STEADY STATE AND TRANSIENT ANALYSIS OF INDUCTION MOTOR DRIVING A PUMP LOAD

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ABSTRACT
The importance of using a digital computer in studying the performance of Induction machine under steady and transient states is presented with computer results which show the transient behaviour of 3-phase machine during balanced and unbalanced conditions. The computer simulation for these operating conditions is obtained from the non-linear differential system of equations which describe the symmetrical Induction machine in the stationary reference frame. It is shown that using the characteristic data available from open- and short circuit tests of the machine: accurate simulations of the machine under steady and transient conditions are possible.

1. INTRODUCTION

The steady state mathematical modelling of Induction machines is not new and has received a considerable attention from researchers dated far back as the machine itself [1, 2, 3]. On the other hand, the transient mathematical modelling of induction machines continues to receive enormous attention and will continue to do so because of the vital effect the transient behaviour of the induction machine has on the overall performance of the system to which it forms a component part. Unlike the steady state modelling, transient modelling proves to be more difficult both in the definition of suitable forms of equations and in the application of appropriate numerical methods needed for the solution of same. However, appropriate mathematical transient models for most machine types were found when the generalised d-q axis theory was developed [4] and the space-vector theory evolved[5]. With the advent of digital computers, digital simulation techniques of these models appear most suitable for the analysis because numerical methods must be used, as the resulting differential equations are non-linear. In papers [6] and [7], analog computers have been used to investigate the transient behaviour of induction machine under various modes of operation. It however, has the demerit of not directly applicable to variable frequency operation from a single de input signal. Nath [8] used digital simulation to predict transient speed, current, and electromagnetic torque of three- phase SCR controlled induction motors under run-up conditions. In general, digital- computer solutions of the basic differential equations describing the transient behaviour of the induction motor have been used to predict the performance of the same motors, and good agreement with comparable reduction in simulation time as against analog-computer, has been realized [9, 1 0]. In this paper, the steady state and transient analysis of induction motor driving a pump load is developed and the computer results are presented for the following modes of operation:

(i) balanced conditions and
(ii) unbalanced stator voltages

2.The Induction Motor Model

The induction motor is modelled as an ideal cylindrical-rotor machine with
(i) Uniform air gap
(ii) Saturation, eddy current, temperature effects neglected
(iii)Skin-effect neglected
 (iv)Identical stator windings

The differential equations governing the transient performance of the induction motor can be described in several ways and they only differ in detail and in their

suitability for use in a given application. If the speed of the motor is assumed constant, then the motor differential equations become linear and analytical method can be used in solving for the motor torque and currents. However, where changes of motor speed have to be accounted for, analytical method becomes highly inadequate as the differential equations are non-linear and could only be solved numerically using digital or analog computers. The d-q axis model of the motor provides a convenient way of modelling the machine and is most suitable for numerical solution. This is preferable to the space-vector motor model which describes the motor in terms of complex variables. Figure 1 shows the d-q equivalent circuits for a 3-phase symmetrical induction machine in arbitrary reference frame. The zero-sequence component, Vo is absent in all cases considered in this paper.

From the below induction models, the differential equations describing the dynamic performance of the motor in arbitrary reference frame are given as in [6] with all the rotor parameters referred to the stator. The prime on the referred values have been omitted here for convenience.

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{qr} \\ V_{dr} \end{bmatrix} = \begin{matrix} (R_s + L_s p) & \omega L_s & L_m p & \omega L_m \\ -\omega L_s & (R_s + L_s p) & -\omega L_m & L_m p \\ L_m p & (\omega - \omega_r) L_m & (R_r + L_r p) & (\omega - \omega_r) L_r \\ -(\omega - \omega_r) L_m & L_m p & -(\omega - \omega_r) L_r & (R_r + L_r p) \end{matrix} \tag{1}$$

$$L_S = L_{lS} + L_m \tag{2}$$

$$L_r = L_{lr} + L_m \tag{3}$$

$$p = \frac{d}{dt} \tag{4}$$

The equation used for the prediction of electromagnetic torque is given by [3],
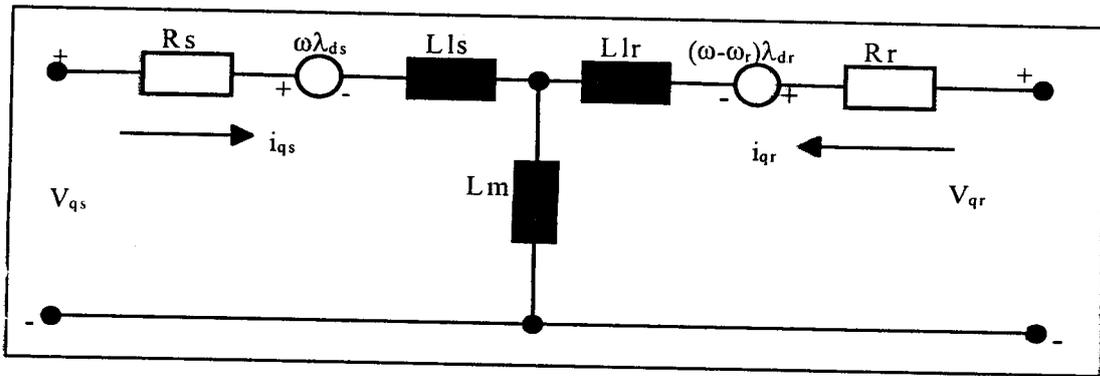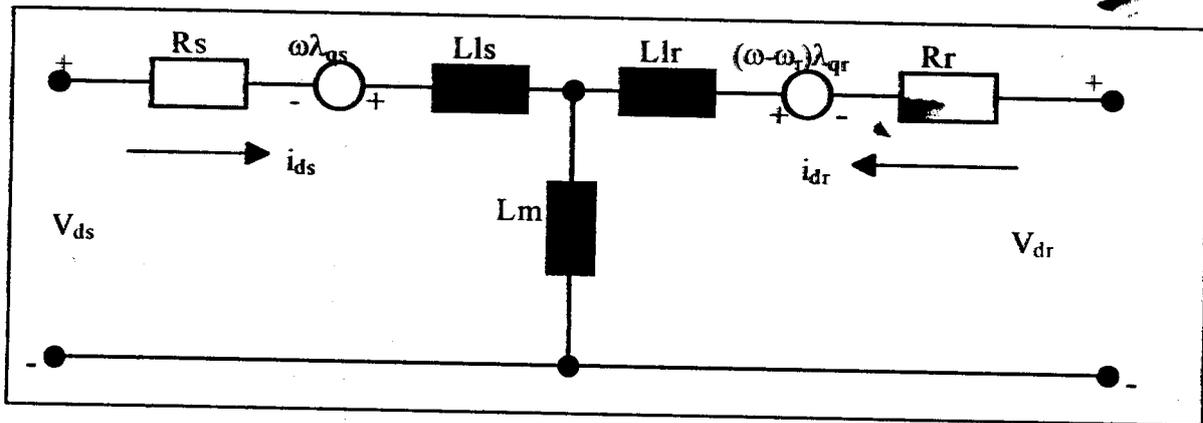
Fig.1a

Fig. 1b

Fig.1 Induction machine models in d-q axis: (a) q-axis model (b) d-axis model

$$T_e = \frac{3}{2}\left(\frac{P}{2}\right)L_m\left(i_{qs}i_{dr} - i_{ds}iqs\right)$$

(5)

And the rotor speed

$$\frac{d\omega_r}{dt} = \frac{P}{2J}(T_e - T_L)$$

(6)

Where,

$$T_e$$

= electromagnetic torque

$$T_L$$

= Applied load torque

J = inertia constant
P = number of poles

The relationship between the actual 3-phase voltage V_{as}, V_{bs} and V_{cs} and the d, q voltages of equation (1) is,

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{os} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix}$$

(7)

The actual 3-phase stator currents can also be obtained by using the inverse transformation to that given in equation (7)
The pump load is assumed to have a torque-sheep characteristic given by

$$T_L = 0.52e^{-0.24} + 1.10e^{-3}(speeds)^2$$

3 Steady State Analysis

The steady state mathematical model equation obtained by equating, all derivative terms in equation (1) to zero and with the machine described in synchronously rotating reference frame(w= w$_e$). By so doing, equation (9) results together with equation(5) which is modified to give equation (10)

$$\begin{bmatrix} U_{so} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s & \omega_e L_s & 0 & \omega_e L_s \\ -\omega_e L_s & R_s & -\omega_e L_m & 0 \\ 0 & (\omega_e - \omega_r)L_m & R_r & (\omega_e - \omega_r)L_r \\ -(\omega_e - \omega_r)L_m & 0 & -(\omega_e - \omega_r)L_m & R_r \end{bmatrix} \begin{bmatrix} i_{qso} \\ i_{dso} \\ i_{qro} \\ i_{dro} \end{bmatrix}$$

(9)

and

$$T_{eo} = \frac{3}{2} \left(\frac{P}{2}\right) L_m \left(i_{qso} - i_{dro} - i_{ds} i_{qro}\right)$$

(10)

where,

i_{qso}, i_{dso}, i_{qro} and i_{dro} are the steady-state currents and w_e, the synchronous speed. Equations (9) and (10) are solved to obtain the steady-state torque-speed characteristic of the motor.

4. Transient State Analysis

The differential equations describing the transient behaviour of the motor in stationary reference frame are obtained by equating ω in equation (1) to zero. Therefore,

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (R_s + L_sP) & 0 & L_mP & 0 \\ 0 & (R_s + L_sP) & 0 & L_mP \\ L_mP & -\omega_r L_m & (R_r + L_rP) & -\omega_r L_r \\ \omega_r L_m & L_mP & \omega_r L_r & (R_r + L_rP) \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix}$$

(11)

Equation (11) can be put in matrix from as follows:

$$[V] = [L]P[i] + \omega_r [G][i] + [R][i]$$

(12)

Where,

$$[V] = \begin{bmatrix} V_{qs} V_{ds} 0 \ 0 \end{bmatrix}^t$$

(13)

$$[R] = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & R_r \end{bmatrix}$$

(14)

$$[L] = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix}$$

(15)

$$[G] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -L_m & 0 & -L_r \\ L_m & 0 & L_r & 0 \end{bmatrix}$$

(16)

$$[i] = \begin{bmatrix} i_{qs} & i_{ds} & i_{qr} & i_{dr} \end{bmatrix}^t$$

(17)

The stator voltages for balanced and unbalanced and unbalanced conditions are represented in equation (18) and equation (19) respectively as:

$$V_{as} = V_m \cos \omega t$$

$$V_{bs} = V_m \cos\left(\omega t - \frac{2\pi}{3}\right)$$

(18)

$$V_{cs} = V_m \cos\left(\omega t + \frac{2\pi}{3}\right)$$

$$V_{as} = V_m \cos \omega t$$

$$V_{bs} = 0.5 V_m \cos\left(\omega t - \frac{2\pi}{3}\right)$$

(19)

$$V_{cs} = V_m \cos(\omega t + \pi/2)$$

5. Digital Representation and Simulation

As can be seen in equation (1), the differential equations involve products of current and current as well as current and speed. As a result, the equations are non-linear anytime the speed varies.

A numerical technique must therefore be applied to solve for the motor speed, currents and torque. For the purpose of digital simulation, equation (11) is represented in state variable form with currents as state variables [11]:

$$P[i] = -[L]([R] + \omega.[G])[I] + [L]^{-1}[V] \qquad (20)$$

One of the contributions of this paper is to show a new tool applied to invert a matrix in a symbolic way. Unlike in [8,9,10], where the authors preferred to invert the matrix L at every integration step, in this paper, in order to minimise the simulation time, a single symbolic inversion of L is carried out so as to obtain an analytical mathematical model of the motor (equation(20)). The symbolic matrix inversion is obtained by means of the software package "Mathematica"[12] as:

$$[L]^{-1} = \frac{1}{L_s L_r - L_M^2} \begin{bmatrix} L_r & 0 & -L_m & 0 \\ 0 & L_r & 0 & -L_m \\ -L_m & 0 & L_s & 0 \\ 0 & -L_m & 0 & L_s \end{bmatrix}$$

$$(21)$$

In order to simulate the induction motor transient model differential equations(6) and (20), MATLAB[13] in-built numerical algorithm, Ode45 , a program that uses Runge-Kutta numerical method, is applied. The linear systems of equations(9) and (10) are also solved to obtain the steady-state speed-torque characteristic of the motor.

6. Simulation results and Conclusions

In order to illustrate the method of solution discussed above, results of the performance studies of an induction motor driving a pump load are presented. The simulations have been carried out using the below motor data gotten from the open and short circuit tests of the induction motor under study:

50hp, 460V, 4Poles, 60Hz

$R_s = 0.087\Omega$

$L_{1s} = 0.8mH$

$L_{1r} = O.8mH$

$Lm = 34.7mH$

$R_r = 0.228\Omega$

$J = 1.660Kgm^2$

Below figures show the steady state and transient state torque-speed characteristics of the motor at no-load and at pump load for balanced and unbalanced stator voltages. In figure 6, the difference between the transient and steady state -speed characteristics is not noticeable. In large horse-power induction motor, however, this difference is highly noticeable and due mainly to the electric transients in the rotor circuits as reported by [3]. The paper has presented a simple method of analysing and simulating the steady state and transient state performances of an induction motor. The simulation results presented in this paper will indeed provide essential information to the Plant Electrical Engineer over the motor performances prior to its design.
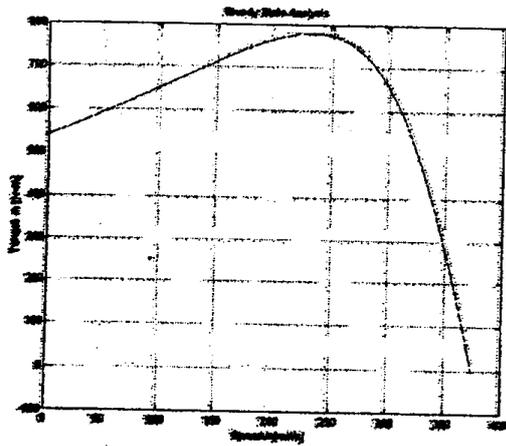
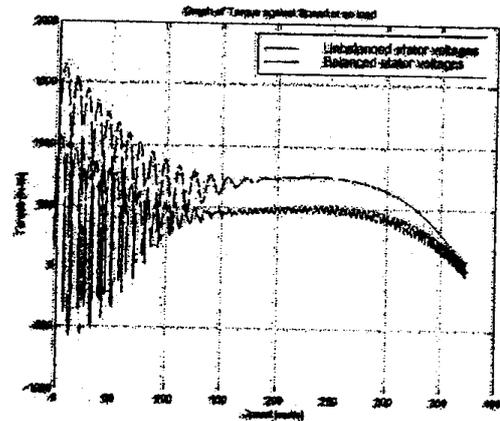Fig.1　Graph of Torque against Speed-Steady State Analysis
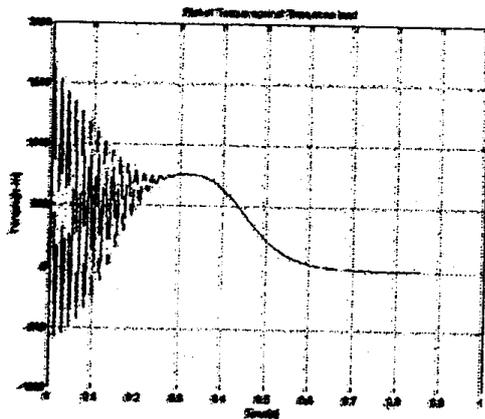
Fig.4　Graph of Torque against Speed at no load.

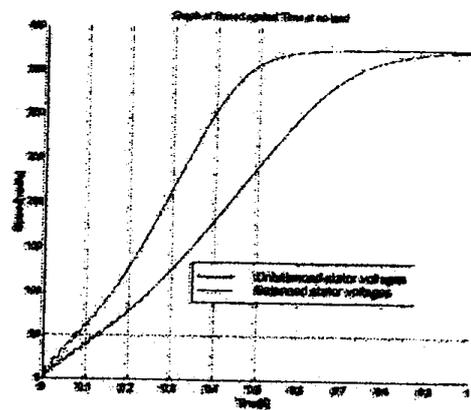ig.2　Graph of Torque against Time-Transient State Analysis
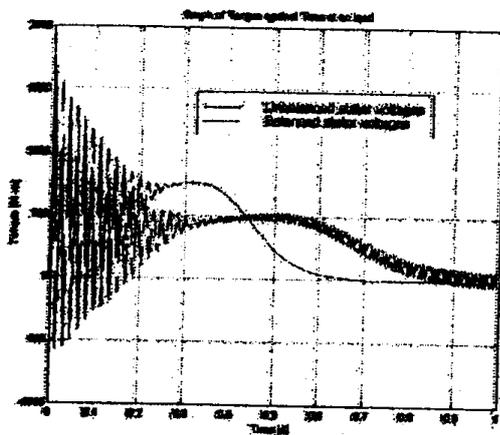
Fig.5 Graph of Speed against Time at no load.

g.3　Graph of Torque against Time at no load.
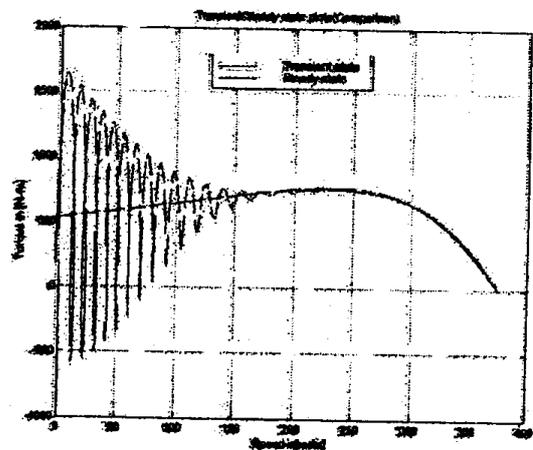
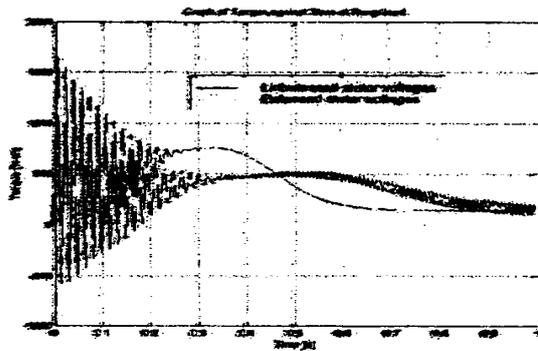Fig.6　Comparison of Steady and Transient states Torque-Speed Characteristic

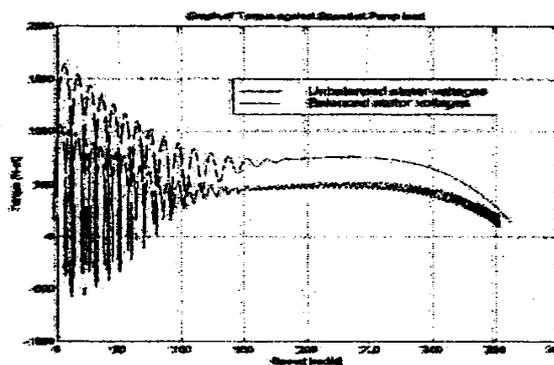**Fig.7 Graph of Torque against Time at load.**

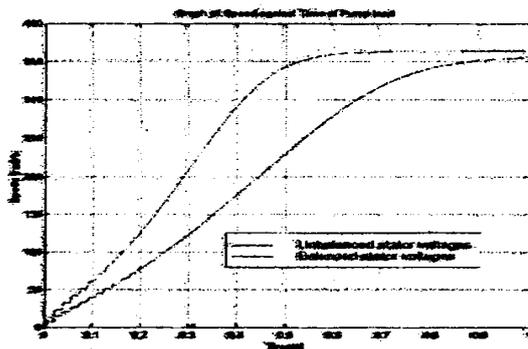**Fig.8 Graph of Torque against Spe Pump Load.**

Fig.9 Graph of Speed against Time at Load.

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