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AN INVESTIGATION INTO THE DECISION MAKERS'S RISK ATTITUDE INDEX RANKING TECHNIQUE FOR FUZZY CRITICAL PATH ANALYSIS

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ABSTRACT
In this paper, the effectiveness of the decision maker's risk attitude index for fuzzy critical path analysis is considered. The fuzzy CPM algorithm was implemented in Visual Basic .Net and used to obtain the project's activities times as well as the total float of project activities. Numerical computation reveals that for a benchmark problem, the decision maker's risk attitude index ranking method produces unrealistic results when the decision maker's attitude towards risk was neutral. However, a modified problem derived from the benchmark problem produced realistic results. The study shows that the utility of the ranking technique may be limited by the risk index associated with the problem. Therefore users of the technique for ranking fuzzy numbers have to proceed with caution.

Keywords: *fuzzy, decision maker, risk attitude, critical path, total float ranking*

1. INTRODUCTION

The project manager is usually faced with a challenging job of successfully managing projects. The complexity associated with project management is increased when the activities in the project have uncertain duration. When the activity times in the project are deterministic and known, critical path method (CPM) [1] has been demonstrated to be a useful tool in managing projects in an efficient manner to meet this challenge [2]. In some projects however, the activity durations cannot be stated in a precise manner. Two classical techniques based on the use of probability theory, namely, the Monte Carlo Simulation technique as well as the Program Evaluation and Review technique (PERT)have been used to address the problem of imprecise activity duration data in project analysis. The applicability of PERT however requires that a basic assumption namely that the three times estimates of the activity duration follow a beta distribution. A number of drawbacks have been identified and documented to be associated with the use of PERT. The estimated probability obtained using PERT is sometimes far from the real implementation probabilities [3]. Additionally, since most activities are being executed for the first time, there exist no previous data based on which the beta distribution criteria can be established. Consequently, the project manager has to state activity duration in a vague form using linguistic language.

The problem of ordering fuzzy numbers plays an important role in decision making under fuzzy environment [4].The use of fuzzy number ranking technique has found useful application in data analysis, artificial intelligence, economic analysis as well as Operations Research [5]. In the fuzzy critical path analysis, fuzzy number ranking technique is used for obtaining the event earliest and latest times in the project. The ranking of fuzzy numbers is a very challenging task since the membership functions representing two fuzzy numbers may overlap. When membership functions overlap occurs, the decision becomes very tricky. A number of researchers have proposed different techniques for ranking fuzzy numbers. The different techniques can be classified into four groups, namely, preference relation, fuzzy mean and spread, fuzzy scoring and linguistic expression. Liou and Wang [6] presented a ranking method based on integral value index. Chu and Tsao [7] presented a method for ranking fuzzy numbers based on the area between the centroid point and original point.

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A number of the existing ranking approaches have inherent drawbacks. Some of the methods are unable to discriminate between two different fuzzy numbers thereby making decision making impossible or complicated. In this paper, we study the Decision makers risk attitude index fuzzy ranking technique for fuzzy critical path analysis of project network with triangular fuzzy durations.

2. FUZZY FUNDAMENTALS

Let R be the space of real numbers. A fuzzy set [8] of numbers is a set of ordered pairs $\left\{\left(x, \mu_{\tilde{A}}(x)\right) \middle| x \in R\right\}$, where $\mu_{\tilde{A}}(x) : R \rightarrow [0,1]$ and is upper semi continuous. $\mu_{\tilde{A}}(x)$ is called the membership function of the fuzzy set. A convex fuzzy set is a fuzzy set in which Eq. (1) and (2) hold

$$\forall x, y \in R, \forall \lambda \in [0,1] \tag{1}$$

$$\mu_{\tilde{A}}\left(\lambda x + (1-\lambda) y\right) \geq \min\left[\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\right] \tag{2}$$

A fuzzy set A is called positive if its membership function is such that $\mu_{\tilde{A}}(x) = 0, \forall x \leq 0$.

A triangular fuzzy set A is a convex fuzzy set which is defined as $A = \left(x, \mu_{\tilde{A}}(x)\right)$ where

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} & a \leq x \leq b \\ \dfrac{c-a}{c-b} & b \leq x \leq c \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The triangular fuzzy set A is given by the set of numbers (a,b,c) where $0 \leq a \leq b \leq c$

3. FUZZY ARITHMETIC

Let $A = (a_1, b_1, c_1)$ and $B = (a_2, b_2, c_2)$ be two flat triangular fuzzy sets. The basic fuzzy arithmetic operations namely, fuzzy addition, fuzzy subtraction, and fuzzy multiplication are:

$$A \oplus B = (a_1 + a_2, b_1 + b_2, c_1 + c_2) \tag{4}$$

$$A \Theta B = (a_1 - d_2, b_1 - c_2, c_1 - b_2) \tag{5}$$

$$A \otimes B = (a_1 a_2, b_1 b_2, c_1 c_2) \tag{6}$$

However, fuzzy subtraction as described in Eq (5) can result in negative fuzzy numbers which are meaningless in fuzzy environment. Consequently, a modified fuzzy subtraction recursive algorithm is used. As an illustration let the result of the fuzzy subtraction of two fuzzy numbers defined previously be as shown in Eq. (7) l_1, l_2, l_3 are computed as shown in Eqs (8) – (10)

$$A \Theta B = (l_1, l_2, l_3) \tag{7}$$

$$l_3 = Max(0, c_1 - c_2) \tag{8}$$

$$l_2 = Max\left(0, Min(l_3, b_1 - b_2)\right) \tag{9}$$

$$l_1 = Max\left(0, Min(l_2, a_1 - a_2)\right) \tag{10}$$

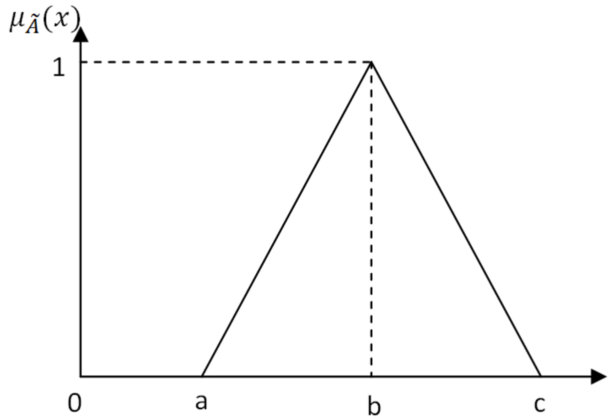

Figure 1: A triangular fuzzy number

4. DECISION MAKER'S RISK ATTITUDE INDEX

Consider a project consisting of a number of activities. The Decision maker's risk attitude index gives an indication of the degree of optimism, pessimism or neutrality of the decision maker.

The Decision Maker's risk attitude index β can be obtained by

$$\beta = \left[\begin{array}{l} \displaystyle\sum_i \sum_j \frac{a_{ij} - b_{ij}}{(a_{ij} - b_{ij}) + (c_{ij} - b_{ij})} \\ A_{ij} \in ACT \end{array} \right] / t \tag{11}$$

where ACT denotes the set of all activities and t denotes the number of activities in a project network.

The ranking index of a fuzzy number $A = (a_i, b_i, c_i)$ is then determined using the value of β in Eq. (12):

$$R(A_i) = \beta \left[\frac{d_i - x_1}{x_2 - x_1 - b_i - d_i}\right] + (1-\beta)\left[\frac{1 - (x_2 - c_1)}{x_2 - x_1 + a_i - c_i}\right] \tag{12}$$

where, $x_1 = \min\{a_1, a_2, \ldots, a_n\}$ and $x_2 = \max\{c_1, c_2, \ldots c_n\}$

Consider two fuzzy numbers A_1 and A_2, we rank the fuzzy numbers with the following rules:

$R(A_1) > R(A_2)$ then $A_1 > A_2$

$R(A_1) < R(A_2)$ then $A_1 < A_2$

$R(A_1) = R(A_2)$ then $A_1 = A_2$

5. FUZZY CRITICAL PATH METHOD

The procedure for fuzzy critical path method uses the concept of fuzzy ranking technique described in the previous section for obtaining the fuzzy earliest start (ES), fuzzy latest start (LS), fuzzy earliest finish (EF), fuzzy latest finish (LF) and event times (E) of the activities in the project network. The forward pass computation produces the fuzzy earliest start, fuzzy earliest finish as well as the event earliest times of project activities. The backward pass produces the fuzzy latest finish, fuzzy latest start and event latest times of project activities. We adopt the following notation for a fuzzy project network. t_{ij} is the duration of an activity between nodes i and j where $i < j$. The fuzzy earliest start of an activity with no predecessor $(p(i) = \phi)$ is (0, 0, 0). The earliest finish time of the activity is given by:

$$EF_{ij} = ES_{ij} + t_{ij} \quad (13)$$

The earliest event time of the starting node of the project is given by (0,0,0). For a node j having a number of predecessor node $(p(i) \neq \phi)$ i, the earliest event time of node j (E_j) is given by

$$\tilde{E}_j = \max_{i \in p(j)} \left[\tilde{E}_i \oplus \tilde{t}_{ij} \right] \qquad i \in p(j) \neq \phi \quad (14)$$

The earliest finish time of an activity between nodes i and j is computed after knowledge of the earliest event time of node i using Eq. (14). The earliest finish time is computed using the expression

$$EF_{ij} = E_i + t_{ij} \quad (15)$$

The fuzzy completion time of the project (T) is given by the Eq. (16), where s(j) denote the set of successor node to node j.

$$T = Max\left[EF_{ij}\right] \ \forall \ i,j : s(j) = \phi \quad (16)$$

The fuzzy event latest times of the terminal node in the project network is equal to the fuzzy project duration computed in Eq. (16). The fuzzy latest start time of an activity with terminal node j and starting node i having S(j)=0 is computed using the expression

$$LS_{ij} = T - t_{ij} \quad (17)$$

The fuzzy latest finish time of an activity with $S(j) \neq \phi$ is given by the expression

$$LF_{IJ} = Min\left[LS_{ij}\right] \ \forall \ i,j : s(j) = \phi \quad (18)$$

Flow charts depicting the solution methodology are shown in Figures 2 and 3.

6. NUMERICAL EXAMPLES

In this section, the methodology presented in previous sections for fuzzy CPM analysis is used to solve two different problems

6.1 Numerical Example 1

Consider the project network by Kumar and Kaur [1] whose precedence relations and activity durations are presented in the Table 1.

The fuzzy CPM analysis algorithm depicted in Figure 2 has been coded in VB.net by the authors and was used to solve the numerical example.

The solution obtained is shown in Table 2.

Table 1: Project Activity data for numerical example 1

Activity	Predecessor	Fuzzy Duration
A	-	2,4,6
B	-	9,13,17
C	A	7,9,11
D	A	12,19,26
E	B,C	6,10,14

The critical activity is D since its total float is (0,0,0). If the duration of activity D is increased, the project completion time will be increased beyond the current project completion time which is (14, 23, 32)

Table 2: Solution for Numerical example 1

Activity	Duration	EST	LST	EFT	LFT	TF
A	2,4,6	0,0,0	0,0,1	2,4,6	1,4,7	0,0,1
B	9,13,17	0,0,0	0,0,1	9,13,17	8,13,18	0,0,1
C	7,9,11	2,4,6	1,4,7	9,13,17	8,13,18	0,0,1
D	12,19,26	2,4,6	2,4,6	14,23,32	14,23,32	0,0,0
E	6,10,14	9,13,17	9,13,18	15,23,31	14,23,32	0,0,1

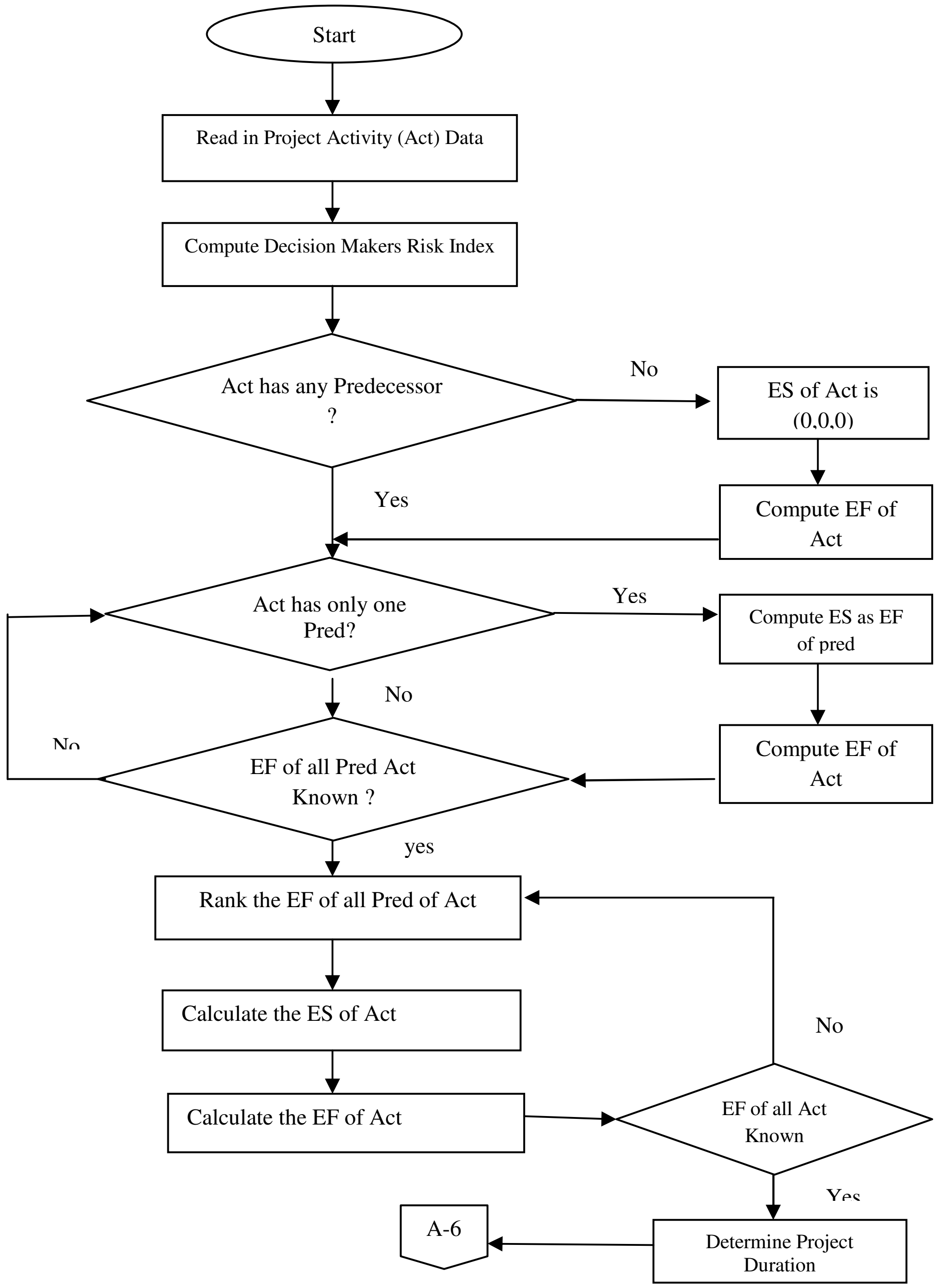

Figure 2: Flowchart of solution methodology

6.2 Numerical Example 2

Consider the following modification to the problem solved by Kumar and Kaur [1] in numerical example 1. The project consists of nine (9) activities. The precedence relations as well as activity durations are presented in Table 3.

A critical activity is one whose total float (TF) is (0,0,0). Its duration must not be increased if the project completion time is to be maintained. From Table 4, it can be seen that two activities namely, A and G have a total float of (0,0,0). Hence, the critical activities are A and G.

Table 3: Project Activity data for numerical example 2

Activity	Predecessor	Fuzzy Duration
A	-	2,4,6
B	-	3,5,6
C	-	1,2,3
D	B	6,8,11
E	A	7,9,11
F	D,E	1,2,2
G	A	12,19,26
H	D,E	6,10,14
I	D,F	1,2,2

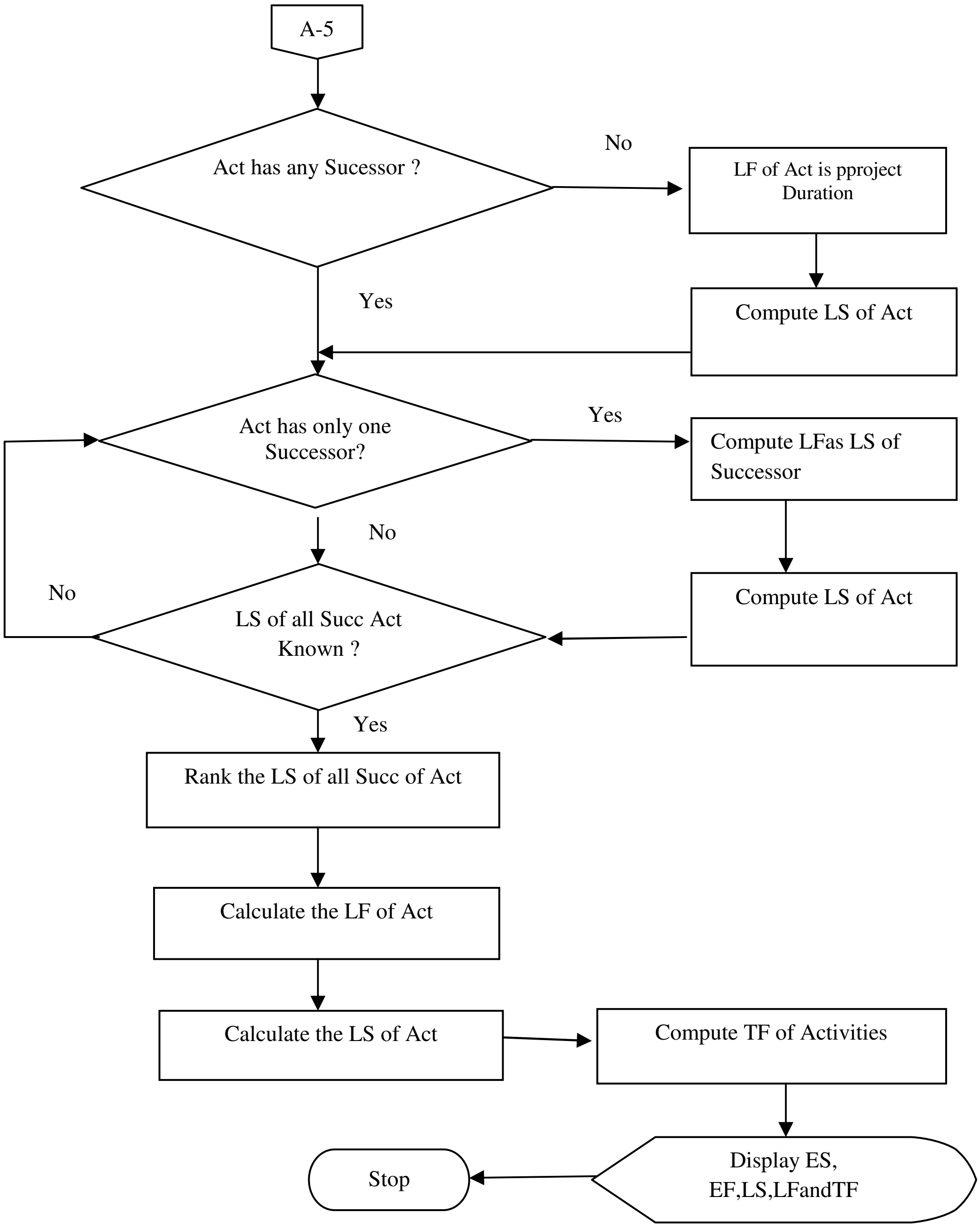

Figure 3: Flowchart of solution methodology

7. DISCUSSION OF RESULTS

The decision makers risk attitude index ranking technique has been used for the fuzzy CPM analysis of two different project networks. The results obtained are quite interesting and will be highlighted hereunder. It can be seen from Table 2 that only activity D has a total float equal to 0,0,0. The implication of this is that it is difficult to identify a critical path in the project network. This is unrealistic from the practical point of view. In numerical example 2 however, it can be seen that two activities namely A and G both have a total float equal to 0,0,0. Consequently, a unique critical path namely A-G is obtained. From the results obtained from both problems using the decision makers risk attitude index, it would appear that the method breaks down under certain circumstances as illustrated using numerical example 1. The problem with numerical example 1 lies in the determination of the latest finish time of Activity A during the fuzzy backward pass. The latest finish time of activity A is the minimum of the latest start times of activities C and D which are 1,4,7 and 2,4,6 respectively. In the determination of the latest finish time of activity B in numerical example 2, we are also faced with a similar problem. The latest finish time of activity B is the minimum of the latest start times of activities F and G which happens to be 1,4,7 and 2,4,6 just like the case of numerical example 1. However, in numerical example 1, the latest finish time of activity A was determined to be 1,4,7 whereas in the case of numerical example 2, the latest finish time of activity B was found to be 2,4,6 using the same ranking technique. It would appear from the results that the same ranking technique ranks two distinct fuzzy numbers differently under different conditions. Numerical computations reveal that the risk attitude index (β) of the problem in numerical example 1 is equal to 0.5 (neutrality) while the risk attitude index (β) for the problem in numerical example 2 is equal to 0.6185

8. CONCLUSION

The Decision makers risk attitude index has been used for fuzzy critical path analysis of two problems with triangular representations of activity duration. It has been shown that in one of the cases considered, the Decision maker's risk attitude index ranking technique produces conflicting results when used to rank two similar fuzzy numbers. Consequently decision makers using the technique have to exercise some caution in its use as misleading or unrealistic results may be produced.

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Table 4: Solution for numerical example 2

Activity	Duration	EST	LST	EFT	LFT	TF
A	2,4,6	0,0,0	0,0,0	2,4,6	2,4,6	0,0,0
B	3,5,6	0,0,0	0,0,1	3,5,6	2,5,7	0,0,1
C	1,2,8	0,0,0	12,19,27	1,2,3	13,21,30	12,19,27
D	6,8,11	3,5,6	2,5,7	9,13,17	8,13,18	0,0,1
E	7,9,11	2,4,6	1,4,7	9,13,17	8,13,18	0,0,1
F	1,2,2	9,13,17	12,19,28	10,15,19	13,21,30	3,6,11
G	12,19,26	2,4,6	2,4,6	14,23,32	14,23,32	0,0,0
H	6,10,14	9,13,17	8,13,18	16,23,31	4,23,32	0,0,1
I	1,2,2	10,15,19	13,21,30	11,19,21	4,23,32	3,6,11