# COMPARISON OF THE MINUTIAE QUADRUPLETS AND MINUTIAE TRIPLETS TECHNIQUES

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#### Abstract

Identifying distorted fingerprint images is a major problem in fingerprint recognition systems. Several techniques, such as the minutiae triplets technique, have been proposed for minutiae matching and indexing. The minutiae triplets technique however is largely affected by minutiae distortions and occlusions and hence can rarely produce a stable feature set. In this paper, the characteristics of the minutiae quadruplets and the minutiae triplets structures are compared. The minutiae quadruplet technique is proposed as a better technique because the features are robust to minutiae distortions and occlusions and it eliminates the known drawbacks of the minutiae triplet technique.

**Keywords:** Clustering, k-means, data retrieval, indexing

#### 1. Introduction

Fingerprint recognition is an interesting pattern recognition problem. Fingerprint recognition may be based on ridge patterns or minutiae details which their arrangement is unique to every single person in existence. Majority of the fingerprint recognition algorithms are based on minutiae matching [1, 2, 3, 4, 5, 6] because it is difficult to extract ridge features from low quality images whereas minutiae can be extracted from low quality images. Local structures have to be defined for minutiae matching. An existing local structure for minutiae matching is the minutiae triplet [1, 2]. The minutiae triplet structure which is a geometric approach used for minutiae matching and indexing cannot in itself handle distorted fingerprint images, if not supported or combined with other approaches that make the work computationally complex and intensive. Hence the minutiae quadruplet structure is proposed as a better local structure for minutiae matching.

#### 2. Cluster Similarity Measures

A minutiae triplet is a group of three minutiae points forming a triangle as shown in Figure 1. The technique of minutiae triplets was first originated by Germain et al. [1] as features for minutiae matching and indexing. Given a minutia triplet in Figure 1, the features they used were the lengths of the three sides,  $S_1$ ,  $S_2$ ,  $S_3$ , the ridge counts between two vertices and the minutiae angles corresponding to the direction of the ridges at the vertices,  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$ . These amount to 9 features.

Lengths of the three sides,  $S_1$ ,  $S_2$ ,  $S_3$ , are sensitive to shear that arises as a result of the deformation of fingerprint ridges in an acquired fingerprint impression. The minutiae positions are consequently distorted making it difficult to match a fingerprint image to

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another impression of the same image in the database. Ridge counts between every two vertices are affected by image quality. It is difficult to estimate fingerprint ridges from poor images and hence the ridge count features are not robust and stable [7].

The local orientation at the vertices,  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$ , can be affected by distortions. The local orientation,  $\theta$ , is the angle a minutia direction makes with the reference x-axis [8]. The angle is absolute to the reference x-axis and not relative to other minutiae in the fingerprint. All acquired fingerprints have some degree of distortion as a result of the shifts in the minutiae points with respect to the x-axis hence the local orientation is adversely affected by minutiae distortions.

Bhanu and Tan [2] improved on Germain et al.'s [1] approach of indexing based on minutiae triplets. They used a total of six features; the minimum and median interior angles,  $\alpha_{min}$ ,  $\alpha_{med}$ , triangle handedness,  $\phi$ , triangle type,  $\gamma$ , triangle direction,  $\eta$ , and maximum side,  $\lambda$ . Two of the features min and med are shown in Figure 1. The six features used in [2] are explained in details.

# Minimum and Median Angles, $\alpha_{min}$ , $\alpha_{med}$

Given 3 interior angles,  $\alpha_{med}$ ,  $\alpha_{min}$ ,  $\alpha_{max}$ , of a scalene triangle,  $\alpha_{max}$  is the maximum angle,  $\alpha_{min}$  is the minimum angle and

$$\alpha_{med} = 180 - \alpha_{max} - \alpha_{min} \tag{1}$$

#### Triangle Handedness, $\phi$

Z is defined in complex number notation as

$$Z_i = x_i + jy_i \tag{2}$$

where  $(x_i, y_i)$  are coordinate location of a minutia point  $P_i$ , for i = 1, 2, 3. With following definitions of Z

$$Z_{21} = Z_2 - Z_1 \tag{3}$$

$$Z_{32} = Z_3 - Z_2 \tag{4}$$

$$Z_{13} = Z_1 - Z_3 \tag{5}$$

The Triangle Handedness  $(\phi)$  is defined as

$$\phi = sign(Z_{21} \times Z_{32}) \tag{6}$$

where  $sign(\cdot)$  is the sign function and  $\times$  is the cross product.

#### Triangle type, $\gamma$

Triangle type,  $\gamma$ , is defined as

$$\gamma = 4\gamma_1 + 2\gamma_2 + \gamma_3 \tag{7}$$

where  $\gamma_i$  is the minutia feature type for  $P_i$ , i = 1, 2, 3 of the triangle. If  $P_i$  is a termination minutiae then  $\gamma_i = 1$ , else it is  $0 \le \gamma_i \le 7$ 

#### Triangle Direction, $\eta$

Triangle Direction,  $\eta$  is defined as

$$\eta = 4\nu_1 + 2\nu_2 + \nu_3 \tag{8}$$

where  $\nu_i$  corresponds to  $P_i$ . If the minutiae is a termination ridge  $\nu = 1$ , else  $\nu$  is 0 for a bifurcation ridge.

#### Maximum side, $\lambda$

Maximum side,  $\lambda$  is defined as

$$\lambda = \max L_i \tag{9}$$

where

$$L_1 = |Z_{21}|, L_2 = |Z_{32}|, L_3 = |Z_{13}|$$
 (10)

However, the features used in [2] are still not robust to minutiae distortions and occlusions because there are a lot of false correspondences arising from the complex features.

Hence, the minutiae quadruplets technique is proposed as a better approach to matching and indexing fingerprints including highly distorted fingerprints, because the quadruplet features are unaffected by minutiae distortions or occlusions. The triplet structure depends on other techniques [9, 10, 11] to handle distorted fingerprints whereas the minutiae quadruplet structure alone can effectively handle distorted fingerprints.

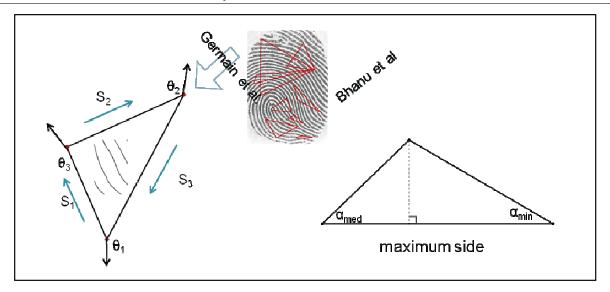


Figure 1: Minutiae Triplets by Germain et al. and Bhanu et al.

### 3. Minutiae Quadruplets

A minutiae quadruplet is a group of four minutiae in a fingerprint [12]. The proposed features are seven in a minutiae quadruplet as shown in Figure 2.

$$S = \{ \varphi_1, \ \varphi_2, \ \delta_1, \ \delta_2, \ \rho_1, \ \rho_2, \ \eta \}$$
 (11)

# Opposite Angle Features ( $\varphi_1, \varphi_2$ )

In a quadruplet, the differences of two opposite angles,  $\varphi_1$ ,  $\varphi_2$ , constitute the two angle features for indexing.

$$\varphi_1 = \theta_1 - \theta_3 \tag{12}$$

$$\varphi_2 = \theta_2 - \theta_4 \tag{13}$$

### Perpendicular Height $(\rho_1, \rho_2)$

The two perpendicular heights of the inner parallelogram formed by joining the midpoints of the four sides of the quadruplet,  $\rho_1$ ,  $\rho_2$ , are length features.

$$\rho_1 = w_1 p h_1 \tag{14}$$

$$\rho_2 = w_1 p h_2 \tag{15}$$

#### **Diagonals** $(\delta_1, \delta_2)$

The two diagonals of the quadruplet,  $\delta_1$ ,  $\delta_2$ , are length features.  $\delta_1$  is the diagonal opposite to the angles  $\theta_1$  and  $\theta_3$ , while diagonal  $\delta_2$  is opposite to angles  $\theta_2$  and  $\theta_4$ .

## Global Feature $(\eta)$

The feature,  $\eta$ , formed by compounding single features from the two polygons – quadruplet and inner parallelogram is a global feature.

$$\eta = 100 \log 10(\tau \nu) \tag{16}$$

where

$$\tau = \sqrt{\overline{w_1 w_2} \cdot \overline{w_1 p h_1}} + \sqrt[4]{\overline{x_1 x_2} \cdot \overline{x_2 x_3} \cdot \overline{x_3 x_4} \cdot \overline{x_1 x_4}}$$

$$(17)$$

$$\nu = \sqrt{\mu} + \lambda \tag{18}$$

where

$$\mu = \{ (s - \overline{x_1 x_2})(s - \overline{x_2 x_3})(s - \overline{x_3 x_4})(s - \overline{x_1 x_4}) - (\overline{w_1 w_2} \cdot \overline{w_1 p h_1} + \sqrt[4]{\overline{x_1 x_2} \cdot \overline{x_2 x_3} \cdot \overline{x_3 x_4} \cdot \overline{x_1 x_4}} - \cos^2(\theta_1 + \theta_3)/2 \}^{0.5}$$

(19)

$$s = (\overline{x_1 x_2} + \overline{x_2 x_3} + \overline{x_3 x_4} + \overline{x_1 x_4})/2$$
 (20)

 $\mu$  is Bretschneider's formula for the area of a quadrilateral of any shape.

$$\lambda = \sqrt{\overline{w_1 w_2} \cdot \overline{w_2 w_3}} \tag{21}$$

# 4. Comparison of the Minutiae Quadruplets and Minutiae Triplets of Bhanu

The minutiae quadruplets and minutiae triplets are compared in Table 1.

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Table 1: A Comparison of the Minutiae Quadruplets and Minutiae Triplets Indexing Techniques.

			ae Quadruplets and Minutiae Triplets Indexing Techniques.
Attributes	Minutiae	Minutiae	Comments
	Quadruplets	Triplets	
Features	7 features used	6 features	Minutiae Quadruplets: The $\varphi_1$ , $\varphi_2$ are able to tolerate minu-
	are the 2 op-	used are	tiae distortions when quadruplets of distorted fingerprints do not
	posite angles	the mini-	have exact angle measurements of $\theta_1$ , $\theta_2$ , $\theta_3$ , and $\theta_4$ as those of
	differences	mum and	the probe. Hence the two differences are better to use than the
	$(\varphi_1, \varphi_2)$ , two	median	four interior angles.
	perpendicu-	interior an-	Either of $\delta_1$ and $\delta_2$ tolerates two to four minutiae distortions be-
	lar heights	gles, $\alpha_{min}$ ,	cause the length does not change with angle distortions. This is a
	of the inner	$\alpha_{med}$ ,	great advantage of the minutiae quadruplet structure in handling
	parallelogram	$\begin{array}{c} \alpha_{med}, \\ \text{triangle} \end{array}$	fingerprints with distorted minutiae.
	$(\rho_1, \rho_2)$ , two	handed-	There is an added advantage of using features from 2 polygons
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		
	_		- the quadruplet and the inner parallelogram. $\rho_1$ and $\rho_1$ are
	the quadru-	triangle	features from the inner parallelogram.
	plets $(\delta_1, \delta_2)$	type, $\gamma$ ,	$\eta$ is a global feature that combines features from both polygons
	and an inte-	triangle di-	in a consistent manner and this feature varies meaningfully for
	grated feature,	rection, $\eta$ ,	any quadruplet's shape and size.
	$\eta$ .	maximum	Minutiae Triplets: The triangle type $\gamma$ , and direction, $\eta$ , are good
		side, $\lambda$ .	features but complex. The use of the longest side in triangles
			to extract subsequent features is one of the drawbacks to the
			triplets approach because the longest side of some triplets can
			change under minutiae distortions as there are all possible shapes
			of triplets in a fingerprint. Consider a triplet in a fingerprint
			impression of a subject which is almost, but not, equilateral; such
			that the three differences between the three sides are negligible;
			however the longest side is . See Figure 3 (a) and Section 5.
			Then consider the same triplet in a second impression of the
			same subject distorted, such that the maximum side changes to
			as shown in Figure 3 (b). This would result in a change in the
			position of the maximum side, the length of the maximum side
			as well as the minimum and median angles and consequently a
			decrease in the number of corresponding triangles.
			The minimum and median angles, $\alpha_{min}$ , and $\alpha_{med}$ , tolerate
			minutiae distortions because the tangent of the angles change
			infinitesimally with minutiae distortions, however, there would
			be a lot of false correspondences because of the non-linear char-
			acteristic of the tangent. Tangent 25° to 45° lie within a range
			of 0.47 to 1.00 while Tangent 45° to 90° lie within 1 and $\infty$ , and
			tangent above 90 is negative.
Geometric	None	Yes	There is no need for a geometric constraint in minutiae quadru-
constraints	110116	169	plets. Geometric constraints were applied in minutiae triplets to
Constraints			
			reduce the number of false correspondences. Minutiae quadru-
A	3.5	3.5	plets do not raise false correspondences.
Attributes	Minutiae	Minutiae	Comments
	Quadruplets	Triplets	
Indexing	Clustering	Hashing	
Scheme			
Complexity	Simple	Complex	Minutiae Quadruplets: There are a variety of simple features that
of Indexing	_	transfor-	can be used in quadruplets for indexing
Algorithm		mations	Minutiae Triplets: The geometric constraints applied are quite a
			number with complex transformations, namely, the relative local
			orientation at mid-points, relative local orientation at vertices,
			relative translation and relative rotation. The algorithm is complex to use.

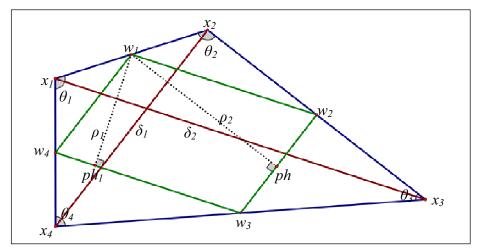


Figure 2: A minutiae quadruplet.

# 5. Drawback of the Maximum Side Feature

Consider a minutiae triplet in a subjects fingerprint image which is almost, but not, equilateral such that the three differences between the three sides are negligible in each case, however, there is a longest side,  $P_2P_3$  in Figure 3(a).

Longest side is  $\overline{P_2P_3}$ 

$$\lambda = Z_{32} = Z_3 - Z_2 \tag{22}$$

where

$$Z_i = x_i + jy_i \tag{23}$$

x, y in each case is a location of point  $P_i$  for i = 1, 2, 3.

The two minimum and median angles are in-between  $\angle P_1P_2P_3$  and  $\angle P_1P_3P_2$  as the angle opposite the longest side  $\overline{P_2P_3}$  is the maximum angle.

Consider the same triplet in a second impression of the same subject's fingerprint distorted such that the longest side changes to  $\overline{P_1P_3}$  in Figure 3 (b).

$$\lambda = Z_{13} = Z_1 - Z_3 \text{(change in size)} \tag{24}$$

The minimum and median angles,  $\alpha_{min}$  and  $\alpha_{med}$ , also change positions and values.

#### 6. Conclusion

In this paper, the minutiae quadruplet technique is proposed as a better local minutiae structure than the minutiae triplet for minutiae matching and indexing fingerprints. The two structures were compared extensively. The drawbacks of the minutiae triplet technique were discussed. The new minutiae quadruplet structure has several advantages over the minutiae triplet structure.

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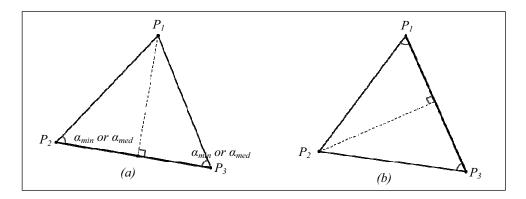


Figure 3: The longest side changes from  $\overline{P_2P_3}$  to  $\overline{P_1P_3}$  due to minutiae distortions.

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