DESIGN OF PREVENTIVE MAINTENANCE SCHEDULING MODEL FOR DETERIORATING SYSTEMS

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Abstract

Maintenance has gained in importance as a support function for ensuring equipment availability. Adoption of effective preventive maintenance culture in an industry can drastically reduce the cost of premature or over delayed maintenance, while accurate maintenance action can sustain continuous and reliable operation of equipment. This paper presents the design of preventive maintenance scheduling models for deteriorating systems based on reliability criterion. It describes how age reduction and reliability improvement can be achieved after an individual preventive maintenance. The models were demonstrated by way of applying them to a specific deteriorating but renewable system. The results obtained gave a successively decreasing maintenance interval above an operational reliability level.

Keywords: preventive maintenance, system improvement, reliability, scheduling

1. Introduction

A system may be made up of a single component, a unit, a multi - unit or combinations of both. Maintenance management for deteriorating systems subject to renewal processes aims at reducing the overall downtime and improving system reliability. Since the operation and effectiveness of industrial systems represent substantial portions of the total output of a process, reliability and maintenance management of such systems have drawn increasing interests geared at raising their availability. An objective of preventive maintenance in industries is to achieve this. However, to earn better recognition of industry, especially with increasing complexity and expanding applications of many industrial machines which brings about new problems, we ought to design maintenance procedures for specified cases or modify existing ones to suite the trend. For instance, Agunwamba [1] described the development of a coordinated preventive maintenance-scheduling model for 294 boreholes. The aim of the model was to determine the optimum number of preventive maintenance for each component per year. Other examples of a coordinated formulation can be seen in Christers and Doherty [2], Sule and Harmon [3] and Aniekan [4]. However, Ritchker and Wilson [5] presented a combined preventive and corrective maintenance model for a multi component system. They restricted attention to a class of policies characterized by two critical numbers "m" and "T", implying that a maintenance activity including repair of all failed units (corrective), and overhaul of all non failed units (preventive) is started if and only if the number of failed units has reached the level of m, or T units of time have passed since the last maintenance activity. Also, Elandt-Johnson [6] and Handlarski [7] variously presented models for scheduling the preventive maintenance such that the total maintenance cost is minimized. Malik [8] maintained that maintenance of goods producing systems is undertaken on the principle of minimum cost whereas that of service producing systems is based on the principle of operational reliability. Accordingly, he presented a preventive maintenance model based on the principle of operational reliability. Many of the pre-scheduled maintenance actions normally taken to improve centrifugal pump availability are often significant to be considered in preventive maintenance scheduling. Furthermore, consideration of the magnitude of the various failure modes resulting in an individual failure will give a better understanding of the resultant failure profile. An attempt will be made to tackle some of these issues in this paper.

We shall assume that the system will continue to be improved to a better working condition through preventive maintenance until replacement becomes necessary.

2. Failure Processes

The failure process of deteriorating systems follows the probability law governing failures. In general, there are two ways of postulating a component failure distribution. In the first instance, we rely on physical reasoning to postulate a form of the conditional failure probabilities. According to Lie et al[9], this method is useful when there is little a priori knowledge. The second method employs empirical evidence observed on components. In this approach, attempts can be made to fit a failure density function to the available data. Obviously, a combination of these methods is preferable if sufficient a priori statistical data is available and where insights into the failure mechanism can be obtained by physical theory. Again, there are alternatives of failure probability distributions for modeling specific system failure cases. They include (but not limited to) the Normal distribution, Lognormal distribution, Exponential distribution, Two-parameter exponential distribution, Weibull distribution and Erlang distribution. The Weibull failure distribution and negative exponential distribution shall be adopted for this paper. The choice of the two failure distributions was informed by the fact that most deteriorating systems exhibit decreasing, constant and increasing failure rates at various stages of their service life. The distribution parameters adequately accounts for such failure behavior. In other words, the most attractive feature in combining both failure distributions is that the result enables one to deal with equipment that deteriorate with age and usage as well as those with constant failure rate.

3. Reliability Models

The system reliability can generally be represented as

$$R(t) = \exp\left[-\int_0^t h(t)dt\right] \tag{1}$$

Where $R(t)$ is the reliability function, $h(t)$ is known as the instantaneous hazard rate or failure rate function and is the system operation time, [10],[11].

Deteriorating systems fail by chance, age and usage which constitute the wear out factors. Hence, a generalized reliability equation for such systems may be written as:

$$R(t) = a_1 e^{-ct} + a_2 e^{-\left(\frac{\omega t}{\theta}\right)^{\beta}} \tag{2}$$

Where ω is the fraction of time it works; a_1 is the coefficient of chance failure and a_2, the coefficient of wear-out failure; β is the shape parameter of the failure distribution; θ is the characteristic life; C is a constant factor; $\omega =$ 0 and 1 for redundant and full time working systems respectively.

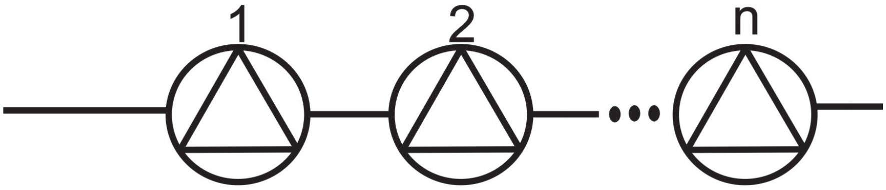

Figure 1: Reliability diagram for centrifugal pumps in series.

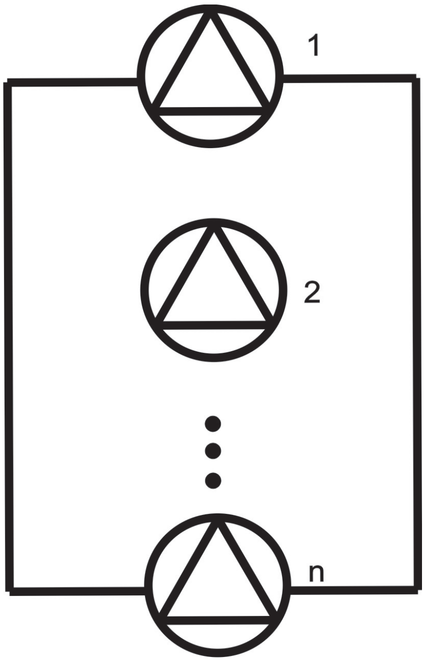

Figure 2: Reliability diagram for centrifugal pumps in parallel.

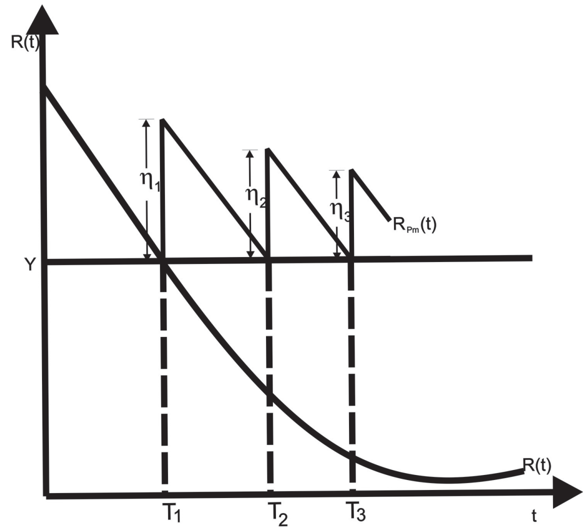

Figure 3: Effect of preventive maintenance on centrifugal pumps

4. Serial and Parallel System Reliability

Individual components or units within a system may be related to one another in either a serial or parallel (redundant) configuration as shown in Fig 1 and Fig 2 respectively. Accordingly,

$$R_s(t) = \prod_{j=1}^{n} R_j(t_i) \tag{3}$$

Where n is the total number of systems (component or unit) connected in series And

$$R_p(t) = 1 - \prod_{i=1}^{n} [1 - R_j(t_i)] \tag{4}$$

Where $R_s(t)$ is the reliability of all units connected in series, $R_p(t)$, the reliability of units connected in parallel, $R_j(t_i)$ is the reliability of the jth unit at ith time interval, n is the total number of systems (components or units) connected in parallel.

Equations (3) and (4) enable us to determine the reliability of all components, units or sub-systems in an industry.

5. System Reliability with Preventive Maintenance

Fig. 3 shows the effect of preventive maintenance on a deteriorating system (centrifugal pump). In a perfect reliability situation, a preventive maintenance done at any time, T_1, say; must make the system survive until the next maintenance point,T_2 say. However, in practical situations, some factors like the proficiency of the maintenance crew, quality of spare parts etc. alter this arrangement. The effect of this non-perfect reliability on the overall reliability of deteriorating but preventively maintained systems cannot be neglected. For this reason, the reliability model represented by equation (2) may be modified to:

$$R_j(t_i) = (1-\varepsilon)\left\{a_1 \exp(-c_j t_i) + a_2 \exp\left(-\frac{\omega_j t_i}{\theta_j}\right)^{\beta}\right\} \quad (5)$$

Where ε is a factor which takes care of prescheduled maintenance actions due to nonperfect maintenance during the last maintenance point.

6. Theoretical Improvement and Age Reduction through Preventive Maintenance

Suppose that when $R_j(t_i) \leq \gamma$, preventive maintenance is done on a unit, where γ is the level of operational reliability, at which preventive maintenance is done on the system. Its survival probability would be improved. We can interpret this improvement by thinking that this t old unit is no longer that old and its post-maintenance age is reduced from t to $t\eta$, and its pre-maintenance reliability $R_j(t_i)$ has become $R_j(t\gamma)$ after the preventive maintenance, where η is the improvement factor. The parameter, η, makes it possible to have a theoretical improvement of deteriorating systems from none to full renewal.

6.1. Assumptions

1) In the industry, the maintenance engineer should be able to estimate how much improvement is done on a system as a result of preventive maintenance done at the end of ith interval but for this paper, there is equal improvement in all maintenance intervals.
2) For practical purposes, η is necessarily more than zero, but less than one.

7. Reliability Improvement Models

If we let $R_j(t_i)$ be the reliability of the jth unit at the end of the ith interval immediately before the preventive maintenance, and $R_j(t_{i+1})$ be the reliability immediately after the preventive maintenance; then, for the first interval, $0 < t \leq T_1$,

$$R_j(t_1) = (1-\varepsilon)\left\{a_1 \exp(-c_j t_1) + a_2 \exp\left(-\frac{\omega_j t_i}{\theta_j}\right)^{\beta}\right\} \quad (6)$$

$$R_j(t_{1+1}) = (1-\varepsilon)\left\{a_1 \exp(-c_j(t_1 \times \eta_{j1})) + a_2 \exp\left[-\frac{1}{\theta_j}(\omega_j t_i \times \eta_{j1})\right]^{\beta}\right\} \quad (7)$$

Where η_{j1} is the improvement in unit j, at the end of the 1st interval.

The reliability of the units during and after the second interval, $T_1 < t < T_2$, will be:

$$\begin{aligned} R_j(t_2) &= (1-\varepsilon)\{a_1 \exp(-c_j(t_1 \times \eta_{j1} + t_2 - t_1)) \\ &\quad + a_2 \exp\left[-\frac{1}{\theta_j}(\omega_j t_1 \times \eta_{j1} + t_2 - t_1)\right]^{\beta}\Bigg\} \end{aligned} \quad (8)$$

$$\begin{aligned} R_j(t_{2+1}) &= (1-\varepsilon)\{a_1 \exp[-c_j(t_1\eta_{j1} + \langle t_2 - t_1\rangle \eta_{j2})] \\ &\quad + a_2 \exp\left[-\frac{1}{\theta_j}(\omega_j t_1 \times \eta_{j1} + t_2 - t_1)\right]^{\beta}\Bigg\} \end{aligned} \quad (9)$$

The reliability of the whole component or units according to their configuration immediately after preventive maintenance at the end of the second interval could be obtained by substituting equation (9) into equations (3) and (4).

When the improvement in the jth unit as a result of preventive maintenance is the same in all intervals, then;

$$R_j(t_i) = (1-\varepsilon)\left\{a_1 \exp(-c_j k'_i) + a_2 \exp\left[-\frac{1}{\theta_j}(\omega_j k'_i\right]^{\beta_j}\right\} \quad (10)$$

Where

$$\begin{aligned} k'_i &= t_1\eta_j + \langle t_2 - t_1\rangle \eta_j + \langle t_3 - t_2\rangle \eta_j + \\ &\quad \langle t_4 - t_3\rangle \eta_j + \cdots + \langle t_{i-1} - t_{i-2}\rangle \eta_j + t_i - t_{i-1} \end{aligned}$$

$$k'_i = t_{i-1} \times \eta_j + t_i - t_{i-1} \quad (11)$$

Similarly,

$$R_j(t_{i+1}) = (1-\varepsilon)\left\{a_1 \exp(-c_j k'_{i+1}) + a_2 \exp\left[-\frac{1}{\theta_j}(\omega_j k'_{i+1}\right]^{\beta_j}\right\} \quad (12)$$

Where

$$\begin{aligned} k'_{i+1} = &\ [t_1 + \langle t_2 - t_1\rangle + \langle t_3 - t_2\rangle + \langle t_4 - t_3\rangle \\ &+ \cdots + \langle t_{i-1} - t_{i-2}\rangle]\eta_j + (t_i - t_{i-1})\eta_j \end{aligned}$$

$$k'_{i+1} = t_i \times \eta_j \quad (13)$$

8. Maintenance Scheduling

The equation $R_j(t_i) = \gamma$ can be solved for $i = 1$ to n, where $R_j(t_{i+1})$ is given by (12), then, t_1 is determined empirically from operational data of system. When t_1 is known, we can find t_2 from

$$t_2 = R_j^{-1}\left(\frac{\gamma}{t_1}\right) \tag{14}$$

Which means t is the inverse of $R(\gamma)$ when t_1 is known [4]. Similarly, we can find $t_3 t_4 \cdots t_n$ for the jth unit successively, where

$$t_{n(j)} = R_j^{-1}\left(\frac{\gamma}{t_{n-1}}\right) \text{ for } n = 2, 3, 4, \cdots \tag{15}$$

where $t_{n-1} = \dfrac{1}{t_{n-1}}$.

Now, if t_1 is the actual time for the first preventive maintenance and t_2 is as given in (14), then the actual time for the second preventive maintenance will be given by;

$$t_{2\,actual} = R_j^{-1}\left(\frac{\gamma}{t_1}\right) + t_{1\,actual} \tag{16}$$

Continuing the procedure for t_n, we obtain;

$$t_n = R_j^{-1}\left(\frac{\gamma}{t_{n-1}}\right) \tag{17}$$

and

$$t_{n\,actual} = t_n + t_{n-1\,actual} \tag{18}$$

9. Illustrative Application to a Centrifugal Pump

The models were applied to the centrifugal pump GR-T4 (see fig.4) in Emenite LTD, Enugu, to obtain the maintenance schedule of fig. 5. Table 1 gives the values of parameters while table 2 gives the maintenance scheduling from zero point. Table 2 was developed by analysis of chance and wear-out failure data on centrifugal GR-T4 pump using standard methods of statistical analysis. See for example, chapter 12 of reference 11. From the analysis, we deduce that the pump's reliability drops from $R(t) = 1.0$ at $t = 0$, to $R(t) = \gamma = 0.66$ or less at $t_1 = 130$ hours. Then, preventive maintenance is carried out on the pump which raises the reliability to a value given by equation (12), where η will be determined by the maintenance engineer. Thereafter, the pump is allowed to work again till the next maintenance point when the reliability falls to γ and given by equation (10). At this point, t_2 is computed using (17) and so on. The pump should be considered for replacement whenever η approaches γ after an individual preventive maintenance.

Table 1: Values of parameters.

c	θ	β	ω	a_1	a_2	γ	η	ε
0.5	185	2	0.2	0.05	0.95	0.66	0.02	0.1

Table 2: Values of parameters.

Preventive maintenance points (i)	Schedule from zero point (t_i)
1	130 hrs
2	226 hrs
3	297 hrs
4	349 hrs
5	387 hrs

10. Conclusion

In this paper, we presented a preventive maintenance scheduling model for deteriorating systems based on operational reliability measures to address some maintenance cases where some actions are taken to increase system effectiveness and availability in between maintenance points. We also attempted to take care of the magnitude of chance and wear out failures in a particular failure phenomenon. With the aid of the models and given the failure data and probabilities for any component, unit or system, optimal decisions can be made on: 1) maintenance schedule; 2) replacement period (when η relatively equals γ after a scheduled preventive maintenance).

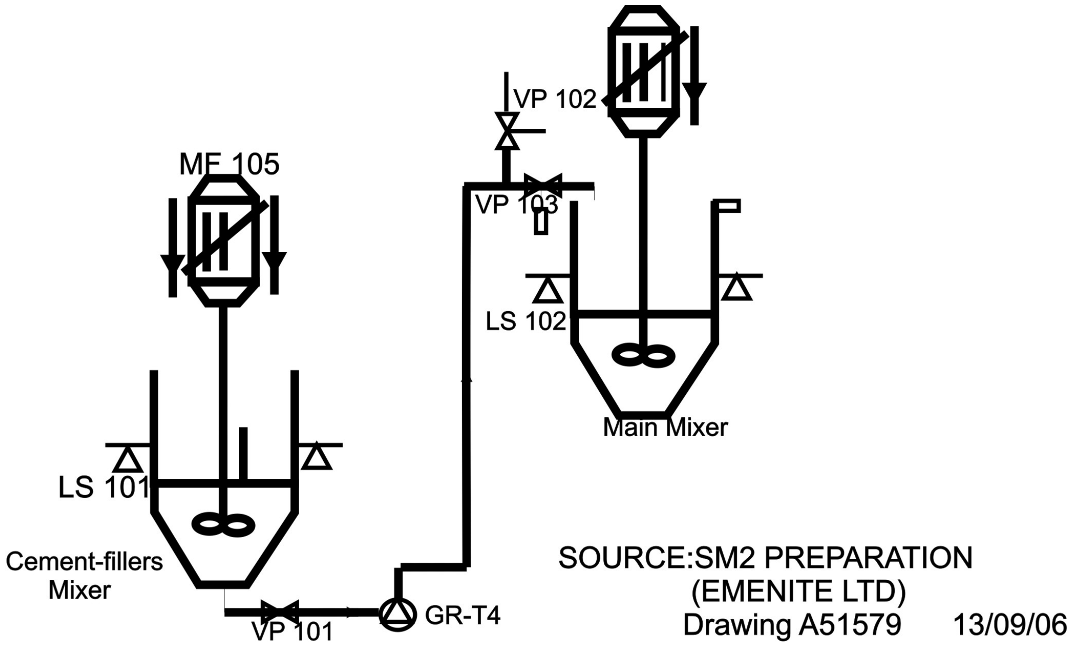

Figure 4: Sketch showing centrifugal pump GR-T4.

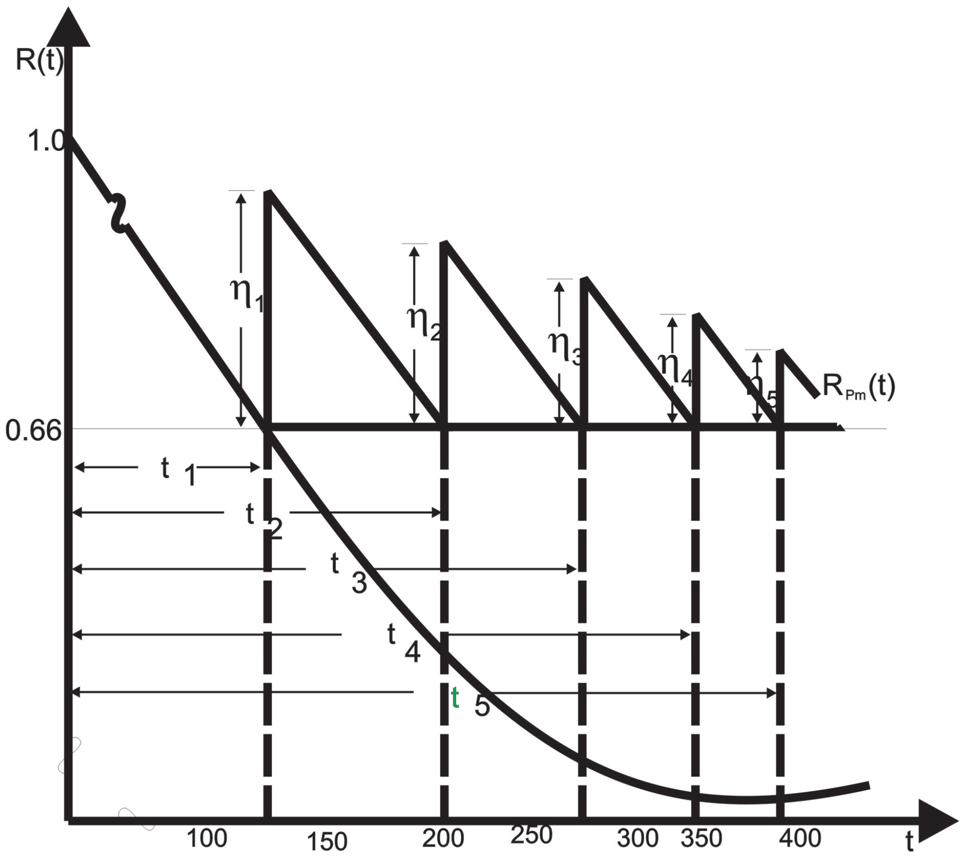

Figure 5: Centrifugal pump preventive maintenance scheduling deterioration and improvement in reliability.

The developed models are recommended for the preventive maintenance scheduling of deteriorating systems after statistical analysis of their field (failure) data, which might bring about new specifications of the model parameters. Our future efforts will concentrate on systems where dependent failures are involved as well as combined criteria for maintenance.

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