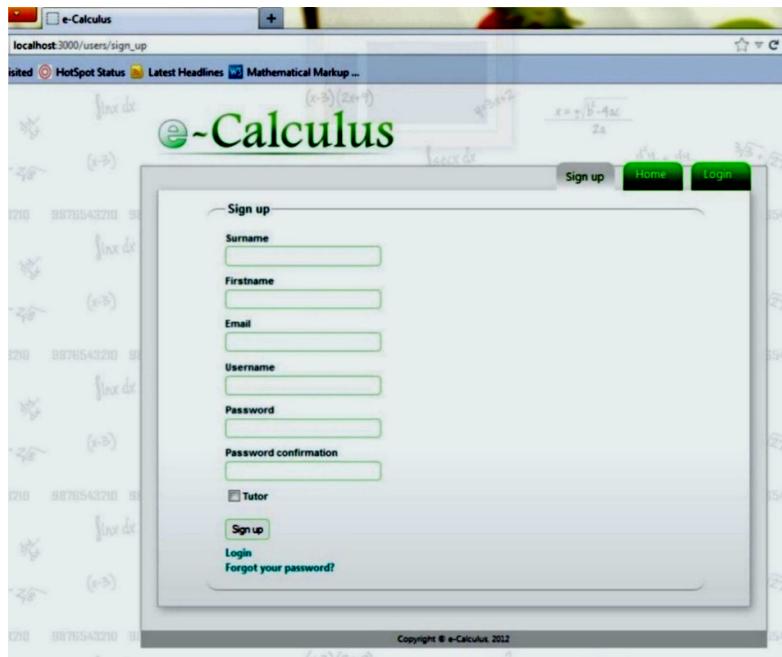


Figure 4: Registration page for users [8]

What differentiates a student and a tutor when signing up is the tutor attribute which accepts input in form of a check box. If tutor is checked, the user is registered as a tutor, but if it is unchecked, the user is registered as a student.

showing the index view of the Topics 'Questions, steps-to-Answers model

D. Student Module

The students access the system after signing in with a registered email address and password. The students can actively engage to solve problems in step by step approach to facilitate their reasoning ability. The link to the tutorial page where the questions in which the students are to answer are displayed, one question at a time in step-wise solution with respect to each topic as shown in Figure 8

C. Administrative Module

The administrative module provides an interface where the tutors are given the privilege to load their questions and their corresponding answers as subject to the appropriate topic. It provides the right to delete and add questions; and determines the way the questions are to be viewed by the students. All the views in Figures 5 to 7 are accessible through the administrative part

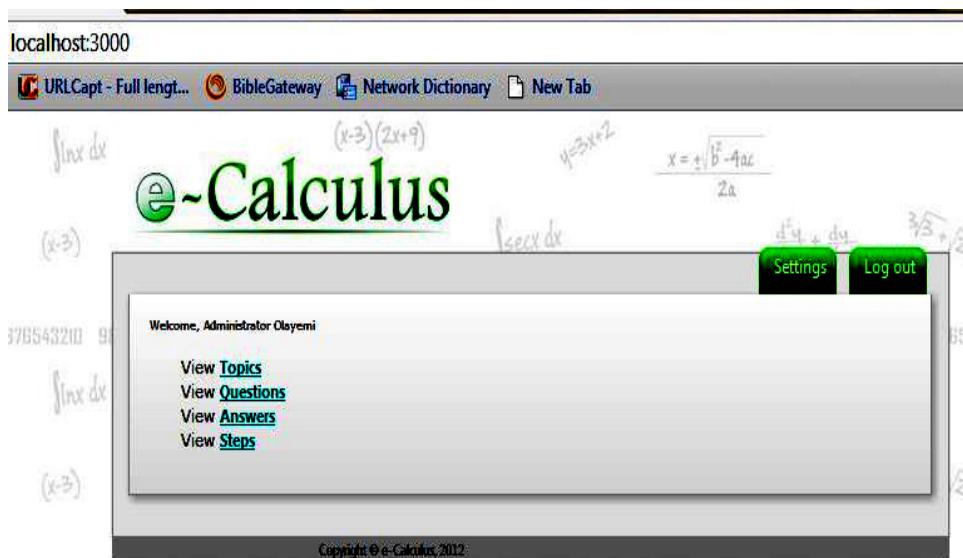


Figure 5: Administrative page [8]

The screenshot shows a web browser window with the URL localhost:3000/questions. The page title is "e-Calculus". The main content area is titled "List of questions" and contains a table of questions categorized by topic:

Topic	Question text	Action Buttons
Limits	$\lim (x^2)-1$ as $x \rightarrow 2$	Edit Answer Show Edit Destroy
Limits	$\lim (2x^2) - 4x$ as $x \rightarrow 3$	Edit Answer Show Edit Destroy
Limits	$\lim 15x^3 - 3x$ as $x \rightarrow -2$	Edit Answer Show Edit Destroy
Limits	$y = (x^2-1)/(x-1)$, Find $\lim y$ as $x \rightarrow 1$	Edit Answer Show Edit Destroy
Limits	$\lim (x^2-9)(x+1)/(x-3)$ as $x \rightarrow 3$	Edit Answer Show Edit Destroy
Derivatives	$y = 2x^2 - 4x$, Find dy/dx	Edit Answer Show Edit Destroy
Derivatives	$y = 3x^2 + 5x - 6$, Find y'	Edit Answer Show Edit Destroy
Derivatives	$y = x^4 + 5x^3 + 4x^2 + 7x - 2$, Find dy/dx at $x = 2$	Edit Answer Show Edit Destroy

Figure 6: Index view of the Topics' Question model [8]

The screenshot shows a web browser window with the URL localhost:3000/steps. The page title is "e-Calculus". The main content area is titled "List of steps" and contains a table of steps:

Answer Id	Step answer	Step score	Hint	Action Buttons
1	$(2^2)-1$	2	substitute 2 in x	Show Edit Destroy
1	4-1	1	Calculate 2^2	Show Edit Destroy
1	3	1	Subtract 1 from 4	Show Edit Destroy
2	$2(3^2) - 4(3)$	2	substitute 3 in x	Show Edit Destroy
2	$2*9 - 12$	1	Expand brackets (3^2) and $4(3)$	Show Edit Destroy
2	18-12	1	Multiply 2 by 9	Show Edit Destroy
2	6	1	Subtract 12 from 18	Show Edit Destroy
3	$15*(-2^3) - 3(-2)$	2	substitute -2 in x => $15*(-2^3) - 3(-2)$	Show Edit Destroy
3	$15*(-8) + 6$	2	expand brackets (-2^3) and $-3(-2)$	Show Edit Destroy
3	-120+6	1	Multiply 15 by -8	Show Edit Destroy
3	-114	1	Calculate $6 - 120$	Show Edit Destroy
4	$(x-1)(x+1)/(x-1)$	3	Factorize $x^2 - 1 \Rightarrow (x-1)(x+1)$	Show Edit Destroy

Figure 7: Index view of the Stepwise-Answers model [8]

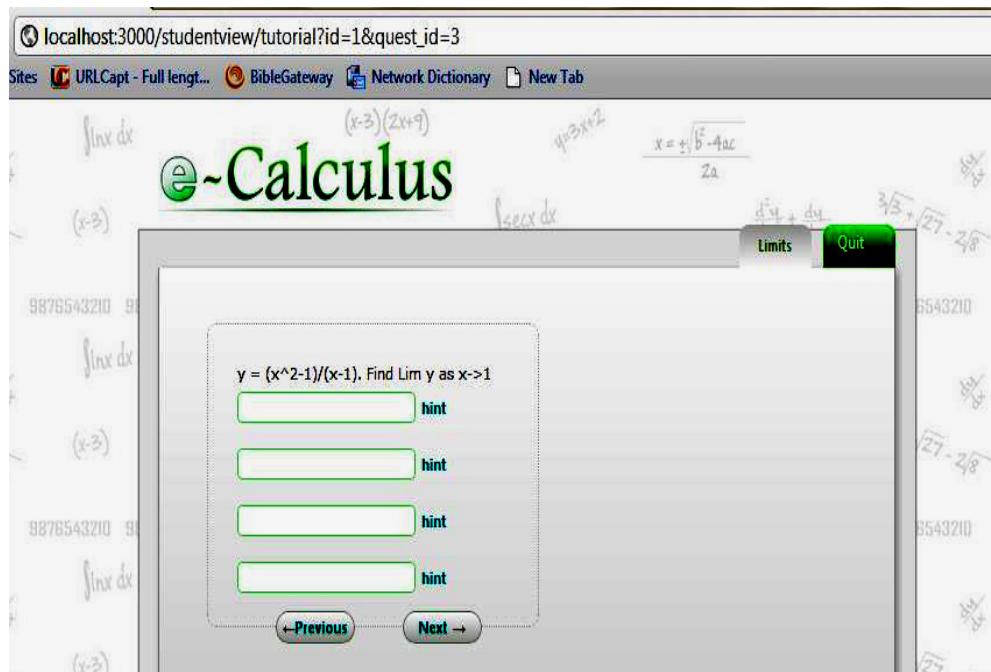


Figure 8: A Stepwise Solving of a limit question[8]

E. Tutor Module

A tutor can access the system with administrative privilege to monitor the activities of the students that are actively engaged in the learning process. The tutor page displays the list of five students and their usage history as they actively engaged in the learning process through stepwise solving approach as shown in Figures 9 – 13. The activities of the students are displayed with respect to the number of questions attempted and answered; and the average time spent on each question. The number of questions attempted for each student is the number of

questions each student visits. The answered questions depict the number of questions the student answered with corrected and uncorrected steps. The average time spent is the function of the mean of all the time spent on each question on the tutorial. The overall score of the students are depicted in Figure 14, which is a sum of all scores derived from each question, where each question score is a sum of the solution steps that were answered correctly. A maximum score of five (5) marks is obtainable from each question with a minimum of one (1) step and a maximum of five (5) steps.

The screenshot shows a web-based application titled "e-Calculus". The main window displays a table titled "Moses Aanuoluwapo's Profile" with the following data:

Topics	Total No. of Questions	No. of Questions attempted	No. of Questions answered	Avg. time spent on a question
Limits	10	10	9	2.5 minutes
Derivatives	10	9	5	3.7 minutes
Integration	1	0	0	0

To the right of the table is a sidebar titled "List of students" containing the names: Moses Aanuoluwapo, Tomiwa Wisdom, Jagun Ize, and Akinyosoye Oluwatosin. At the top of the window, there are buttons for "Student", "Back", "Help", and "Log out".

Figure 9: The first Student report page [8]

The screenshot shows a web-based student report interface. At the top right, there are green buttons for 'Back', 'Help', and 'Log out'. Below them is a sidebar titled 'List of students' containing four entries: 'Moses Aanuoluwapo', 'Tomiwa Wisdom' (which is highlighted in blue), 'Jagun Ize', and 'Akinyosoye Oluwatosin'. The main content area is titled 'Tomiwa Wisdom's Profile' and contains a table with the following data:

Topics	Total No. of Questions attempted	No. of Questions answered	Avg. time spent on a question
Limits	10	8	2.3 minutes
Derivatives	10	4	3.2 minutes
Integration	1	0	0

Figure 10: The second Student report page [8]

The screenshot shows a web-based student report interface. At the top right, there are green buttons for 'Back', 'Help', and 'Log out'. Below them is a sidebar titled 'List of students' containing four entries: 'Moses Aanuoluwapo', 'Tomiwa Wisdom', 'Jagun Ize' (which is highlighted in blue), and 'Akinyosoye Oluwatosin'. The main content area is titled 'Jagun Ize's Profile' and contains a table with the following data:

Topics	Total No. of Questions attempted	No. of Questions answered	Avg. time spent on a question
Limits	10	4	1.3 minutes
Derivatives	10	3	3.7 minutes
Integration	1	0	0

Figure 11: The third Student report page [8]

The screenshot shows a web-based student report interface. At the top right, there are green buttons for 'Back', 'Help', and 'Log out'. Below them is a sidebar titled 'List of students' containing four entries: 'Moses Aanuoluwapo', 'Tomiwa Wisdom', 'Jagun Ize', and 'Akinyosoye Oluwatosin' (which is highlighted in blue). The main content area is titled 'Akinyosoye Oluwatosin's Profile' and contains a table with the following data:

Topics	Total No. of Questions attempted	No. of Questions answered	Avg. time spent on a question
Limits	10	5	2.2 minutes
Derivatives	10	7	2.6 minutes
Integration	1	1	2.4 minutes

Figure 12: The fourth student report page [8]

Topics	Total No. of Questions	No. of Questions attempted	No. of Questions answered	Avg. time spent on a question
Limits	10	8	5	3.6 minutes
Derivatives	10	5	5	2.9 minutes
Integration	1	1	0	1.6

List of students

- Moses Aanuoluwapo
- Tomiwa Wisdom
- Jagun Ize
- Akinyosoye Oluwatosin
- Dada Chrtstianah

Figure 13: The fifth Student report page [8]

Student Name	Tutorial Score
Moses Aanuoluwapo	55
Tomiwa Wisdom	54
Jagun Ize	52
Akinyosoye Oluwatosin	72
Dada Chrtstianah	49

List of students

- Moses Aanuoluwapo
- Tomiwa Wisdom
- Jagun Ize
- Akinyosoye Oluwatosin
- Dada Chrtstianah

Figure 14: Overall Students report page [8]

5. Conclusion

In this paper, an online-based model was developed to track the activities of the students as they solve calculus questions in mathematics. This was carried out in order to obtain the level of their understanding in the concepts network of calculus. The questions are tackled by the students in stepwise manner supported with various hints to provide steps to the solution. The system was tested with five students and the outcome of students' result showed that the system performs better for the desired designed objective. Further research should focus on

extending the scope to other topics in the mathematics subject.

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