THE SEQUENCE OF WET AND DRY DAYS AT IBADAN AND ONNE (SUB-HUMID ZONE OF NIGERIA)

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ABSTRACT

Twenty-three years daily rainfall data were collected for Ibadan while twenty years data were collected for Onne. These data were analysed and modelled for the dependence of wet days following dry days; dry days following wet days; dry days following dry days and wet days following wet days using the first order Markov chain rule. The results were produced as monthly and decadal transition probability of dry days, wet days and their dependence, and this was used to model the dependence of wet and dry days in the study area. This model was tested using Ten years daily rainfall record each of Akure and Calabar for the sub-humid and humid areas and was found to perform well in its predictability. The study shows that, for the purpose of water resources planning as well as agriculture operations in the two area under study and for most humid and sub-humid stations, there is need to take into consideration the state (i.e. wetness or dryness) of the previous 2 day especially in the rainy months.

1.0 INTRODUCTION

Most water resources planning programme and agricultural acnvities particularly (crop production) are dependent on the availability or non-availability of rainfall. The absence or presence of rain at certain times determines the degree of success or failure of these processes. Major farm operations or farm management strategies depend on the current weather, and even the weather of the next few hours [1]. Analyses of historical rainfall data have been found to assist in understanding the pattern of rainfall distribution and occurrence. Such analyses have been carried out on yearly monthly, decadal, weekly and daily basis in several studies. Sivakumar et al [1], however noted that for agricultural purposes, the daily analysis produces the optimal result. Analyses of daily rainfall analysis have equally being approached from various perspectives, which include temporal distribution and pattern, total amount, intensity and spatial coherence. These analyses are applied in communication study, civil and water works and also in agricultural planning but the dependence of the dry and wet day sequence poses a greater challenge to agricultural production. A careful and systematic examination of historical rainfall records has been found to assist in the understanding of the dependence of wet and dry days, rainfall amount and their temporal distribution, planning and management of rainfall dependent events such as rainfed and irrigated agriculture etc. [2, 3]. Studies on rainfall dependence are not totally new. The Poisson point process model has been severally used to model different aspect of rainfall process including; the arrival of storm events. [4, 5], rainfall depth, and distribution of dry periods [6] and flood size and frequency of occurrence. [7], just to mention a few. However due to the statistical dependence in storm arrival due to clustering in time and space, the Neyman- Scott point process was later proposed. This model was a clear improvement of the Poisson model [8] and was based on Discrete Autoregressive Moving Average (DARMA) process. One major limitation of these models was the inability to allow for an explicit representation of the different types of rainfall e.g. convective or frontal. This differentiation is however important because to these different types corresponds different relations between the depth and the duration statistics, convective cell being more intense and shorter frontal one for instance [9].

Similarly using the Markovian chain analysis method, Daniel [10] derived a stochastic dependence model for daily precipitation and this was used to fit dry, near normal and wet subset of monthly precipitation using 30 forecasted value rainfall values. Foufoula and Dennis [11] developed a Markov renewal model to predict the occurrence of rainfall. This model is a clear deviation from the DARMA model often criticised of lacking physical motivation and continuous memory. Fasheun [12] used the Markov chain rule to model the monthly rainfall data for Ibadan and reported an exponential relationship between frequency of dry spell and their duration. His result was proposed for farm operation planning in Ibadan and other sub- humid zone of Nigeria. Oladipo and Kwaghsaa [13] studied the dependence of wet and dry days at 3 stations in the Savannah area of Nigeria using the first order Markovian model. The adequacy of the model was tested using X_2 test procedure and they concluded that a higher degree of Markov chain (i.e. Second order and above) is necessary for accurate prediction of rainfall dependence in the Savannah areas.

The Markov chain model has been widely adopted for modelling because of its useful application to real life problem and the ease of interpretation of result for a very clear understanding. Another major plus for the Markov model application is the ease of application to available daily total record in most developing nations. Most other rainfall prediction models require the use of rainfall intensity data, which are very scarce in most synoptic data stations of the developing nations.

This paper examines sequence of wet and dry days in Ibadan and Onne (sub-humid and humid station in Nigeria) using the first order Markov chain model. The result is produced as monthly transition probability that shows the month-to- month variation-in probability of the sequence of wet and dry days and their dependence.

2.0 MATERIALS AND METHOD

2.1 Description of the Study Areas

Ibadan and Onne constitute the area under study and are both located in the southern part of Nigeria. Ibadan lies on latitude 7°30’ and longitude 3° 54' while Onne lies on latitude 4°43' N and longitude 7°1'E. The mean annual rainfall for Ibadan and Onne are 1250 mm and 2400 mm respectively, Ibadan is classified under the sub- humid zone while Onne is classified in the humid tropical zone. Other agro-ecological features and weather characteristics of both sites are presented in Table 1 [14, 15].

Table 1: Summary. of Agro-ecological and Weather Characteristics of Ibadan and Onne

Agro-ecological features	Ibadan	Onne
Longitude	3^o 54 E	7^o 01 E
Latitude	7^o 30 N	4^o 43 N
Climatic zonation	Sub humid	Humid
Rainfall pattern	Bimodal	Unimodal
First rainy season	April July	March-November
Second rainy season	August-November	March-November
Soil type	Ferric luvisols	Thionoc Fluvisols
Vegetation	Mild hot farmland	Swamp farmland
Maximum temperature	30.5^{o}C 3.5	29.5^{o}C 1.5
Minimum temperature	21 ^{o}C ± 1.5	22.5 ^{o}C ± 1.5
Mean annual rainfall	1250 mm	2400

Source: Jagtab & Alabi [15]

The choice of Ibadan and Onne for the study was informed by the availability of long-term. reliable daily meteorological data and the representation of two different zones of the humid tropics, which is the scope of this present study.

2.2 Data Source and validation

Available meteorological data for the two stations at Ibadan (7°30N, 3°54'E) and Onne (4°43'N,7°1'E) were obtained from the weather station of the International Institute for Tropical Agriculture, Ibadan, Nigeria. Twenty-three (23) years data were collected for Ibadan (1974-1996), and twenty (20) years data were collected for Onne (1977-1996). The meteorological parameters collected on the daily basis include: rainfall, pan evaporation, wind speed, minimum and maximum air temperature, solar radiation and minimum and maximum relative humidity in percentage (%). All the parameters were found to be consistent except the solar radiation; whose measuring unit was changed in Ibadan in 1990 and in 1989 at Onne. There were however, few cases of missing data due to instrument failure. Table 2 shows the type and accuracies of instruments used for the meteorological measurement at both sites.

Data validation was done in the case of single missing data by weighing the surrounding (nearest) data and the use of regression method [16]. Long period missing data were substituted with data of a year with similar trend. This however occurred only once.

Table 2: Instrumentation at both sites and their accuracies

Parameter	Instrument	Accuracy	Unit
Evaporation	U.S. Class A Pan	0.01	Mm
Temperature	Casella Min. & Max Thermometer	0.1	°C
Rel. Humidity	HMP35c Temp/RH Probe	2%	%
Solar Radiation	L1200X Pymanometer	3%	MJ/m
Rainfall	Universal Self-recording Rainguage	0.1	D
Wind Speed	MET-ONE 014A Wind Speed Sensor	1.5%	M/s

2.3. Markov Chain Analysis

The short-term correlation of rainfall incidence of successive days has frequently been modeled with Markov Chain [12, 13]. This method assumes that the occurrence of rainfall or otherwise is dependent on whether the previous was wet or dry. This represents the application of the first order, two states Markovian chain rule. In this study, a day is conventionally defined wet if the rainfall on that day was at least I millimetre(1mm) and dry otherwise [13]. This threshold was chosen since the' case under consideration is on the occurrence of rainfall or dryness, wherein any contribution from 1 mm and above is significant.

If P (X_i /X_{i-1}, X_{i-2}……..X_{i-j}) is the probability that a day i will be wet (or dry) "given that" (/) each days of i-1, i-2… i-j had a particular condition (wet or dry), then the transitional probabilities can be obtained for dry condition (D) as:

$$P(D)_I/D_{i-1}) = \frac{No.of\ days\ day\ i\ and\ day i-1\ were\ dry}{No.of\ days\ the (i-1)th day\ was\ dry} \quad (1)$$

$$P(D_i/D_{i-1,}D_{i-2}/D_{i-3} = \frac{No.of\ days\ day\ i\ and\ day i-1, i-2, i-3\ were\ dry}{No.of\ days (i-1)th, (i-2)th, (i-3)th day\ were\ dry} \quad (2)$$

where

P (D_i) is the initial probability for the sequence and it is the probability that the (i) th day of the year will be dry irrespective of what occurred on the preceding days.

It is given as the relative frequency of dry days such that

$$P(D_i) = \frac{No.of\ years\ day i\ was\ dry}{No.of\ years\ of\ record} \quad (3)$$

In a similar manner, the probabilities of wet conditions (W) can be obtained as:

$$P(W/D_{i-1}) = 1 - P(D_i/D_{i-1}) \quad (5)$$

$$P(D_i/W_{i-1}) = \frac{[P(D_i) - P(D_{i-1})^* P(D_i/D_{i-1}]}{P(W_{i-1})} \quad (6)$$

$$= \frac{[P(D_i) - P(D_{i-1})^* P(D_i/D_{i-1}]}{[1 - P(D_{i-1})]} \quad (7)$$

$$P(W/W_{i-1}) = 1 - P(D_i/W_{i-1}) \quad (8)$$

For the two sites, these relationships were used to compute the various probabilities of the sequence of wet and dry days and the results are presented as monthly transition probability table.

The system assumes that if a day is in state 0 (dry) at time t, the probabilities of that system to be in a state 1 (wet) and state 0 (dry) at time (t+l) are *A* and *1-A*, respectively. Similarly, if the system is in state 1 (wet) at time t, the probability of it to be in state 0 (dry) and state 1 (wet) at time (t+1) are 1-*B* and *B*, respectively. These transition probabilities can be written in matrix array as

$$P = \begin{pmatrix} 1-A & A \\ B & 1-B \end{pmatrix} \quad (9)$$

Which in turn can be summarized as

$$p = p_0(t)^* p_1(t-1) \quad (10)$$

After a sufficiently long period, the system settles down to a steady state in which the state occupation probabilities are independent of the initial conditions. The relationship between the length of the wet *(W)* and dry *(D)* species in this model is defined as

$$L(W) = \frac{A+B}{A} \qquad (11)$$

$$L(D) = \frac{A+B}{B} \qquad (12)$$

and the seasonal cycle C is defined as

$$L(C) = L(W) + L(D) \qquad (13)$$

3.0 RESULTS AND DISCUSSION

3.1 Transition Probability

Using the first order Markov probability equations (1 to 8) on the available twenty three years daily rainfall data for Ibadan and twenty years data for Onne to model the dependence of wet and dry days, the following transitional probability matrix were obtained:

$$P = \begin{pmatrix} 0.82 & 0.18 \\ 0.66 & 0.34 \end{pmatrix}$$

for Ibadan and

$$P = \begin{pmatrix} 0.78 & 0.22 \\ 0.60 & 0.40 \end{pmatrix}$$

for Onnne.

Thus in Ibadan, if Jan 1st of a year is taken as the reference date i.e. day (0) and given January 2nd as a dry day, there is a probability of 0.82 that January 3rd is a dry day. Also given that January 2nd is wet, there is a 0.34 probability that January 3rd will be wet. In Onne, if Jan 1st of a year is taken as the reference date i.e. day (0) and given January 2nd as a dry day, there is a probability of 0.78 that January 3rd is a dry day. Given that January 2nd is wet there is a 0.40 probability that January 3rd will be wet. Interpolation for subsequent days could be done manually or with the use of appropriate software. A major requirement for the successful application of this model is the adequate definition of the number of states desired. This determines the order of the Markov equation to be used. In the present study, a 2 state is desired namely 'Wet' or 'Dry' The interactive equation (2) when resolved gave

$P(3) = (0.75 \;\; 0.25)$ for Ibadan and

$P(3) = (0.78 \;\; 0.22)$ for Onne.

This connotes that the system has settled to a condition of statistical equilibrium after 3 days. That is, the states of occupation probabilities are independent of the initial condition after 3 days.

Table's 3a and 3b show the monthly transition probability for the 2 cities and it clearly reflects the pattern of rainfall distribution in the two Cities. The variation in the number of days to attain equilibrium was observed to coincide with the changes in the season. In the month of July and August in Ibadan, which is often referred to as "little dry season" for example, the system attained stability after 4 days.

Table 3a: Monthly Transition Probability Table for Ibadan (Obtained from the analysis of daily data of (1973- 1996)

Month	$P(D_i / D_{i-1})$	$P(W_i / D_{i-1})$	$P(W_i / W_{i-1})$	$P(D_i / W_{i-1})$
January	0.98	0.02	0.02	0.98
February	0.86	0.14	0.17	0.83
March	0.86	0.14	0.17	0.83
April	0.63	0.37	0.25	0.75
May	0.57	0.43	0.28	0.72
June	0.55	0.45	0.49	0.51
July	0.56	0.44	0.58	0.42
August	0.58	0.42	0.51	0.49
September	0.44	0.56	0.56	0.44
October	0.57	0.43	0.51	0.49
November	0.94	0.06	0.19	0.81
December	0.97	0.03	0.01	0.99

Table 3b: Monthly Transition Probability Table for Onne (Obtained from the analysis of daily data of(Obtained from the analysis of daily data of(1977-1996) (1977-1996)

Month	P(Di/ D_i-1)	peWi/ D_i-1)	P(W_i / W_i-1)	P(D_i / Wi-1)
January	0.95	0.05	0.02	0.98
February	0.86	0.14	0.10	0.90
March	0.75	0.25	0.25	0.75
April	0.57	0.43	0.36	0.64
May	0.46	0.54	0.45	0.55
June	0.47	0.53	0.61	0.39
July	0.44	0.56	0.76	0.24
August	0.37	0.63	0.75	0.25
September	0,29	0.71	0.76	0.24
October	0.45	0.55	0.51	0.49
November	0.75	0.25	0.28	0.72
December	0.92	0.08	0.30	0.70

$P(W/Wi\_l)$ = Probability ofa wet day following a wet day;
$P(W/D_{i\_l})$ = Probability of a dry day following a wet day;
$P(D/W_{i\_l})$ = Probability of a wet day following a dry day.

The transition probability of dry day following dry day increased from 0.57 in October to January at Ibadan and in Onne 0.45 in October to 0.95 in January after which it decreased to 0.55 and 0.47 in June respectively. This is a reflection of the seasonal patterns of rainfall distribution in the sub-humid and humid climatic zones of Nigeria. This corroborates the findings of Fasheun [12].

3.2 Probabilities of Dry and Wet days

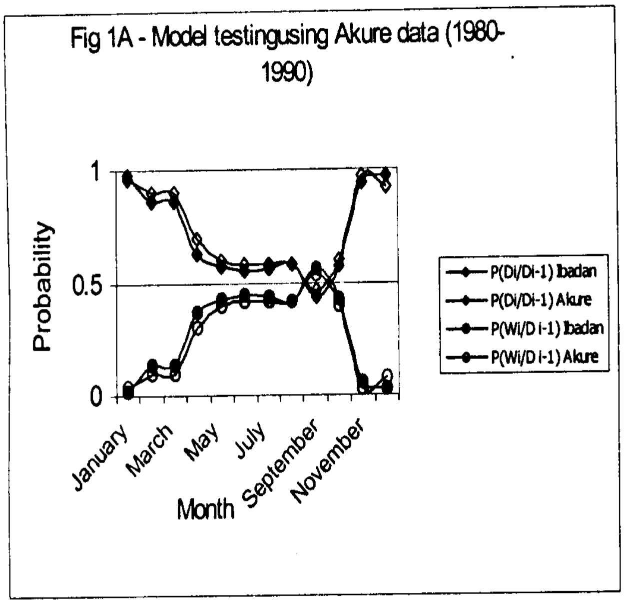
The transitional probability of a wet day $P(W_i)$ following a wet day $P(W/W_{i-1})$, a wet day following a dry day $P(W/D_i\text{-}1)$, a dry day following a dry day $P(D/D_i\text{-}1)$ and a dry day following a wet day $P(D/W_i\text{-}1)$ for the 2 study area equally shown in tables 3a and b. In Ibadan, most of the months had lower than 50% probability ($p<0.5$) of being wet. The number of months having the probability greater than 50% ($p>0.5$) of obtaining a wet day after a wet day is low. Remarkably in Ibadan, the months of June and October show this trend, thus it can be concluded that continuous and persistent (every day) rainy events are most likely in these months. The probability of having a dry day following a wet day was very high in the month of September. However, the probability of a wet day following a dry day was more pronounced i.e. $(P(D_i/W_{i-1})) > 0.5)$ for most of the months. In Onne, the probability of a dry day $(P(D/D_{i-1}))$ following a dry day was less pronounced. The probability of a wet day following a dry day was high in most of the months i.e. $(P(W/D_{i-1}>0.5)$ between April and November, this implies that, there were more likelihood of rain event most times of the year. The probability of continuous (daily) rainfall was very high $(P(W/W_{i-1}>0.5)$ between the months of June and October and was at its peak $(P(W/W_{i-1})$, 0.7) between July and September. This implied that rainfall event was most likely on a daily basis in this months.

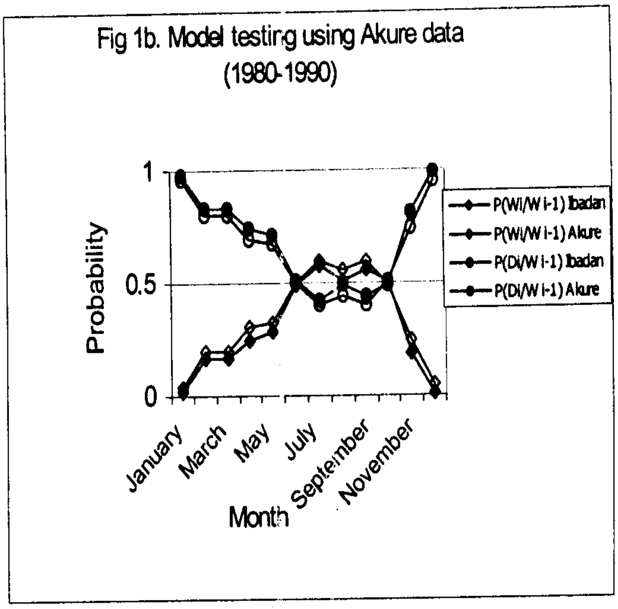
Generally, the likelihood of rainfall events being independent of the previous day condition using initial probability value was higher in Ibadan than Onne. Again, Ibadan showed the classical bi-modal feature of the "little dry season" while the Unimodal feature was shown by Onne.

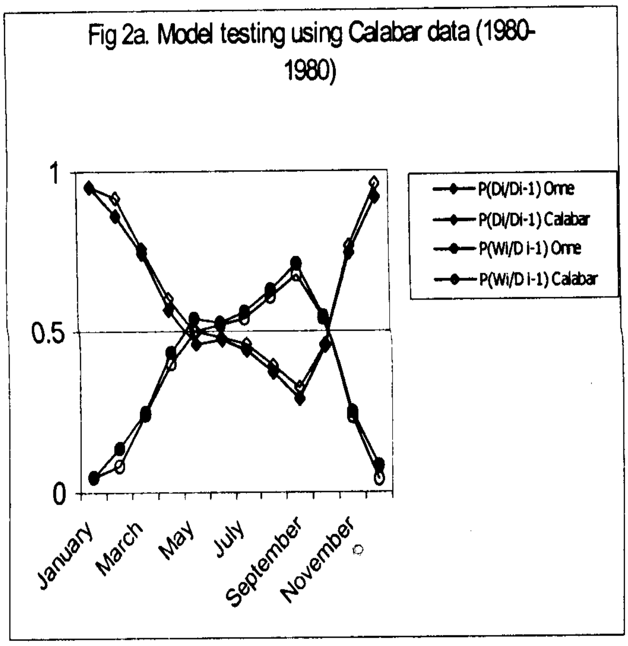
The pattern of the transition probabilities was similar to that obtained from the initial condition, though there were evidences of intra-seasonal fluctuations. Generally, as the rainy season progressed, the probability of a dry being wet was greater when the previous day was wet than if it was dry $[(P(W_i / W_{i-l} > P(W_i / D_{i-l})]$. Conversely, the probability of a day being dry was smaller for a previous dry day than if the previous day was wet $[P(D_i/D_{i-1})<P(W_{i-1})]$. The difference between $P(W_i/W_{i-1})$ and $P(W_i/D_{i-1})$ decreased as the rainy season progressed suggesting that the probabilities of a day being wet became less dependent on the condition of the previous day as the rains became more steady. In the same vein, the difference between $P(D/D_{i-1})$ and $P(W_i/D_{i-1})$ decreased as the rainy season progressed corroborating the observation that the probabilities of having a wet day became less dependent on the condition of the previous day as the rains became more steady. However, the difference between $P(D/D_{i-1})$ and $P(D_i/W_{i-1})$ increased during the rainy season to show that the conditional dependence of dry days was inevitable even during the core of the rainy months of June and October in Ibadan and July to October in Onne.

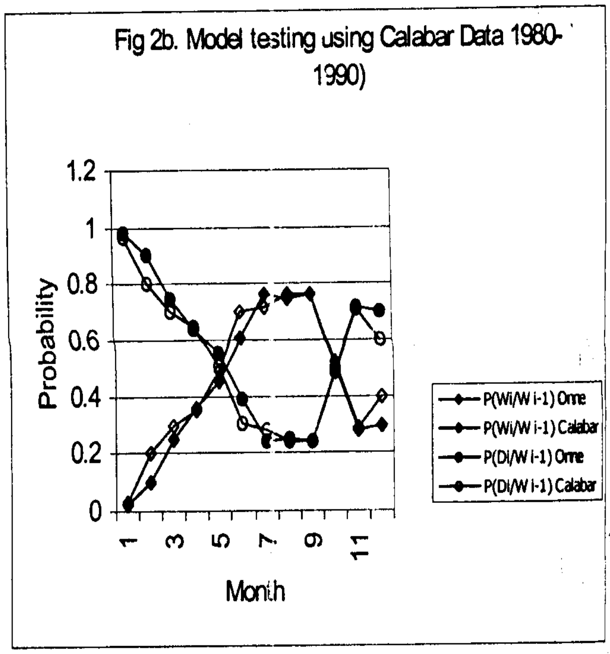
4.0 CONCLUSION

The first order Markovian chain was used to model the dependence of dry and wet days and it shows that the condition of wetness or dryness for a given day have significant influence on the state (wetness or dryness) in the following day and the third day in both humid and sub-humid areas of Nigeria as reflected in the transition probability of the test station, Ibadan and Onne as well as the validation sites Akure and Calabar (Fig 1a, 1b, 2a, 2b). This is a deviation from the result of a similar study by Oladipo and Kwaghsaa [13], for the savannah areas of Nigeria, which shows that there is no significant influence of the state of a previous day on the state in the next day. A higher order Markovian analysis will be needed to investigate the dependence for three to fours days after the initial condition.









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